Unit-5
Kinematics of Gears
Subject: Kinematics of Machinery

Mr. D. S. Lengare
Assistant Professor, Automobile Engineering Department
Government College of Engineering & Research, Awasari khurd.

Introduction

Gears are the machine elements that transmit motion by means of successively engaging teeth.

Advantages :-

- It transmits exact velocity ratio
- It may be used to transmit large power
- It has higher efficiency
- It has reliable service
- It has compact layout
- It is used when the distance between driver & follower is very small

Introduction

Disadvantages :-

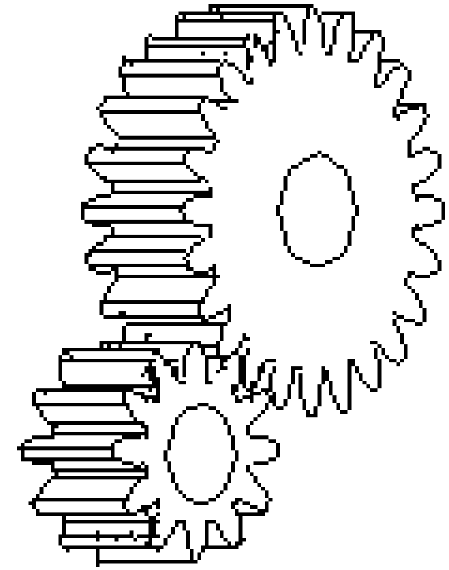
- Manufacturing of gears require special tools and equipments
- The error in cutting teeth may cause vibrations and noise during operation

Gear Classification

- **Connecting Parallel Shafts**
  - Spur Gears
  - Helical Gears / Herringbone Gears
  - Rack & Pinion Gears
- **Connecting Intersecting Shafts**
  - Bevel Gears
  - Helical Bevel Gears
- **Connecting Non-parallel Non-intersecting Shafts**
  - Spiral Gears
  - Worm & Worm Gear

Spur Gears

- Used for parallel shafts
- Have straight teeth
- Suitable for low to medium speed application
- Relatively high ratios can be achieved (7)
- Steel, Brass, Bronze, Cast Iron & Plastics

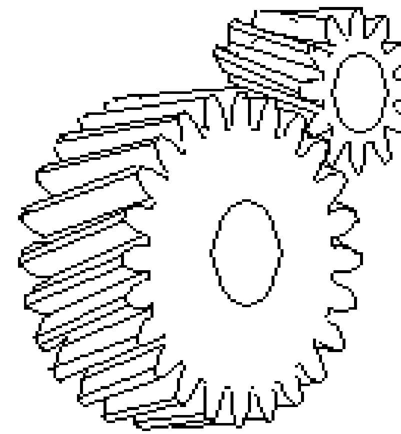


- **Advantages:** Spur gears are easy to find, inexpensive, and efficient.
- **Limitations:**

During engaging, the teeth collide, and this impact makes a noise. It also increases the stress on the gear teeth.

It cannot be used when a direction change between the two shafts is required.

Helical Gea



- Teeth are at an angle (Helix angle 7 to 230)
- Gradual engagement of teeth reduces shocks & Stresses
- More smooth & quiet operation
- Used for high speed transmission
- Tooth strength is greater because the teeth are longer
- Greater surface contact on the teeth thus carry more load than a spur gear
- Used in automobiles

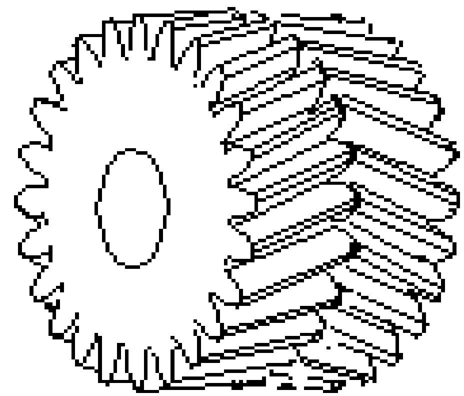
Helical Gears

- **Disadvantage:**

  -Longer surface of contact reduces the efficiency of a helical gear relative to a spur gear

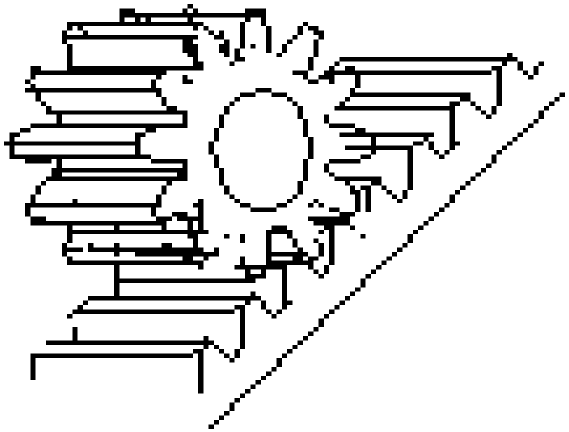
  -They induce axial thrust in one direction on bearing

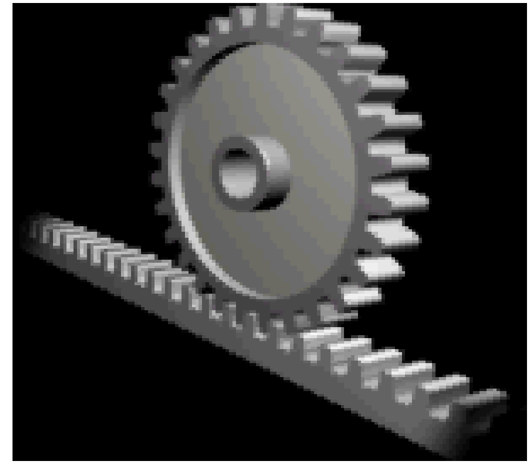
Herringbone Gears



- Two helical gears of identical pitch & of opposite hand
- Axial thrust of two gears act in opposite direction, thus…
- Problem of axial thrust is eliminated

Rack & Pinion Gears

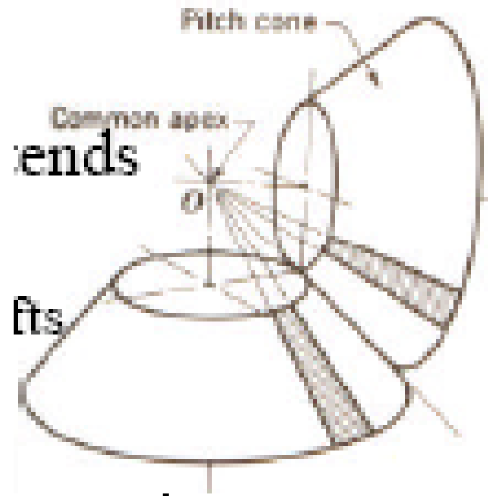




- ***Rack:*** Straight Gear with infinite diameter
- Used to convert rotational motion to translational motion by means of a gear mesh
- **Application:** Rack and pinion steering system used on many cars in the past.

  Used in machine tools

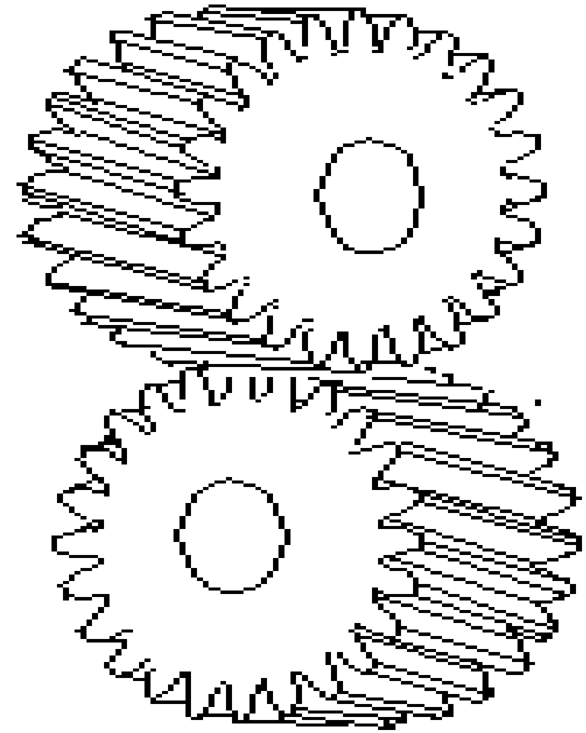
Bevel Gears



- Used for intersecting shafts (90) in same plane

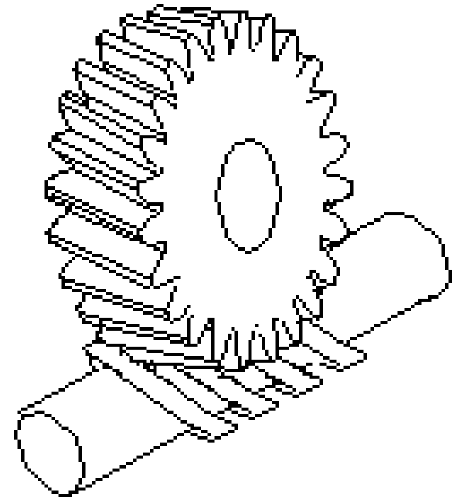
- ***Straight*** Bevel Gear / ***Spiral*** Bevel Gear
- For one to one ratio
- Used to change direction

Spiral Gears

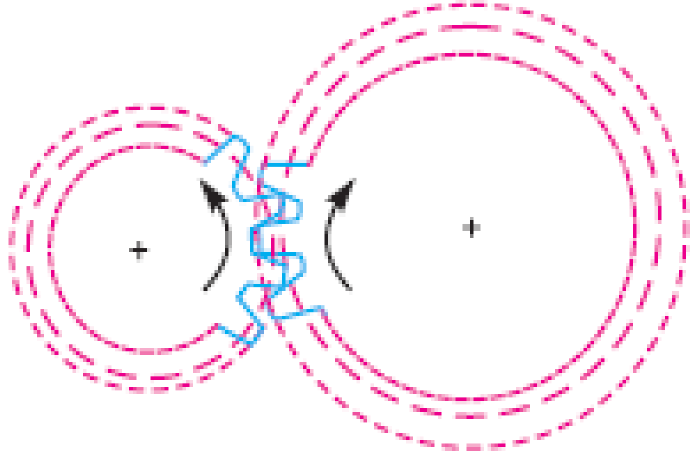


- Skew Gears / Crossed Helical Gears
- Used for Non-parallel & Non-intersecting shafts
- Point contact between mating teeth
- Low load transmission

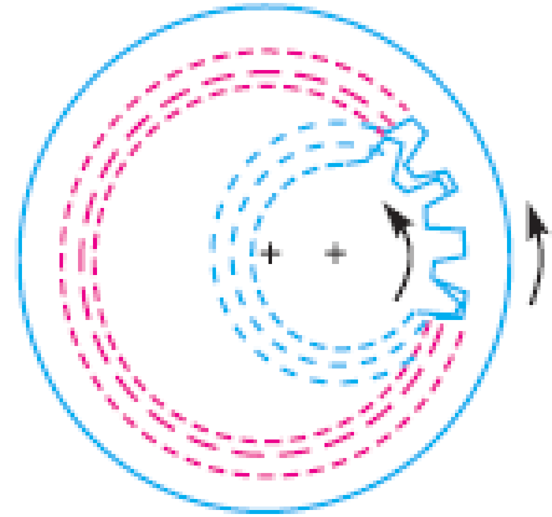
Worm & Worm Wheel



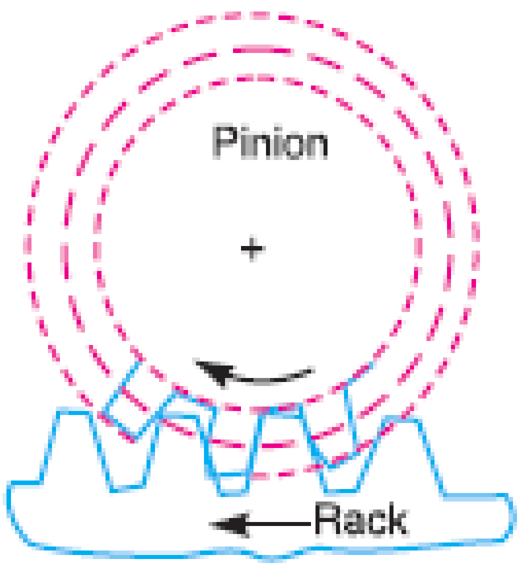
- Used for Non-parallel & Non-intersecting shafts
- Large speed reduction upto 100:1
- Worm can easily turn the gear but…
- Gear cannot turn worm
- This locking feature acts as a brake
- Used in conveyor systems



(*a*) External gearing.



(*b*) Internal gearing.

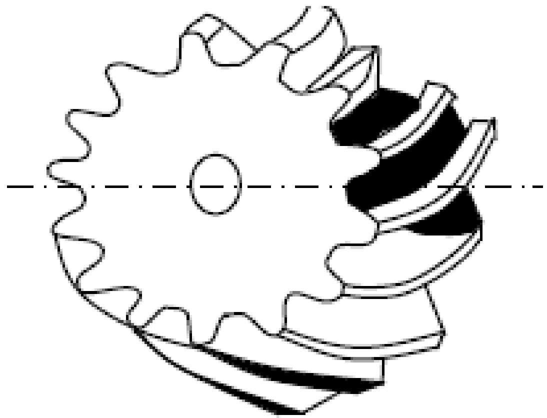


Internal gears

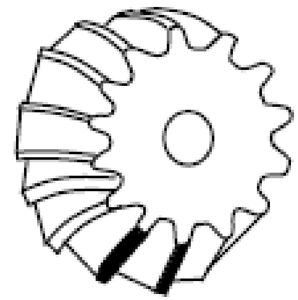

Rack and pinion

Helical Gears

Double helical gears



(a) Spiral Bevel Gear



(b) Zerol Bevel Gear

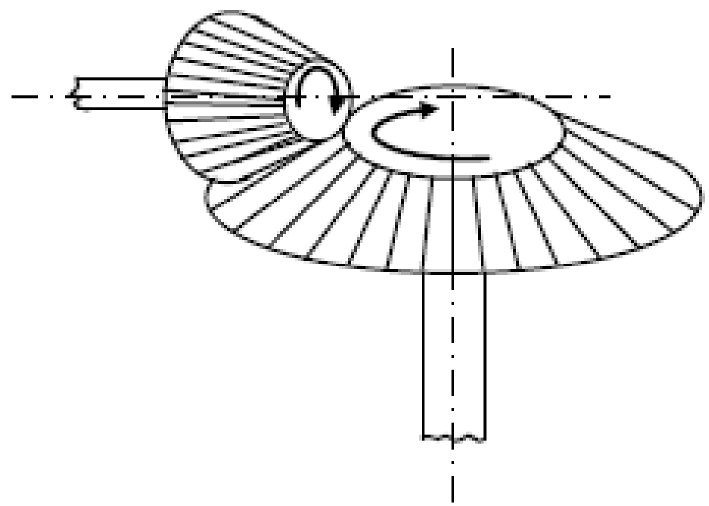
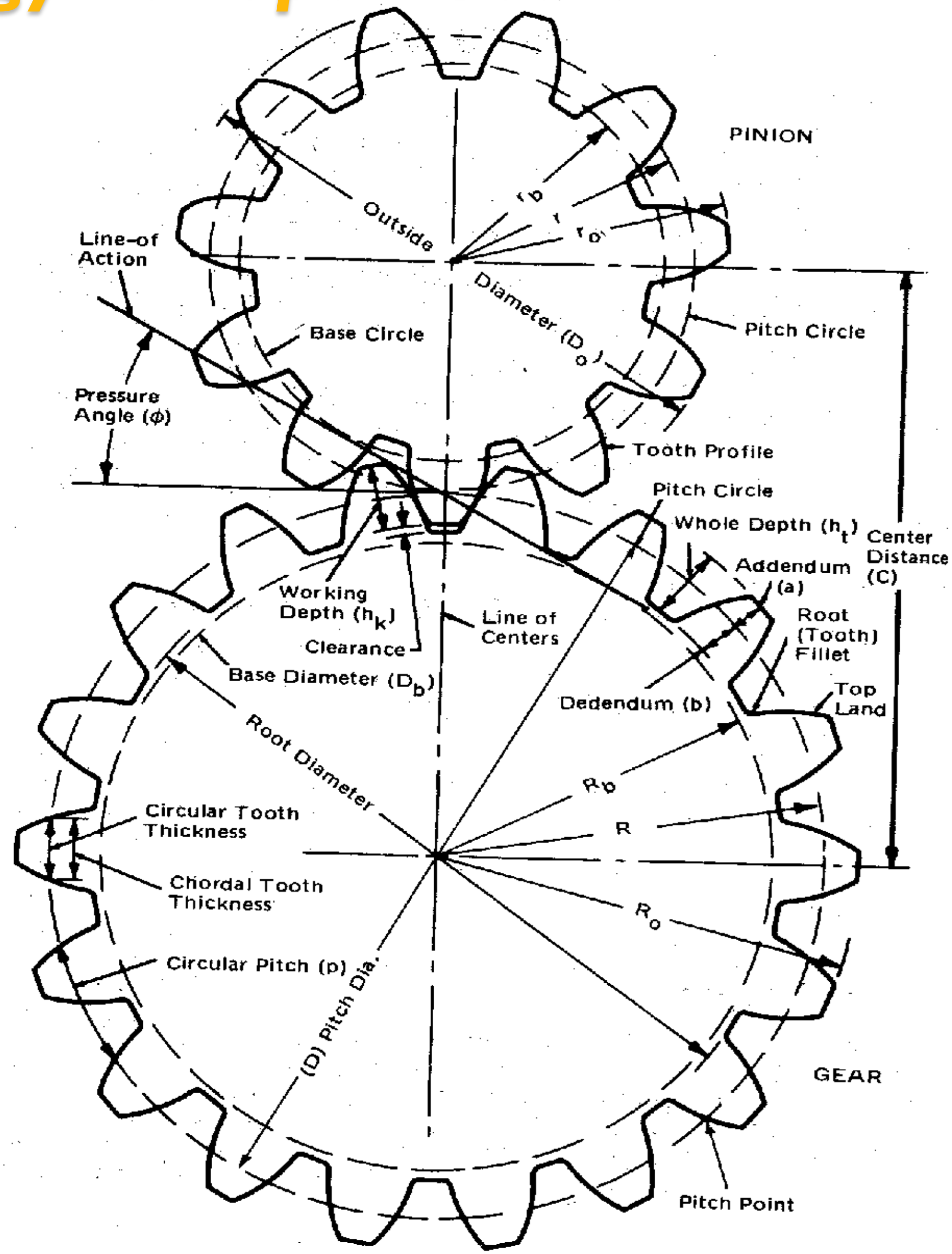
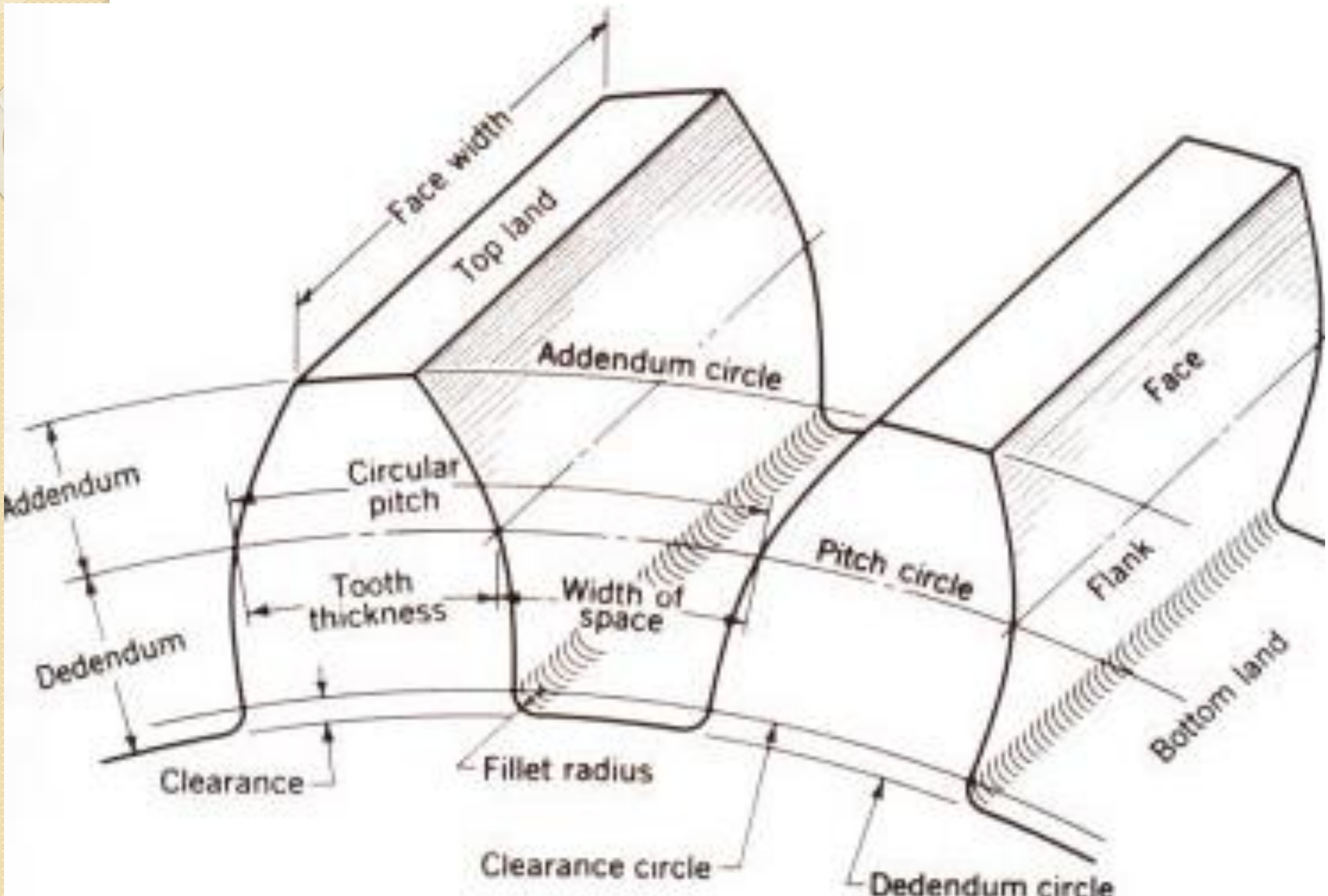


Figure 3.20 : Straight Bevel Gears

Terminology For Spur Gears



Gear Tooth Terminology



- **Pitch Circle :** An imaginary circle which by pure rolling action would give the same motion as actual gear
- **Pitch Circle Diameter :** Diameter of pitch circle
- **Pitch Point :** Common point of contact between two pitch circles
- **Pressure Angle:** Angle between ***common normal*** to two gear teeth at the point of contact & ***common tangent*** at pitch point
- **Addendum:** The radial distance between the PC and the top of the teeth
- **Addendum Circle:** Circle drawn from top of tooth & concentric with PC
- **Dedendum:** The radial distance between the bottom of the tooth to PC
- **Dedendum Circle:** Circle drawn from Bottom of tooth & concentric with PC
- **Circular pitch** (pc): Distance measured on circumference of PC from a point of one tooth to the corresponding point on next tooth

$$pc = \frac{\pi D}{T}$$

D= Diameter of PC

T= No. of teeth

$$pc = \frac{\pi D1}{T1} = \frac{\pi D2}{T2} \qquad \textbf{OR} \qquad \frac{D1}{D2} = \frac{T1}{T2}$$

- **Diametral pitch** (Pd): The ratio of the number of teeth to PCD.

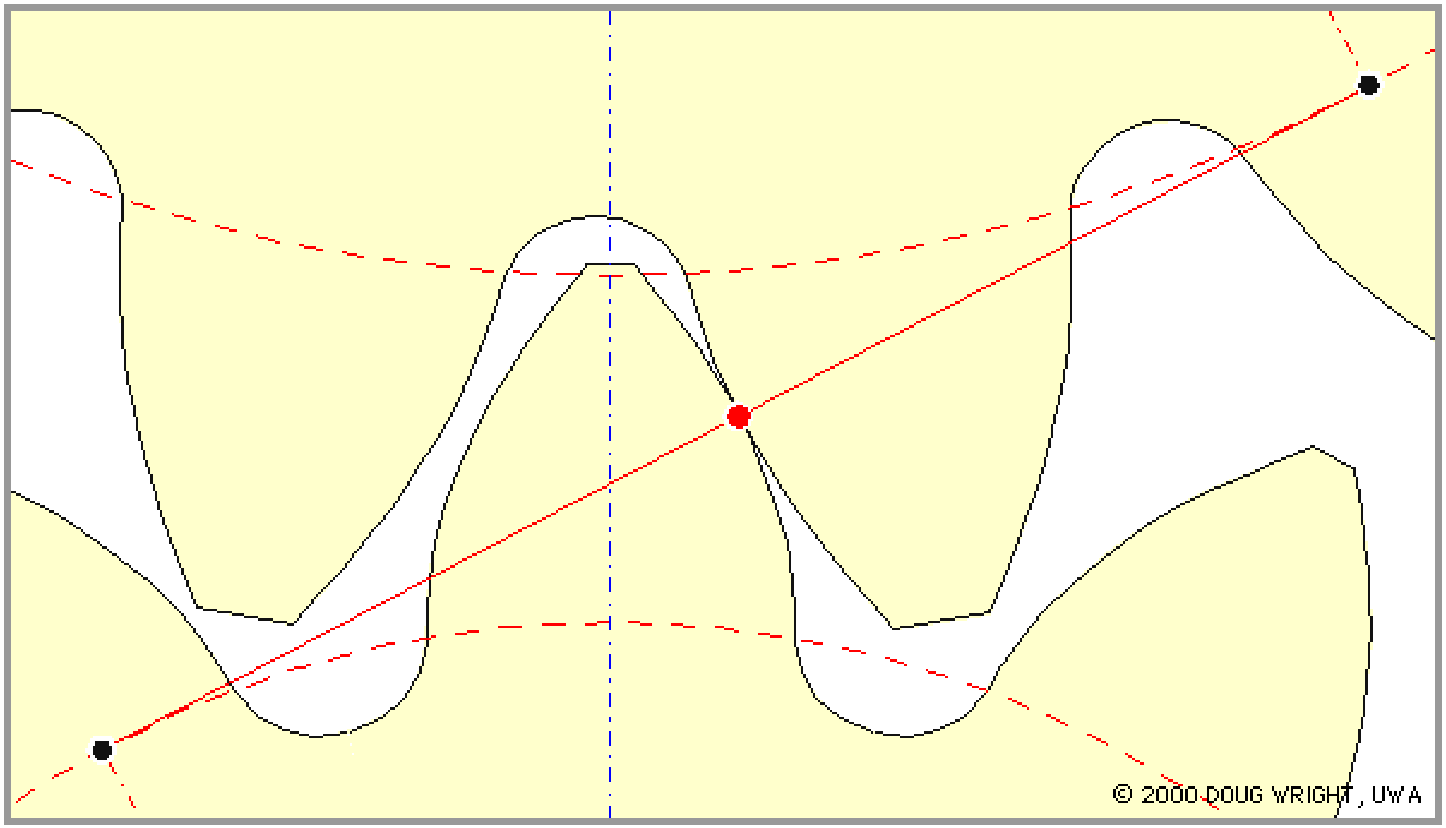
$$pd = \frac{T}{D} = \frac{\pi}{pc}$$

- **Module (m):** Ratio of Pitch circle Diameter (mm) to No. of Teeth

$$m = \frac{D}{T}$$

- **Clearance**: Difference between the *dedendum* of one gear and the *addendum* of the mating gear
- **Total depth** : Radial distance equal to sum of *addendum* & *dedendum* (working depth plus clearance)
- **Working depth** : Depth of engagement of two gears, i.e., the sum of addenda of two mating gears
- **Tooth Thickness :** Width of teeth measured along *PC*
- **Tooth Space**: Distance between adjacent teeth measured along *PC*
- **Backlash**: Difference between tooth thickness and tooth space on *PC*

- **Face of a tooth**: Surface of gear tooth above *PC*
- **Flank of a tooth**: Surface of gear tooth below *PC*
- **Top Land :** Surface of top of tooth
- **Face width :** Width of gear tooth measured parallel to its axis
- **Fillet Radius :** connects root circle to profile of tooth
- **Path of contact** : path traced by a point of contact of two teeth frm *beginning* to *end* of engagement
- **Length of path of contact :** Length of common normal cut-off by addendum circles of wheel & pinion
- **Arc of contact :** Path traced by a point on pitch circle frm beginning to end of engagement
- **Arc of Approach :** path of contact frm beginning of engagement to pitch point
- **Arc of Recess :** path of contact frm pitch point to end of engagement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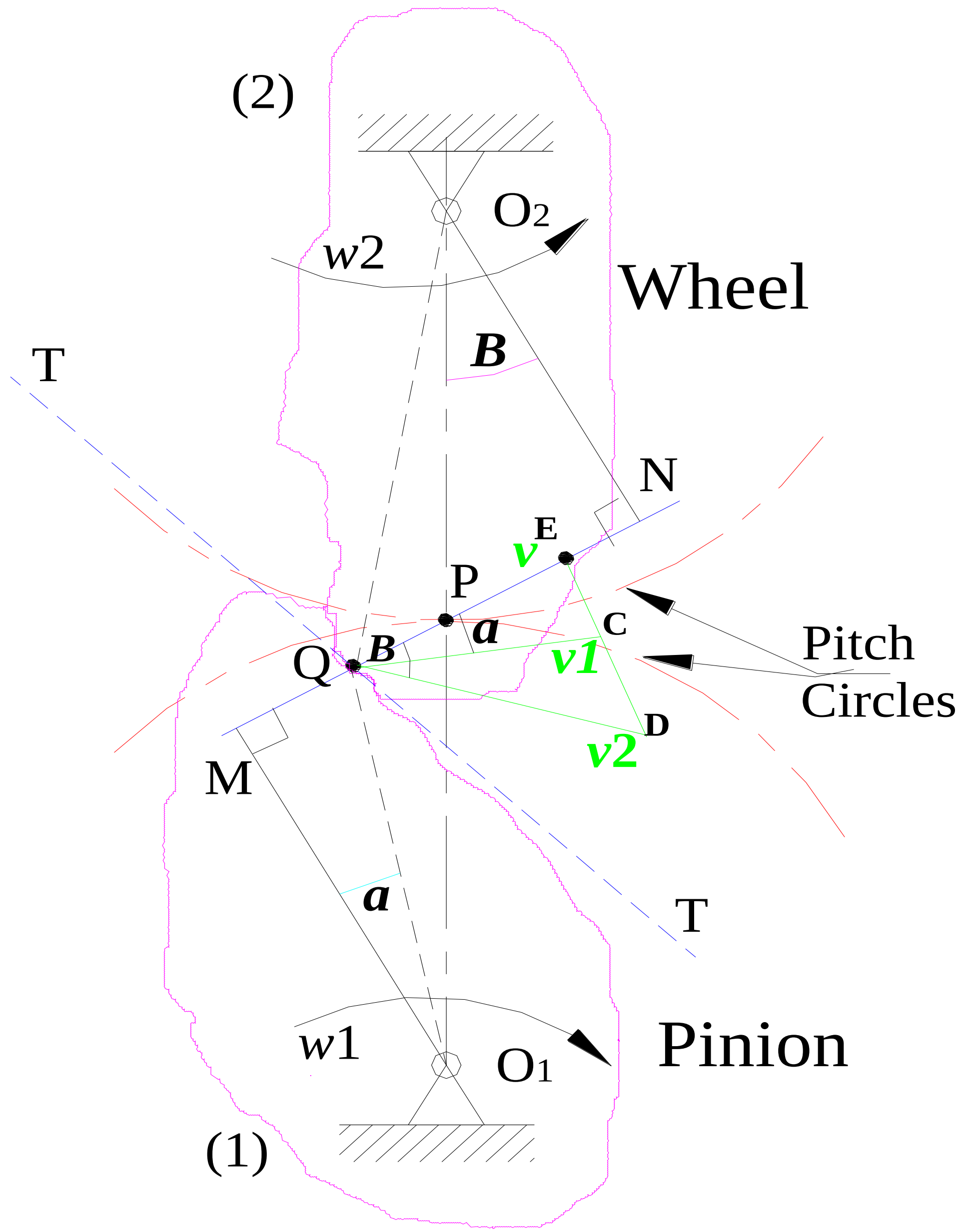

(2)
O2
w2
Wheel
B
T
N
E
v
P
C
a
Q
B
v1
Pitch
Circles
D
v2
M
T
a
w1
O1
Pinion
(1)

Law of Gearing

Let,

O1,O2 – Centres of wheel 1 & 2 resp.

Q – Point of contact of two teeth

T T – Common tangent at the point of contact Q

MN – Common normal at the point of contact Q

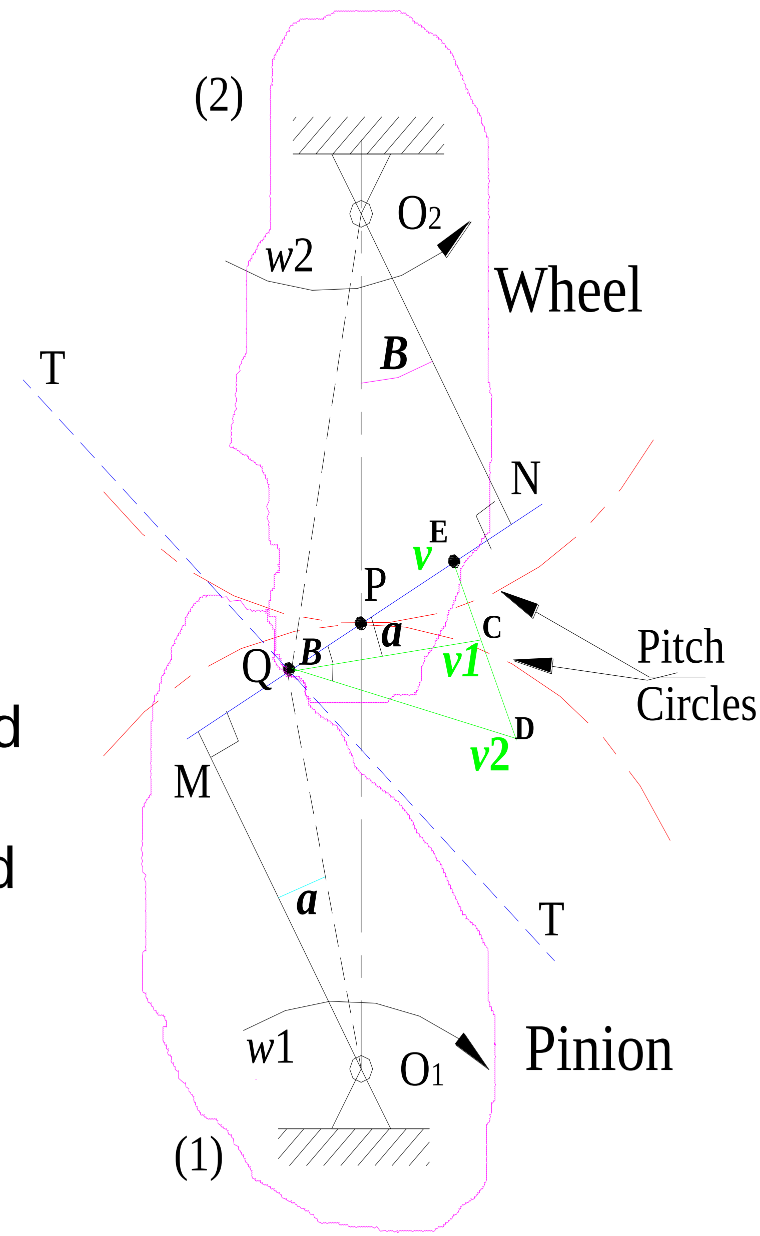
O_1M, O_2N – Perpendicular to MN

QC – Direction of Q when considered on wheel 1

QD – Direction of Q when considered on wheel 2

v_1 – Velocity of Q along QC

v_2 – Velocity of Q along QD



Law of Gearing

If the teeth are to remain in contact, then… the components of velocities along the common normal MN must be *equal*

$$\therefore v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 * O_1Q)\cos\alpha = (\omega_2 * O_2Q)\cos\beta$$

$$(\omega_1 * O_1Q)\frac{O_1M}{O_1Q} = (\omega_2 * O_2Q)\frac{O_2N}{O_2Q}$$

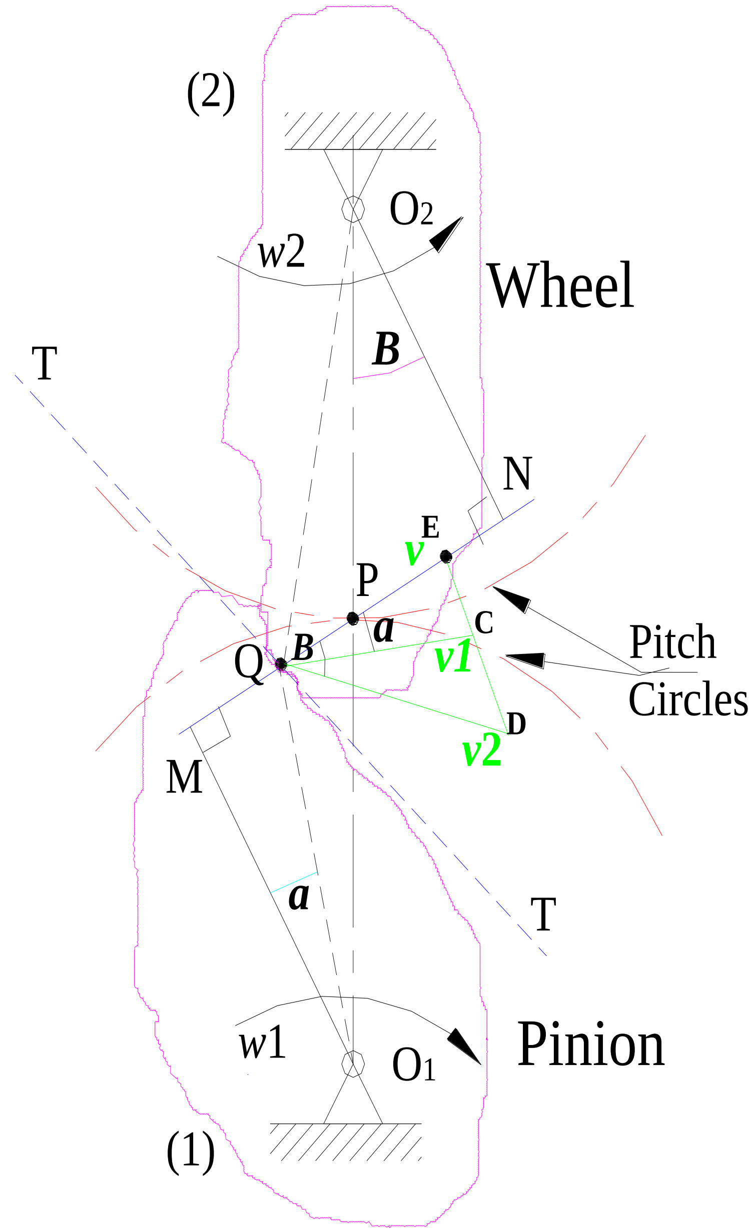
$$\omega_1 * O_1M = \omega_2 * O_2N$$

$$\therefore \frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$$

$$Also...\Delta O_1MP \approx \Delta O_2NP$$

$$\therefore \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$



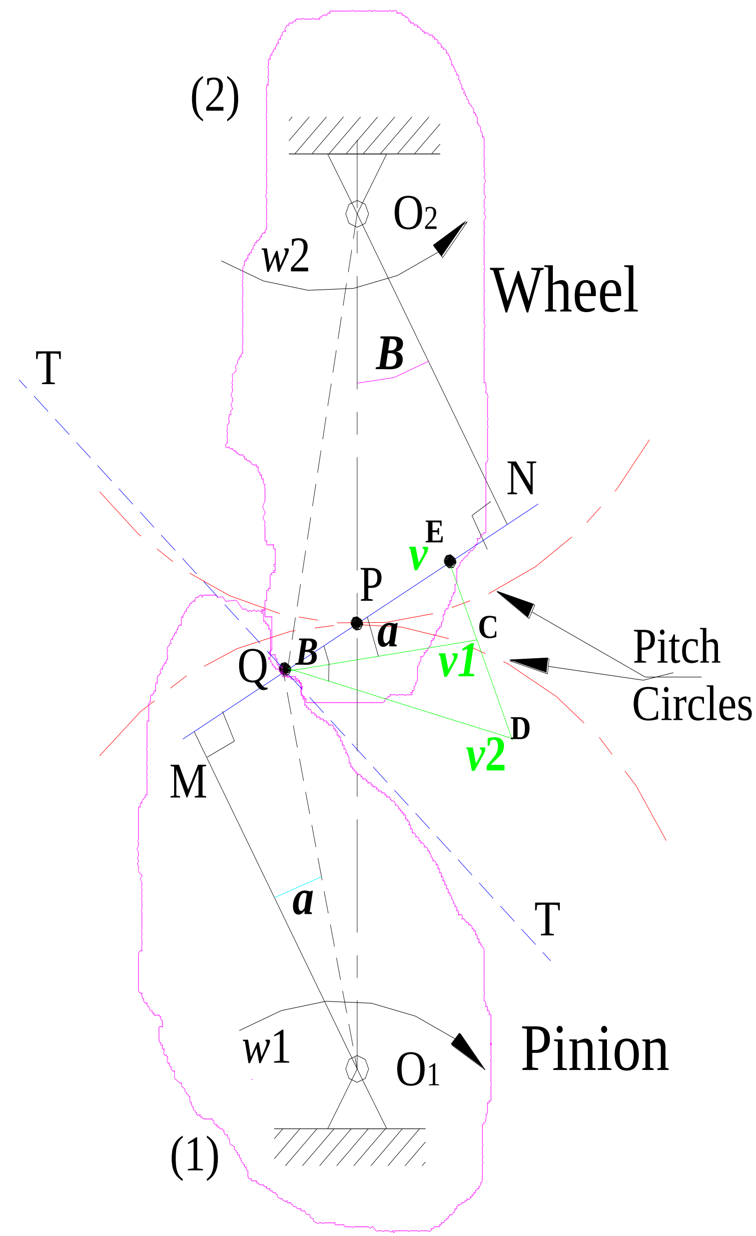
Law of Gearing

Angular velocity ratio is inversely proportional to ratio of distances of point P from centres O_1 & O_2

OR

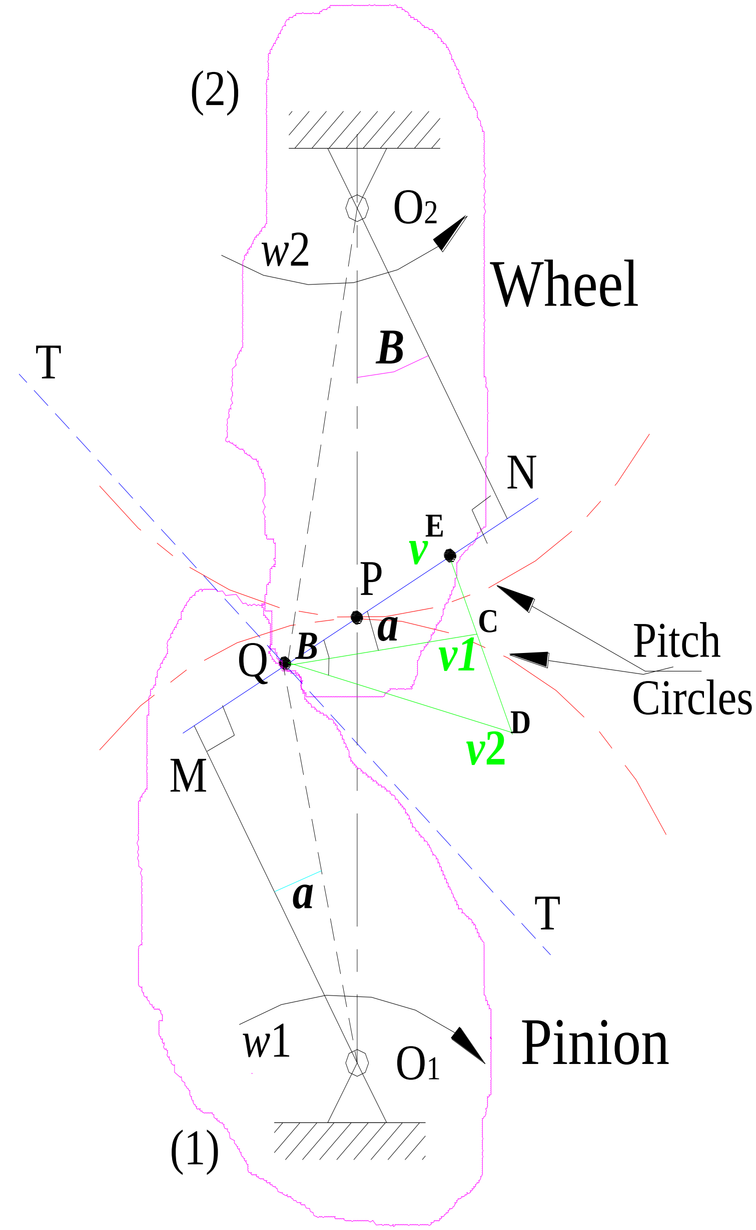
Common Normal at the point of contact Q intersects the line of centres at point P which divides the Centre distance inversely as the ratio of angular velocities

To have constant angular velocity For all positions of wheel, point P Must be the fixed point



Law of Gearing

"Common normal at the point of contact between a pair of teeth must always pass through the pitch point"



Velocity of Sliding

It is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact

EC – velocity of point Q (on wheel 1) along TT

$$\Delta QEC \approx \Delta O_1MQ$$

$$\therefore \frac{EC}{MQ} = \frac{v}{O_1Q} = \omega_1$$

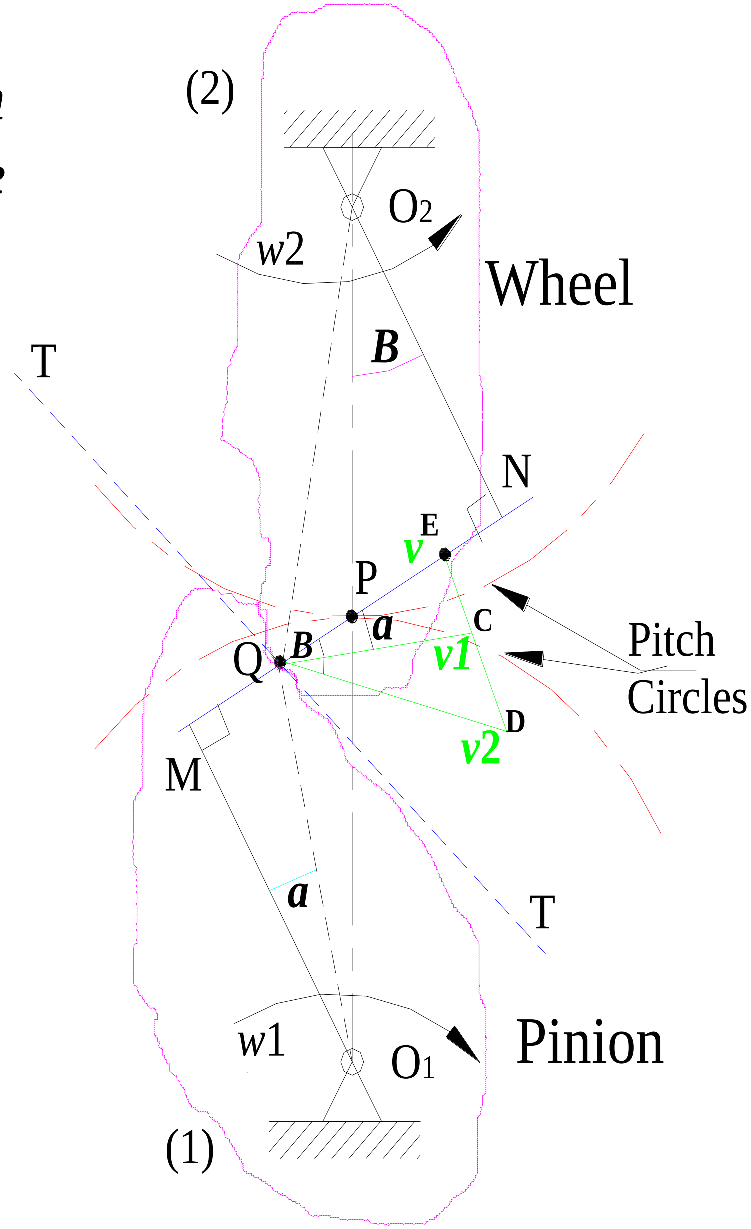
$$EC = \omega_1 * MQ$$

ED – velocity of point Q (on wheel 2) along TT

$$\Delta QED \approx \Delta O_2NQ$$

$$\therefore \frac{ED}{QN} = \frac{v_2}{O_2Q} = \omega 2$$

$$ED = \omega 2 * QN$$



Velocity of Sliding

Let,

v_s ***= Velocity of sliding at Q***

$$vs = ED - EC = v2\sin\beta - v1\sin\alpha$$

$$= (\omega_2 * O_2Q)\frac{QN}{O_2Q} - (\omega_1 * O_1Q)\frac{MQ}{O_1Q}$$

$$= \omega_2.QN - \omega_1.MQ$$

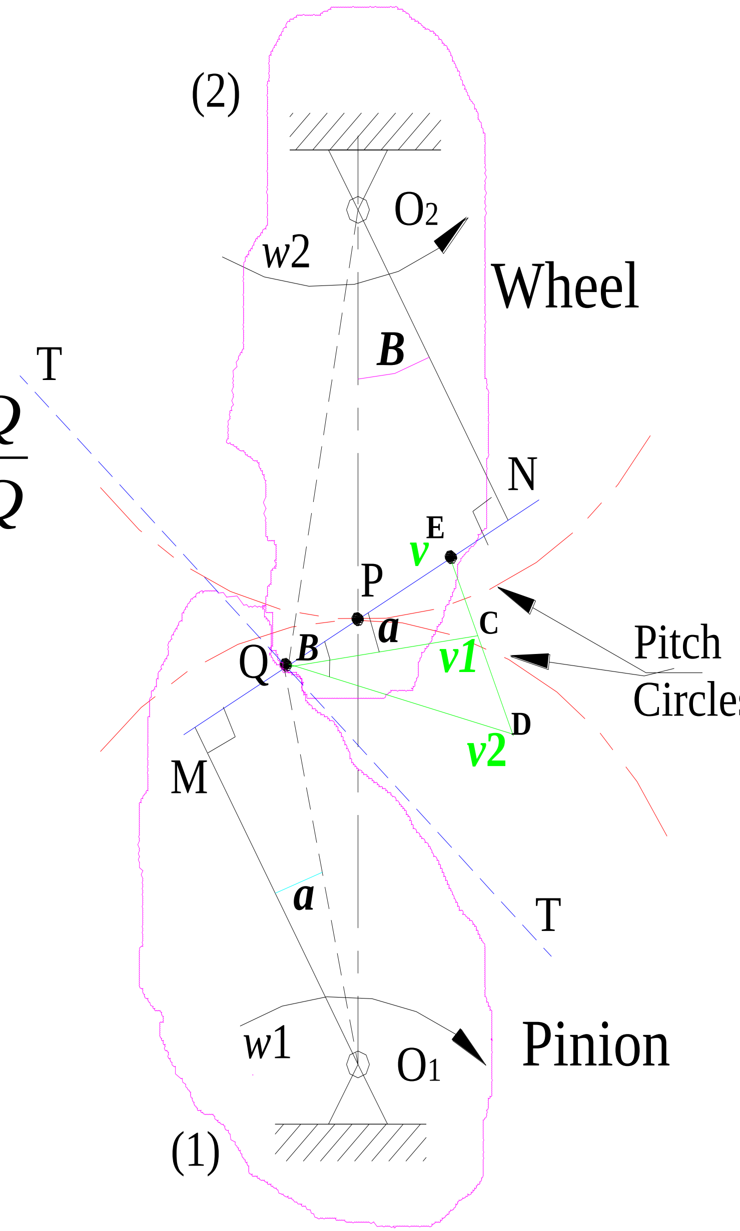
$$= \omega_2(QP + PN) - \omega_1(MP - QP)$$

$$= (\omega_1 + \omega_2)QP + \omega_2.PN - \omega_1.MP$$

$$But...,\ \frac{\omega_1}{\omega_2} = \frac{O_2P}{O_1P} = \frac{PN}{MP}$$

$$i.e.,\ \omega_1.MP = \omega_2.PN$$

$$\therefore v_s = (\omega_1 + \omega_2)QP$$



Velocity of sliding is proportional to distance of point of contact to pitch point

Forms of Teeth

1. ***Conjugate teeth***

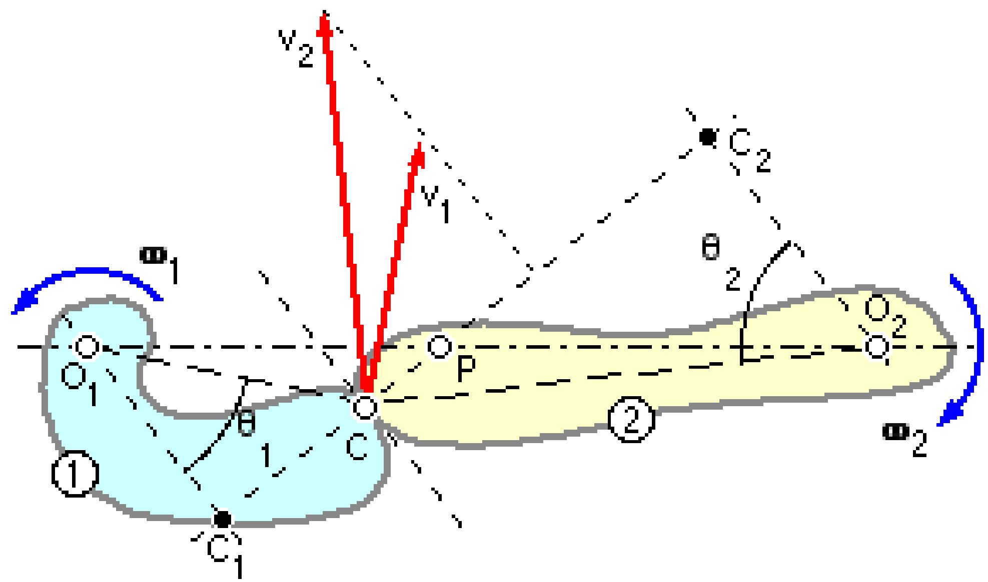
2. ***Cycloidal teeth***

3. ***Involute teeth***

Conjugate action

It is essential for correctly meshing gears, the size of the teeth (the module) must be the same for both the gears.

Another requirement - the shape of teeth necessary for the speed ratio to remain constant during an increment of rotation; this behavior of the contacting surfaces (ie. the teeth flanks) is known as ***conjugate action.***

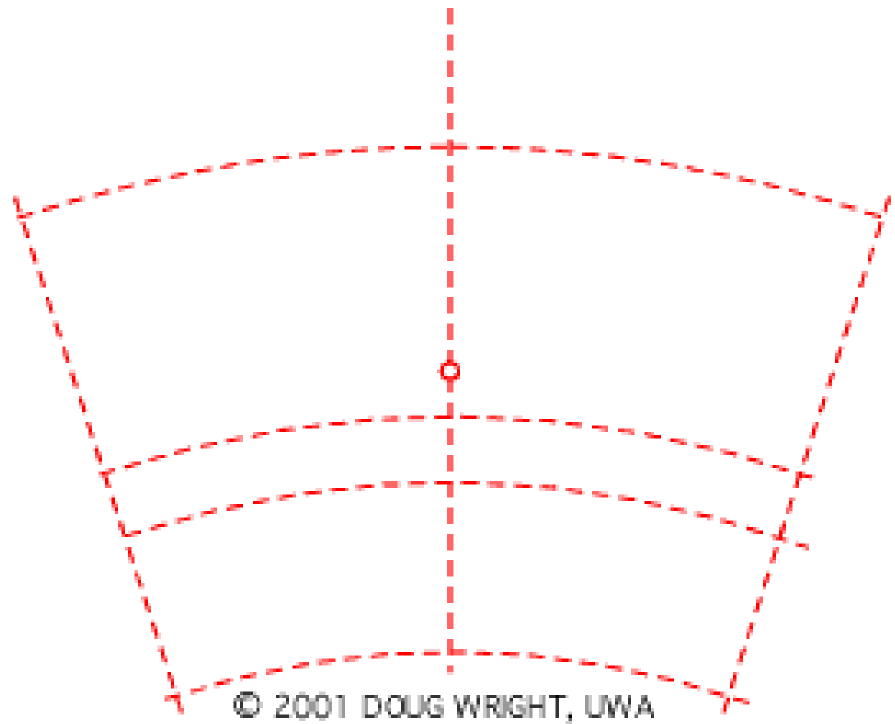


Drawback :-

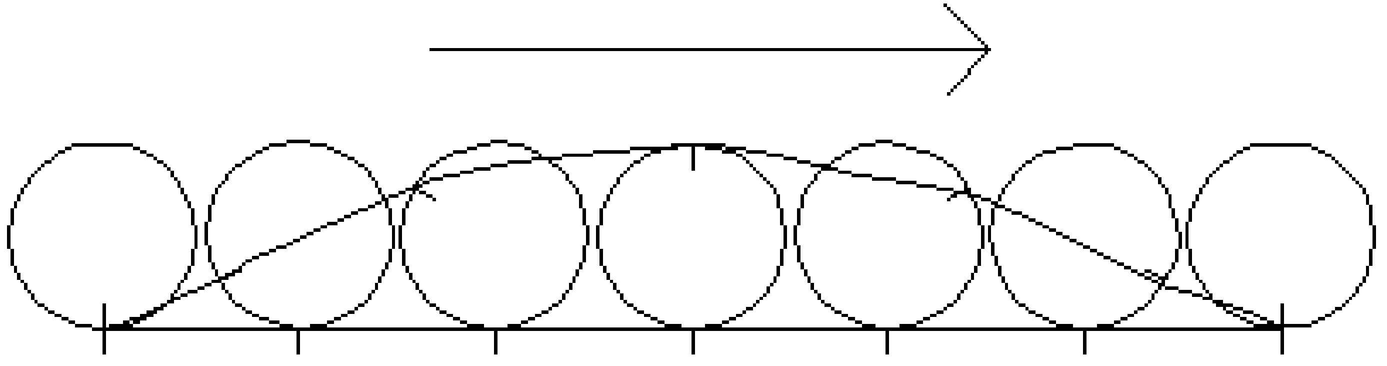
Difficulty to manufacture,
Standardisation &
Cost of production

Tooth Profiles

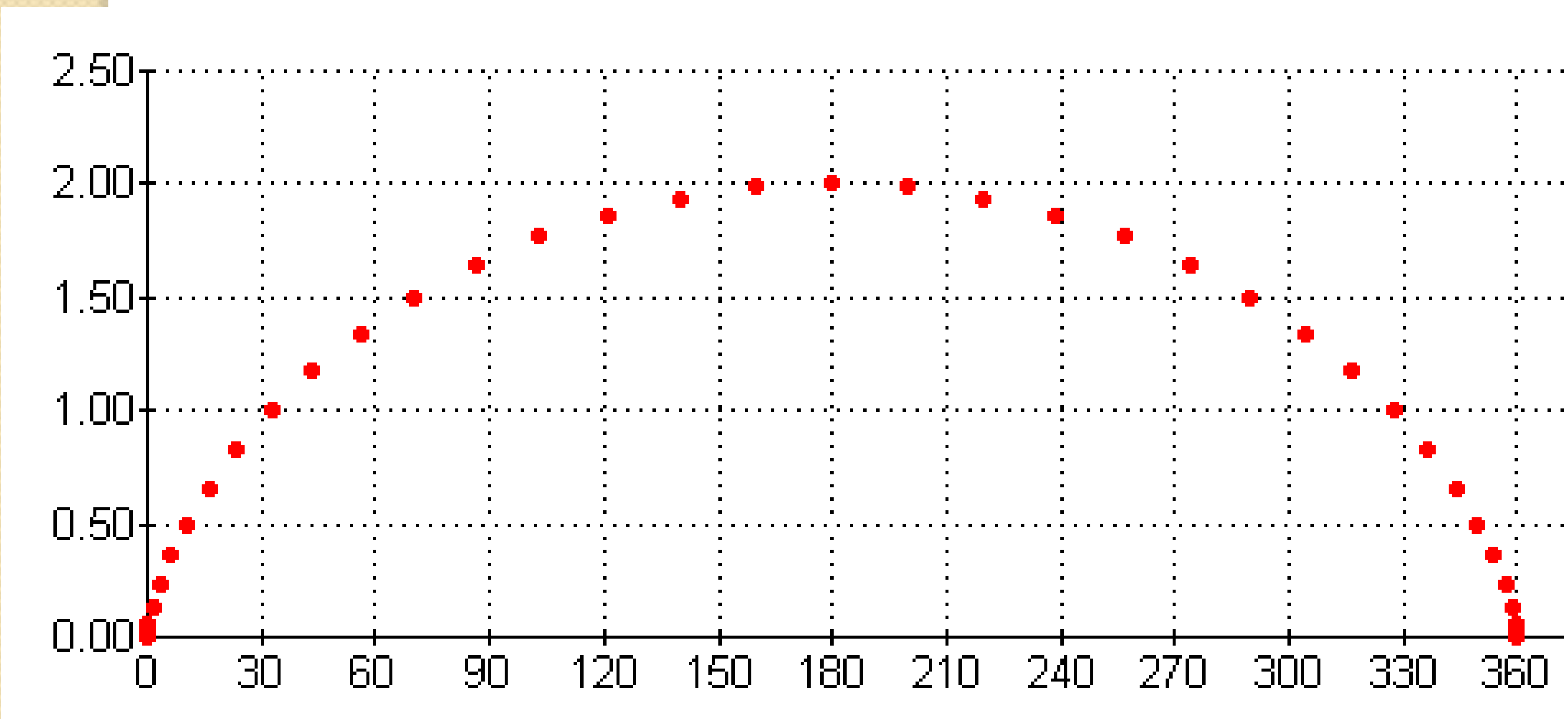
Although many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the ***cycloidal*** and ***involute*** profiles.



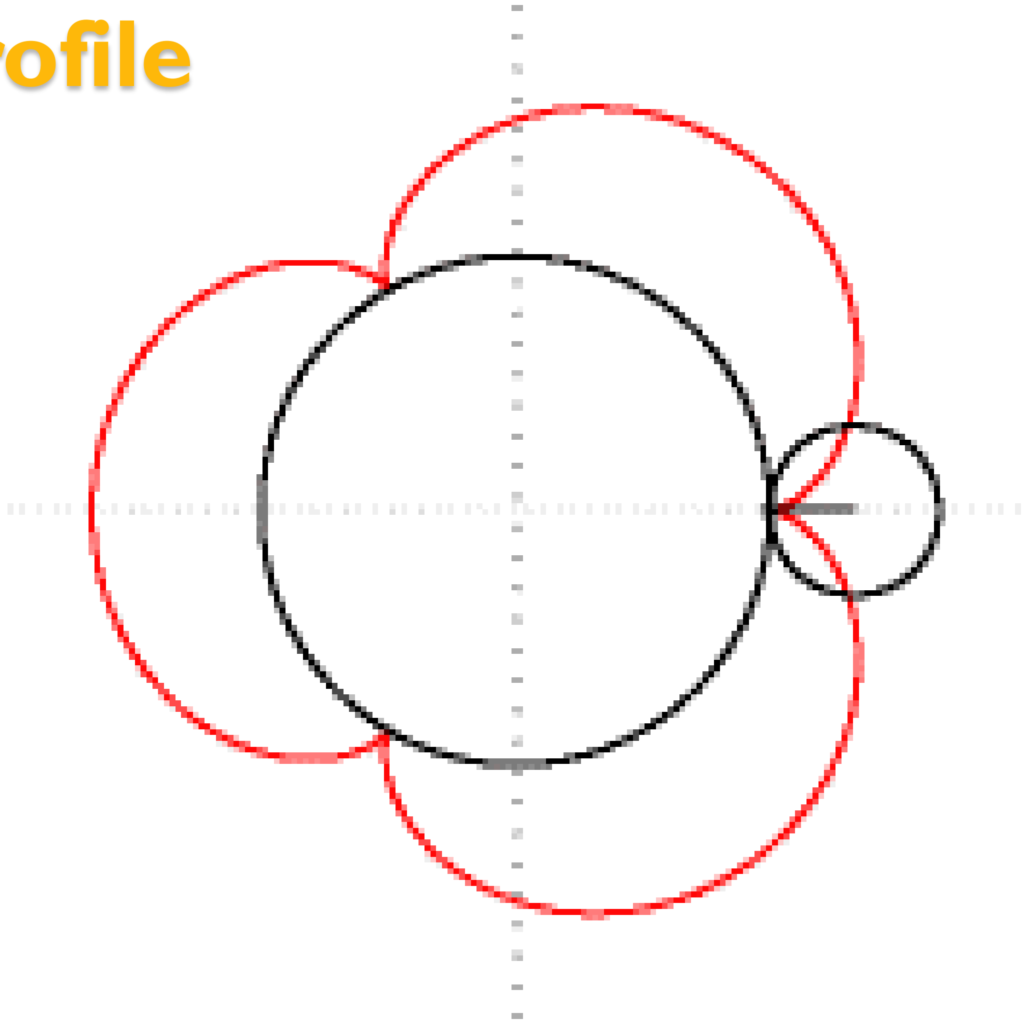
Cycliodal Profile



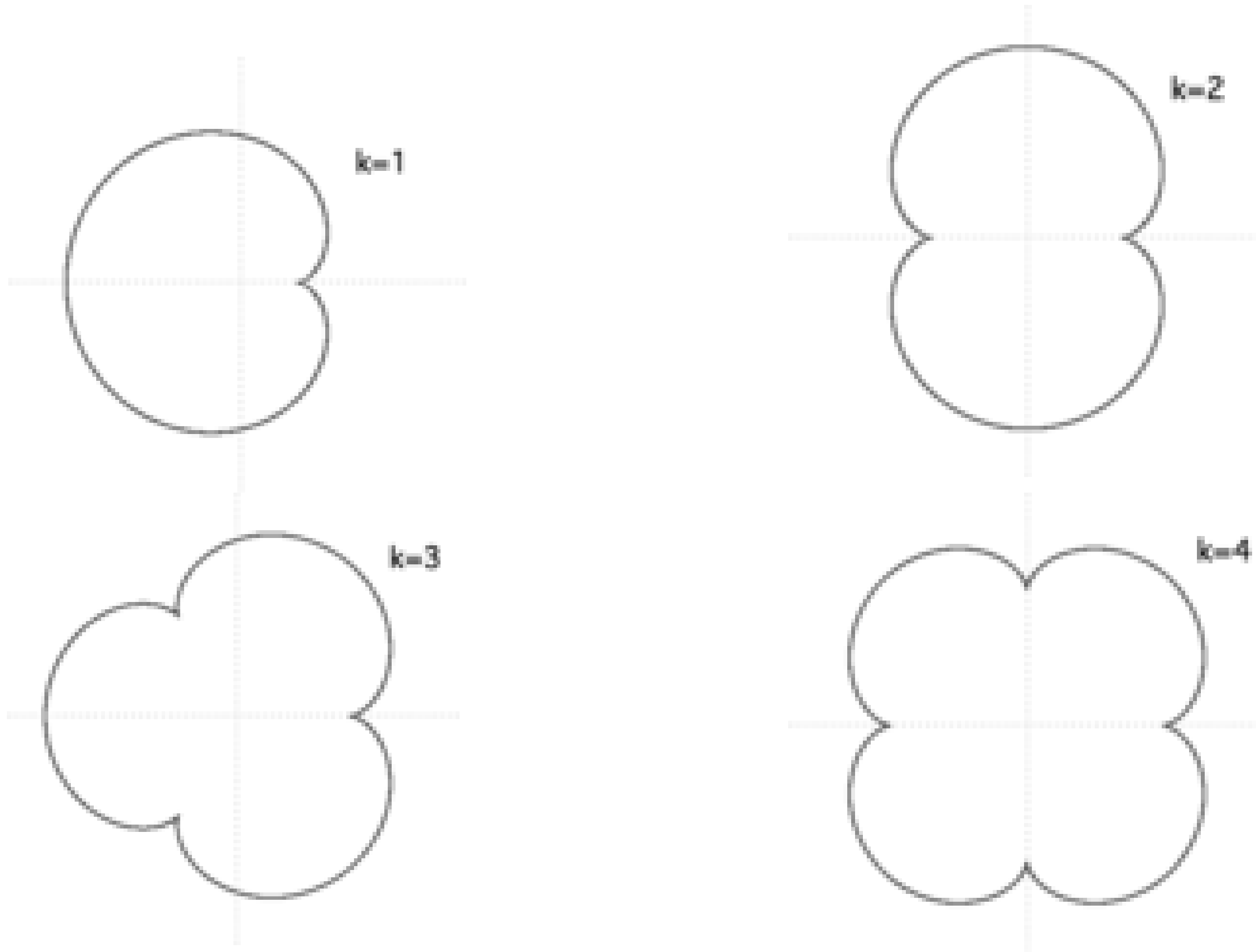
Cycliodal Profile



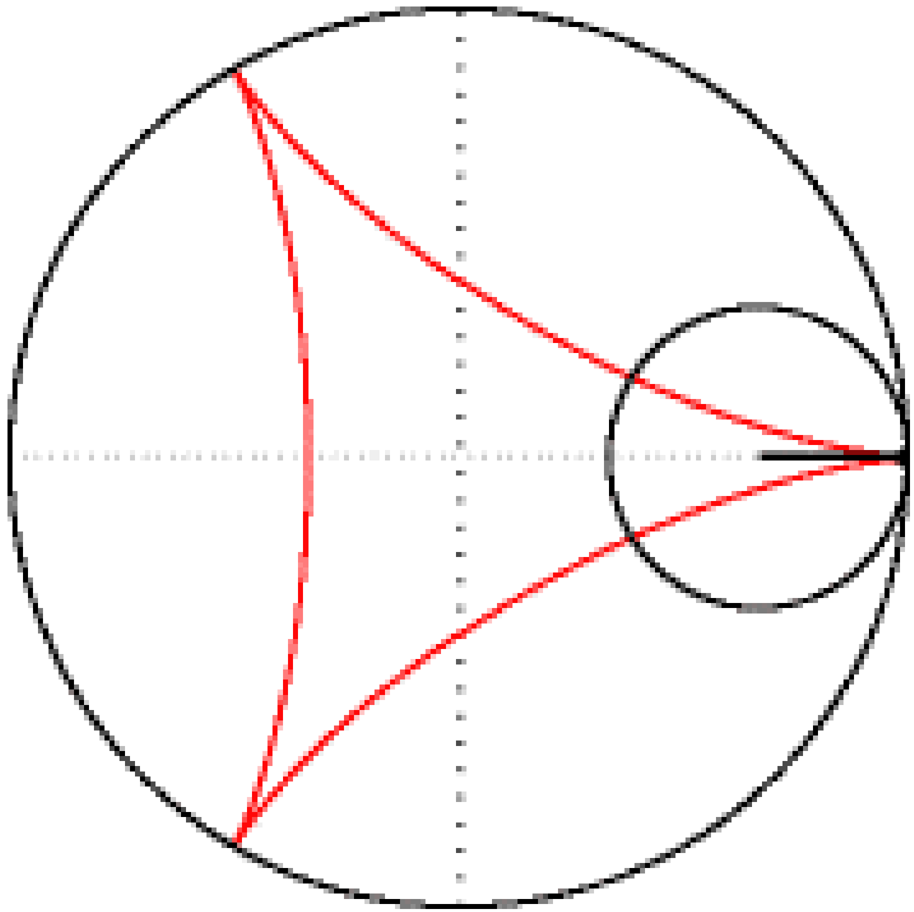
Epicycliodal Profile



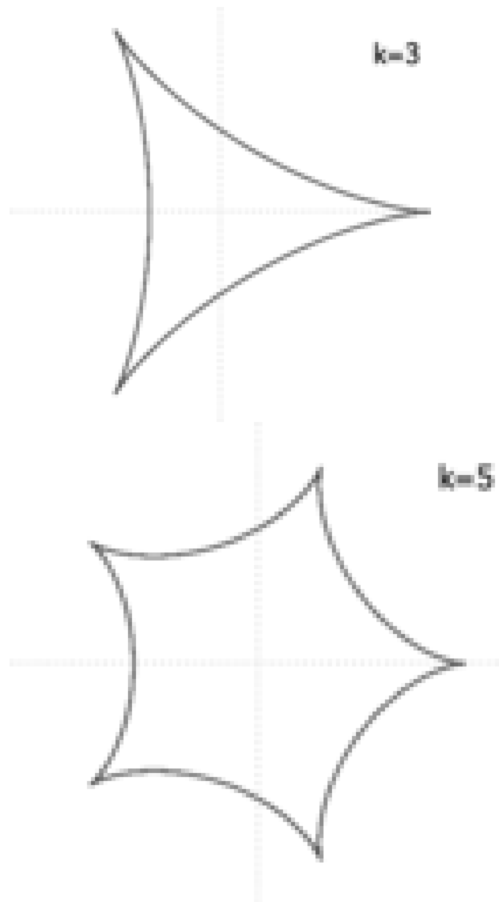
Epicycliodal Profile



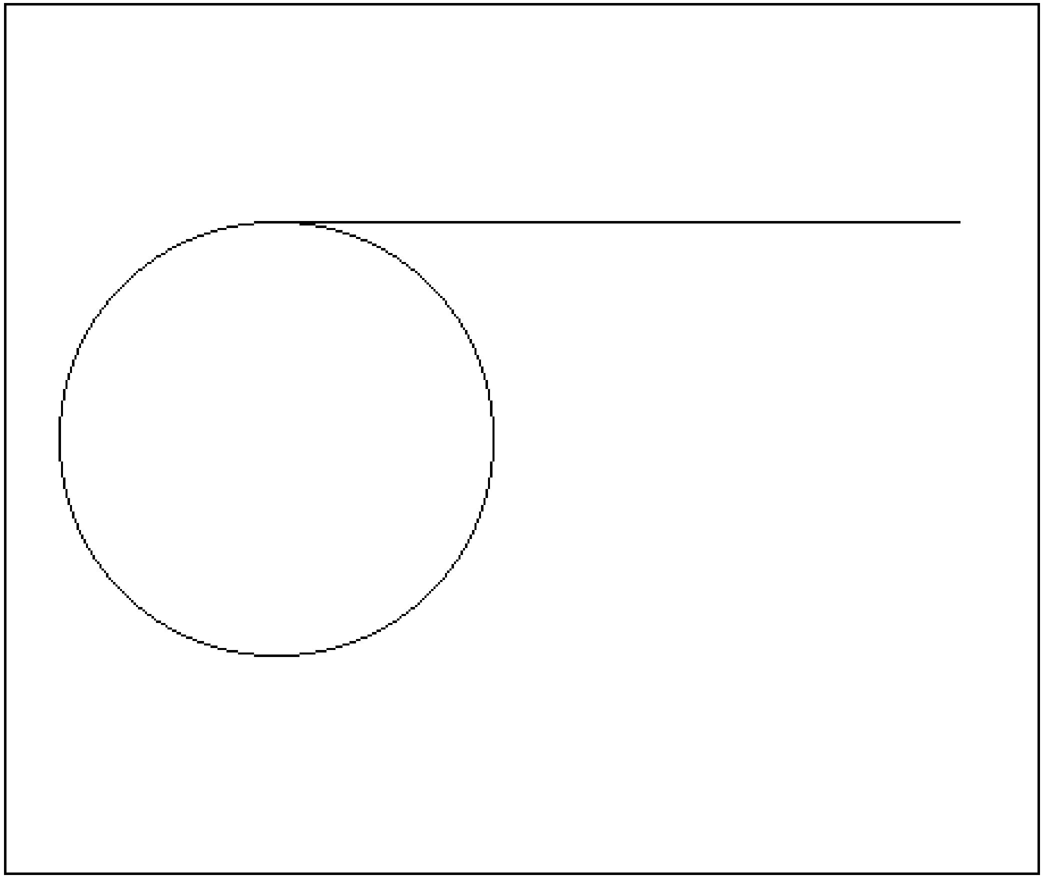
Hypocycliodal Profile



Hypocycliodal Profile

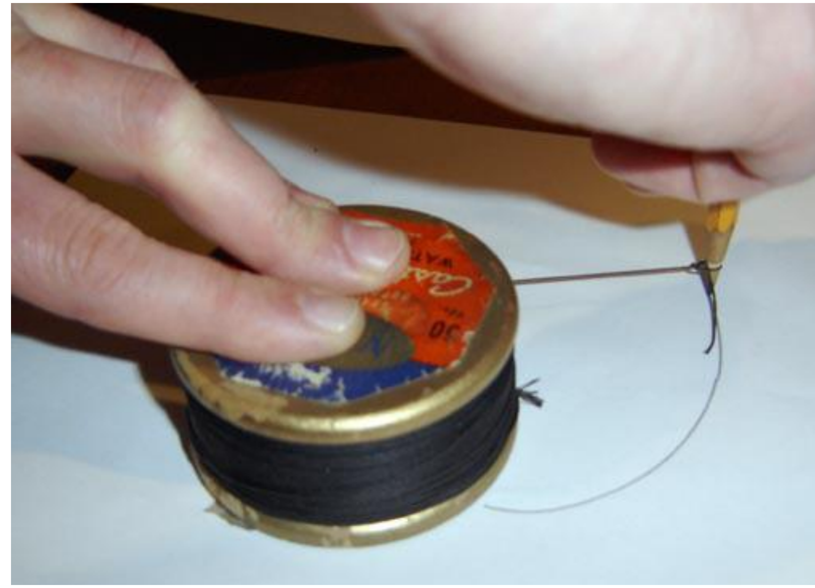


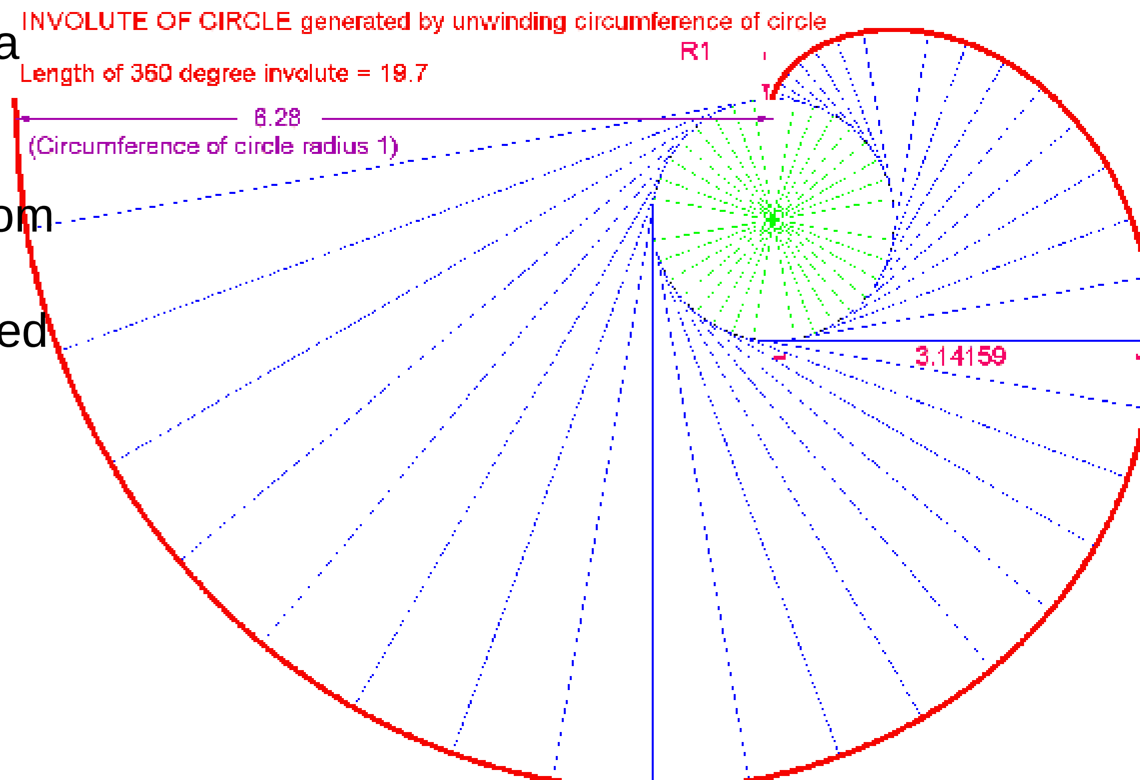
Involute Profiles



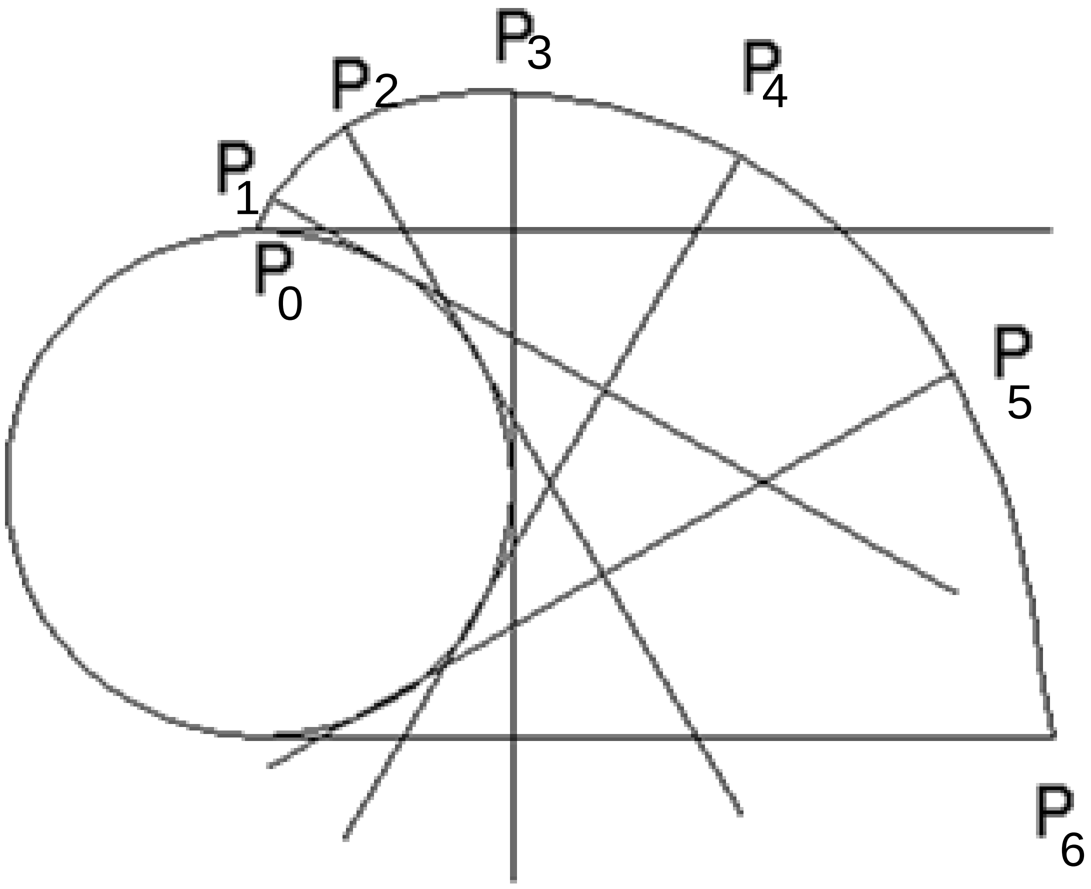
□ **Generation of the Involute Curve**

This **involute curve is the** path traced by a point on a line as the line rolls without slipping on the circumference of a circle. It may also be defined as a path traced by the end of a string which is originally wrapped on a circle when the string is unwrapped from the circle. The circle from which the involute is derived is called the **base circle.**

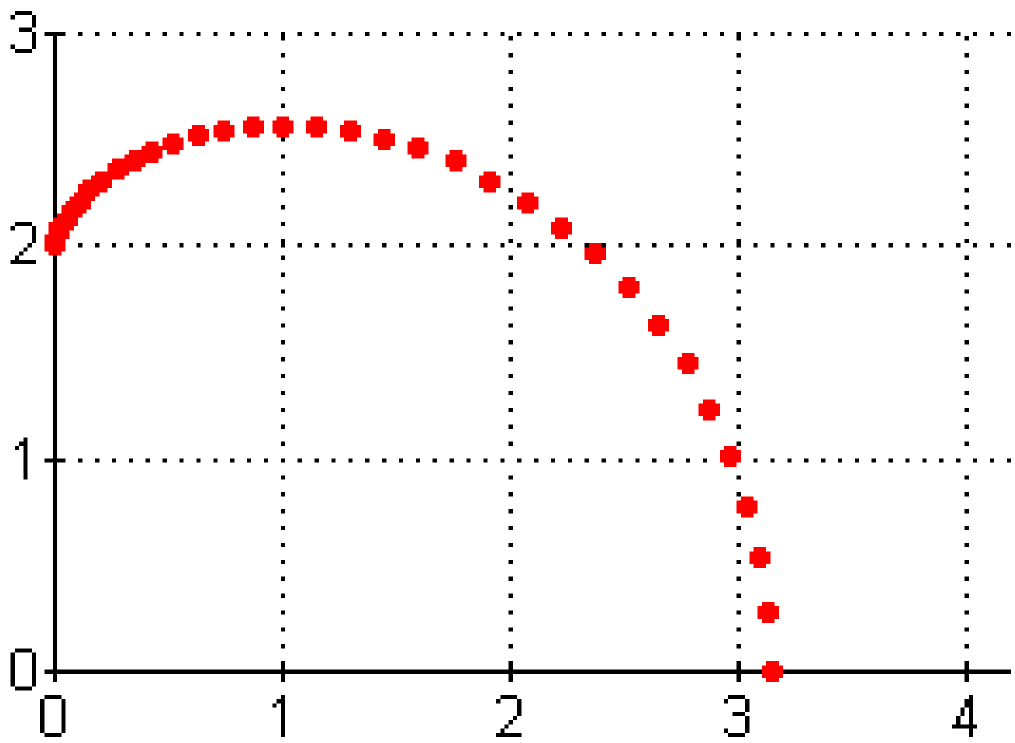




Involute Profile

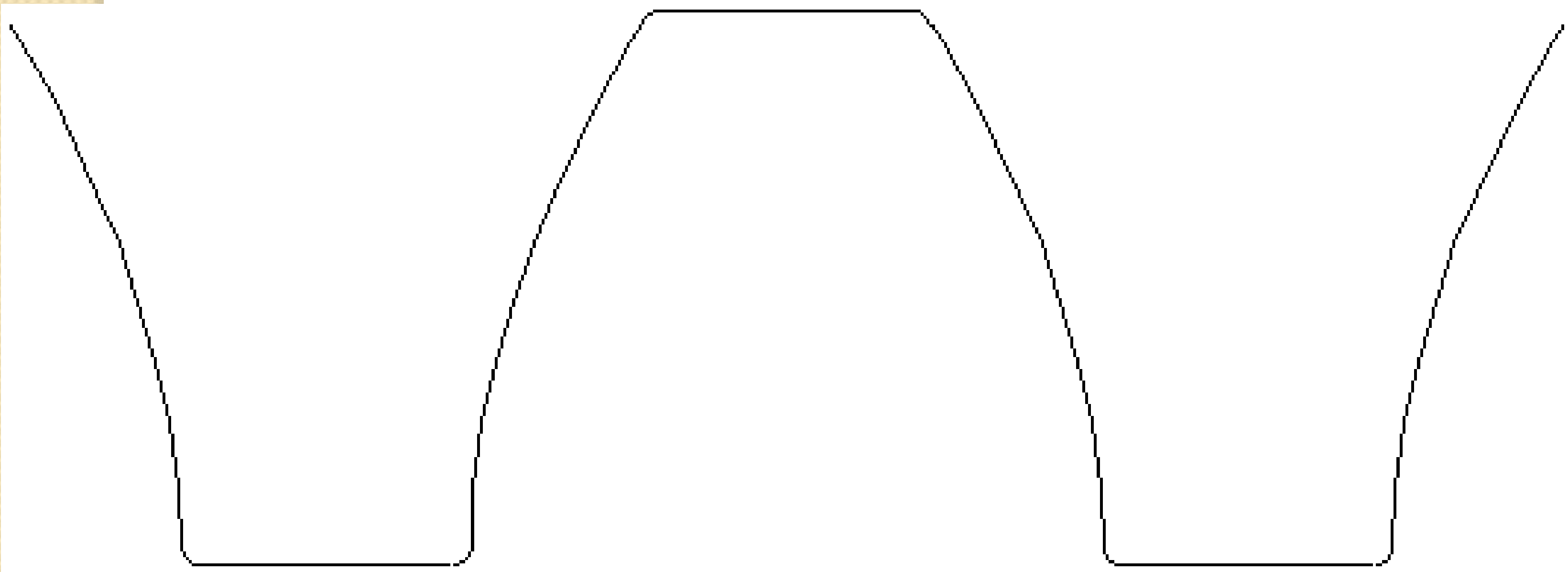


Involute Profile



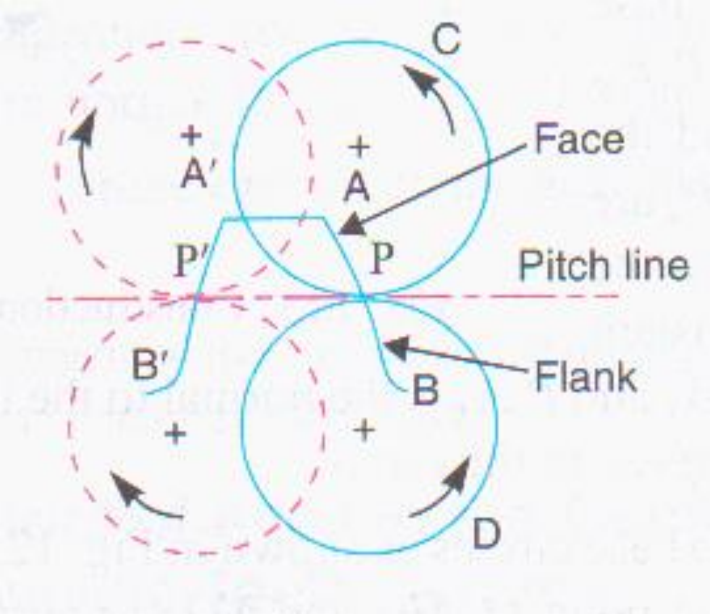
The portion of the Involute Curve that would be used to design a gear tooth

Involute Profile

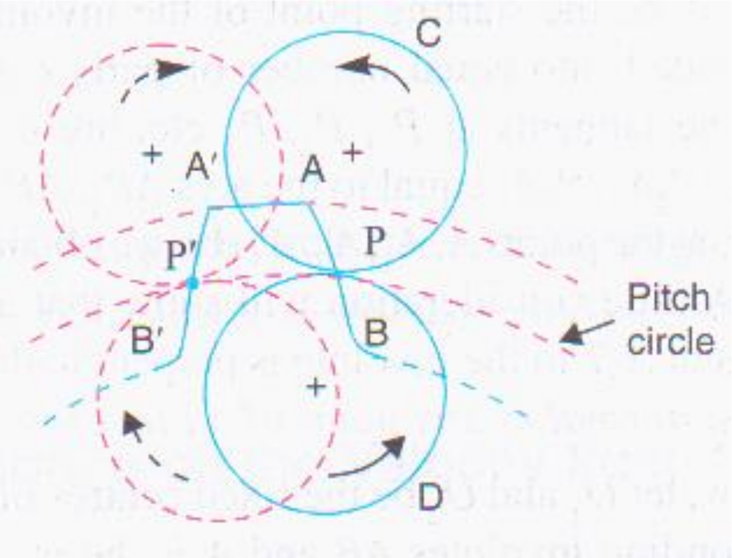


http://auto.howstuffworks.com/gear8.htm

Cycliodal Profile

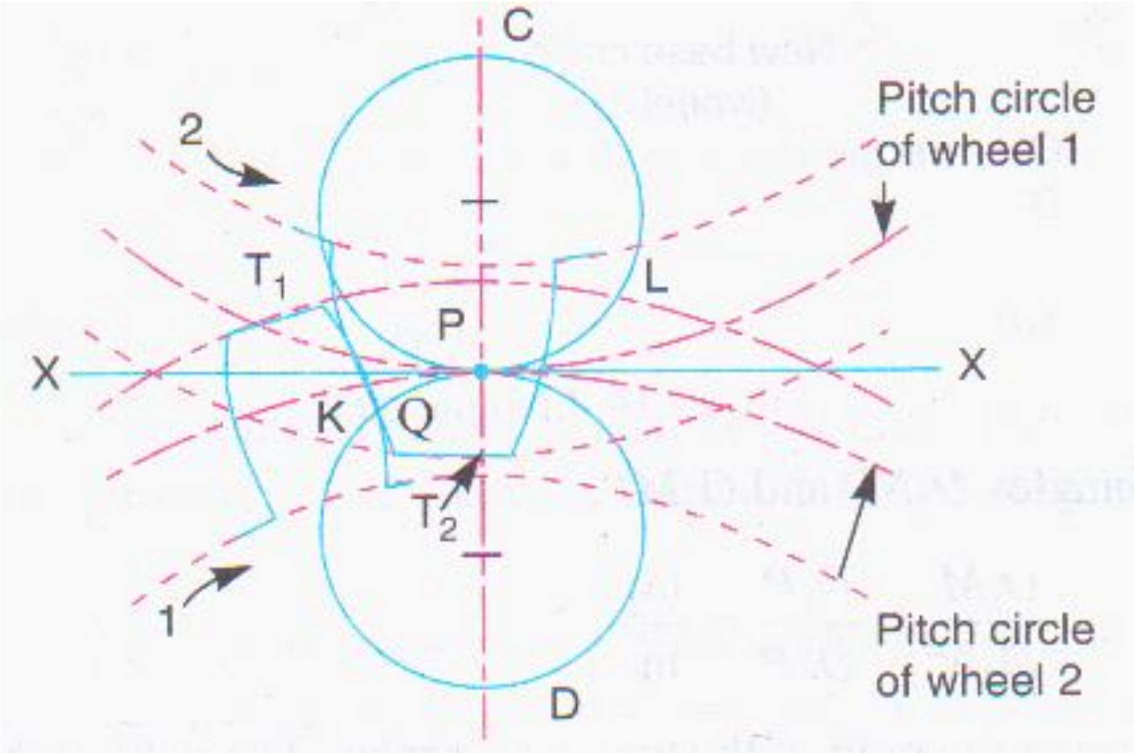


(a)

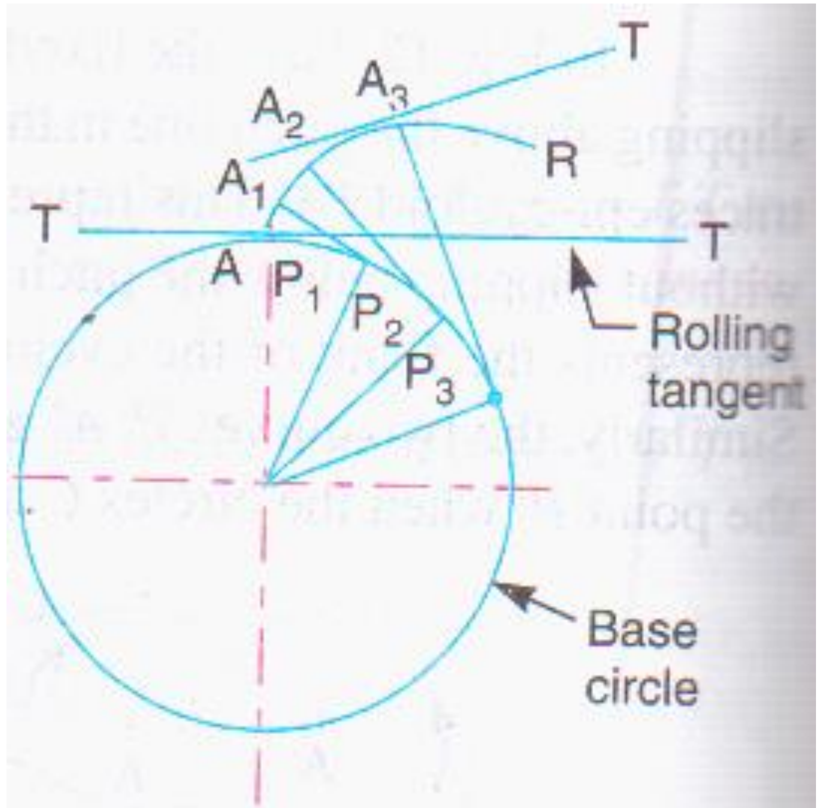


(b)

Cycliodal Profile

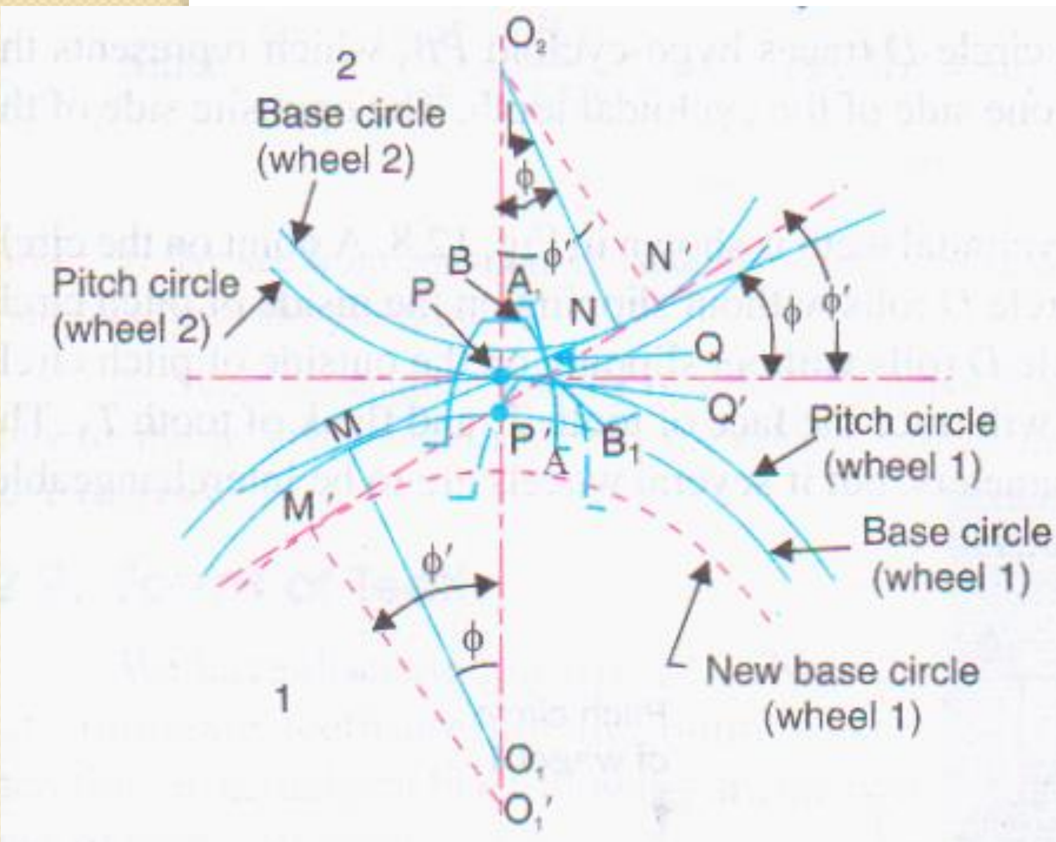


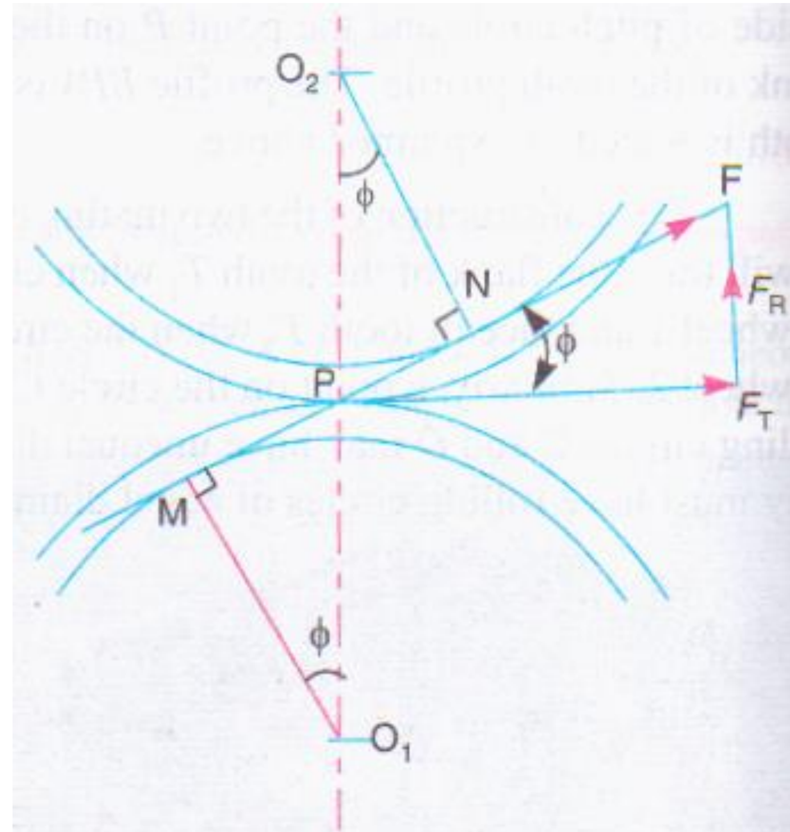
Involute Profile



Normal at any point of an involute is a tangent to the circle.

Involute Profile





(b)

Involute Profile

From similar triangles 0_2NP and O_1MP.

$$\therefore \frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1}$$

radii of the two base circles,

$$O_1M = O_1P\cos\phi,$$

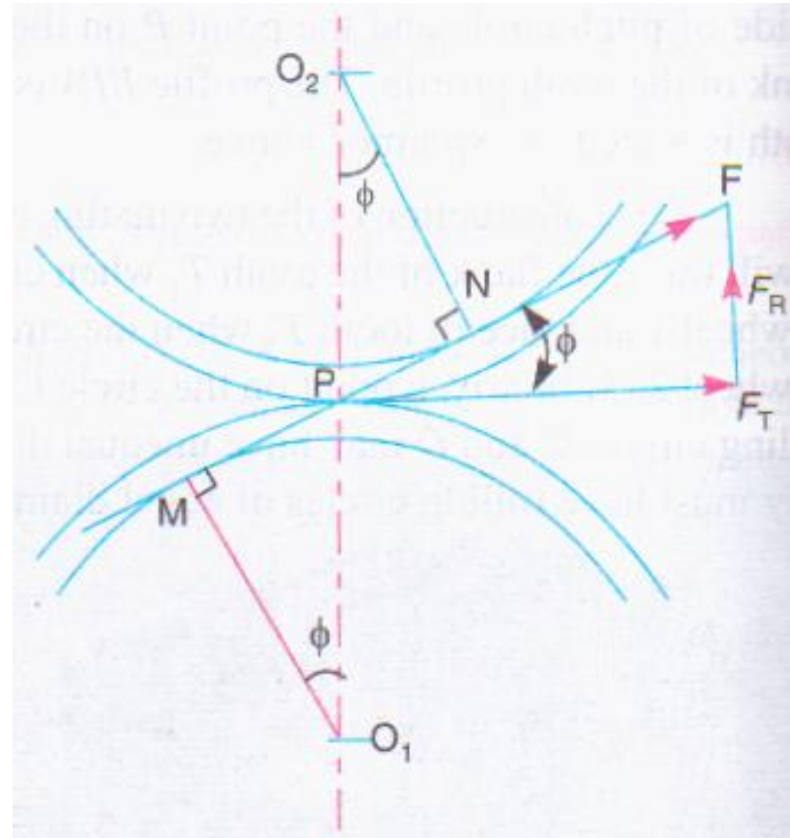
$$O_2N = O_2P\cos\phi$$

Also the centre distance between the base circles,

$$O_1O_2 = O_1P + O_2P$$

$$= \frac{O_1M}{\cos\phi} + \frac{O_2N}{\cos\phi}$$

$$= \frac{O_1M + O_2N}{\cos\phi}$$



(b)

Involute Profile

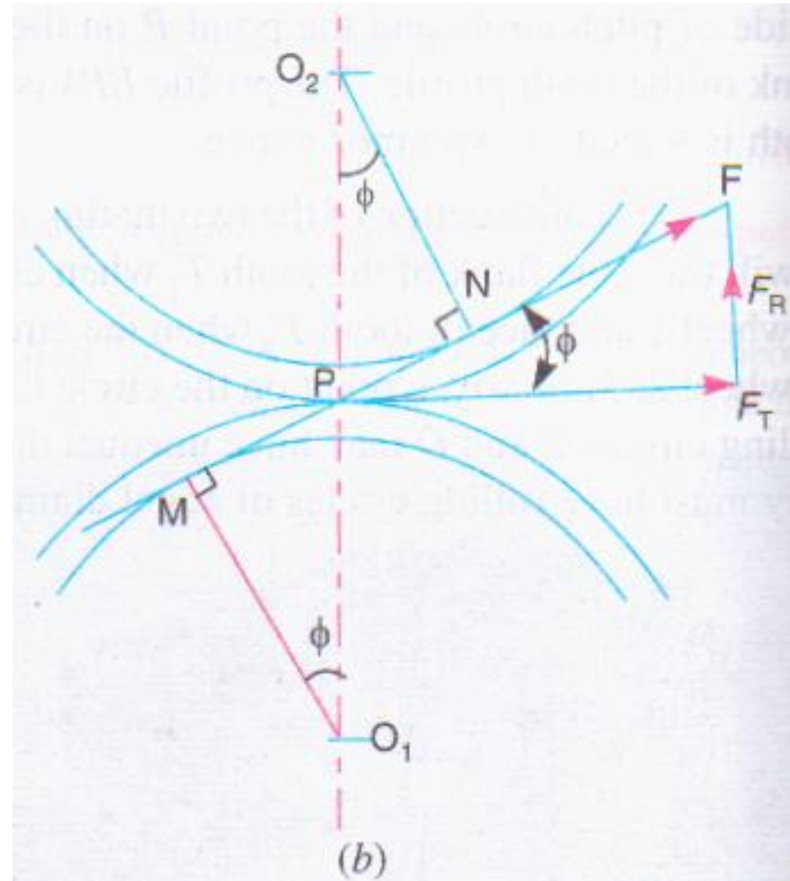
If Fis the maximum tooth pressure

Tangential force. $F_T = F \cos \phi$

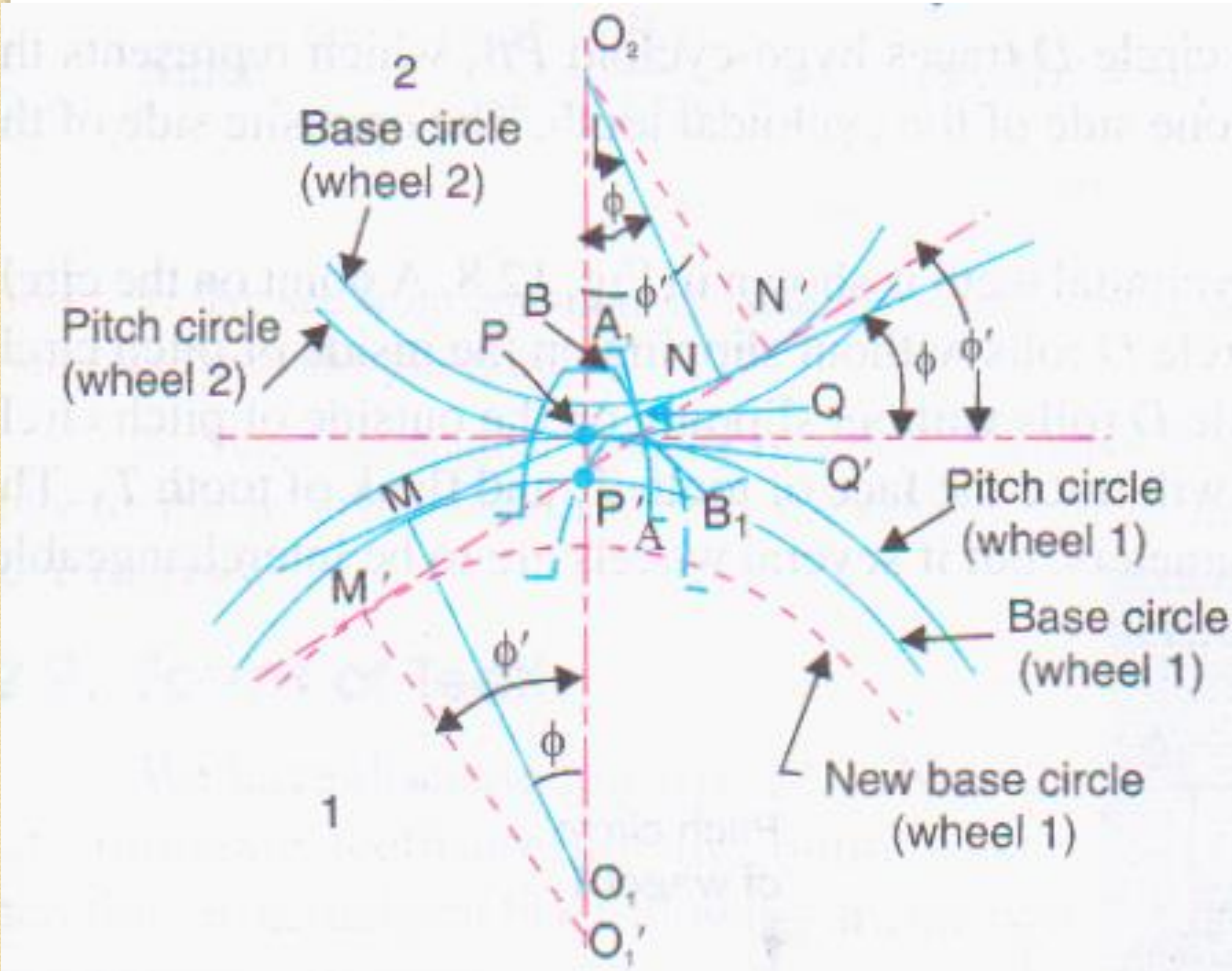
Radial or normal force $F_R = F \sin \phi.$

Torque exerted on the gear shaft

$$= F_T \times r,$$



Effect of Altering the Centre Distance



Comparison Of Involute & Cycloidal

- *Center distance* between a pair of involute gears can be varied without changing velocity ratio

- The pressure angle from start of engagement to the end of engagement remains constant. Thus, smooth running of gears

- *Center distance* between cycloidal gears is to be kept constant to keep constant velocity ratio

- The pressure angle varies from start of engagement to the end of engagement. Thus, less smooth running of gears

Comparison Of Involute & Cycloidal

- Teeth generated by single curve. Thus, easy for manufacturing
- Teeth generated by double curves (epicycloid & hypocycloid). Thus, difficult for manufacturing
- Strength is less due to radial flanks
- Strength is more due to wider flanks
- Have interference problem
- Do not have interference problem
- More wear of tooth surface
- Less wear of tooth surface

Properties of Involute tooth profile

1. A normal drawn to an involute at pitch point is a tangent to the base circle.

2. Pressure angle remains constant during the mesh of an involute gears.

3. The involute tooth form of gears is insensitive to the centre distance and depends only on the dimensions of the base circle.

Properties of Involute tooth profile

4. The radius of curvature of an involute is equal to the length of tangent to the base circle.

5. Basic rack for involute tooth profile has straight line form.

6. The common tangent drawn from the pitch point to the base circle of the two involutes is the line of action and also the path of contact of the involutes.

Properties of Involute tooth profile

7. When two involutes gears are in mesh and rotating, they exhibit constant angular velocity ratio and is inversely proportional to the size of base circles. (Law of Gearing or conjugate action)

8. Manufacturing of gears is easy due to single curvature of profile.

System of Gear Teeth

The following four systems of gear teeth are commonly used in practice:

1. $14\frac{1}{2}^{O}$ Composite system

2. $14\frac{1}{2}^{O}$ Full depth involute system

3. 20^{O} Full depth involute system

4. 20^{O} Stub involute system

System of Gear Teeth

The $14\frac{1}{2}^{O}$ *composite system* is used for general purpose gears.

It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion.

The teeth are produced by formed milling cutters or hobs.

The tooth profile of the $14\frac{1}{2}^{O}$ *full depth involute system* was developed using gear hobs for spur and helical gears.

System of Gear Teeth

The tooth profile of the 20^o *full depth involute system* may be cut by hobs.

The increase of the pressure angle from 14½^o to 20^o results in a stronger tooth, because the tooth acting as a beam is wider at the base.

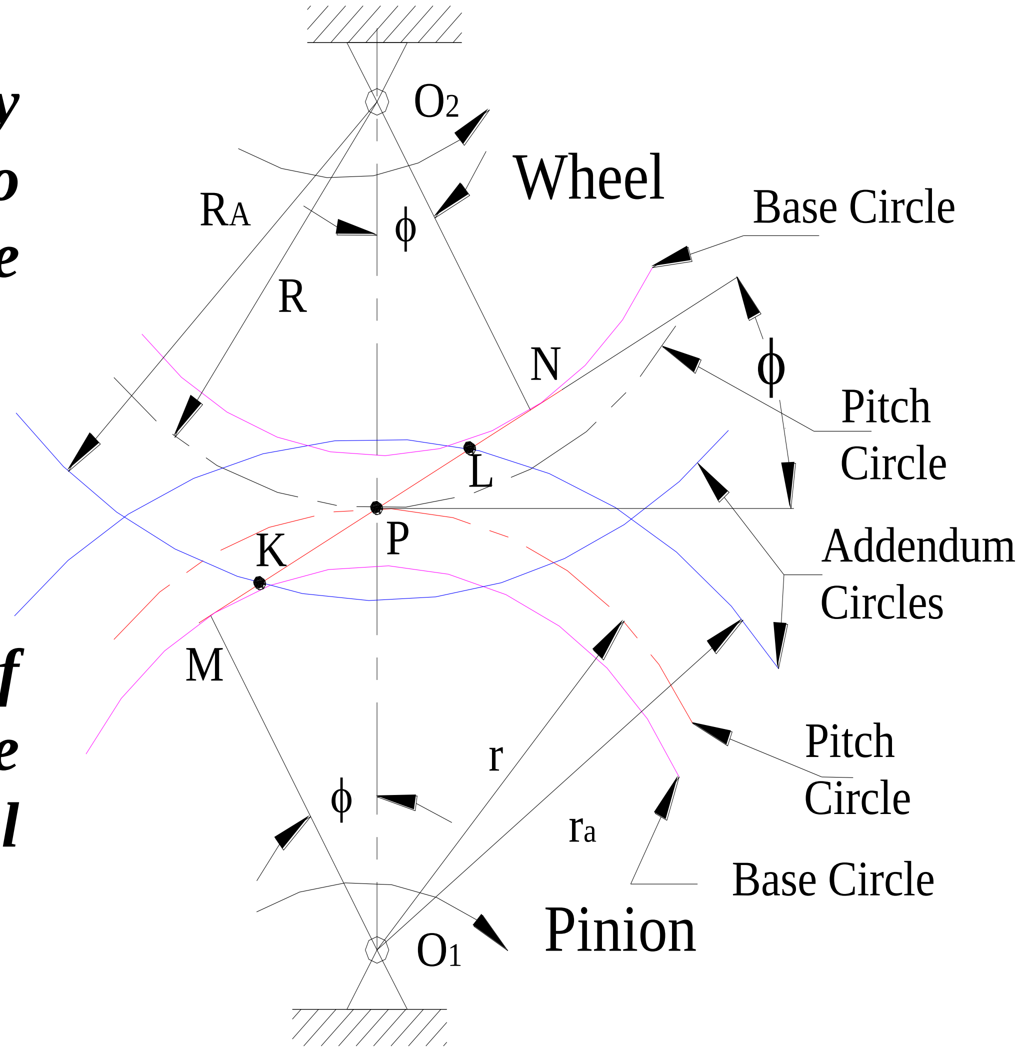
The *20^o stub involute system* has a strong tooth to take heavy loads.

Path of contact

It is the path traced by the point of contact of two teeth from beginning to the end of engagement

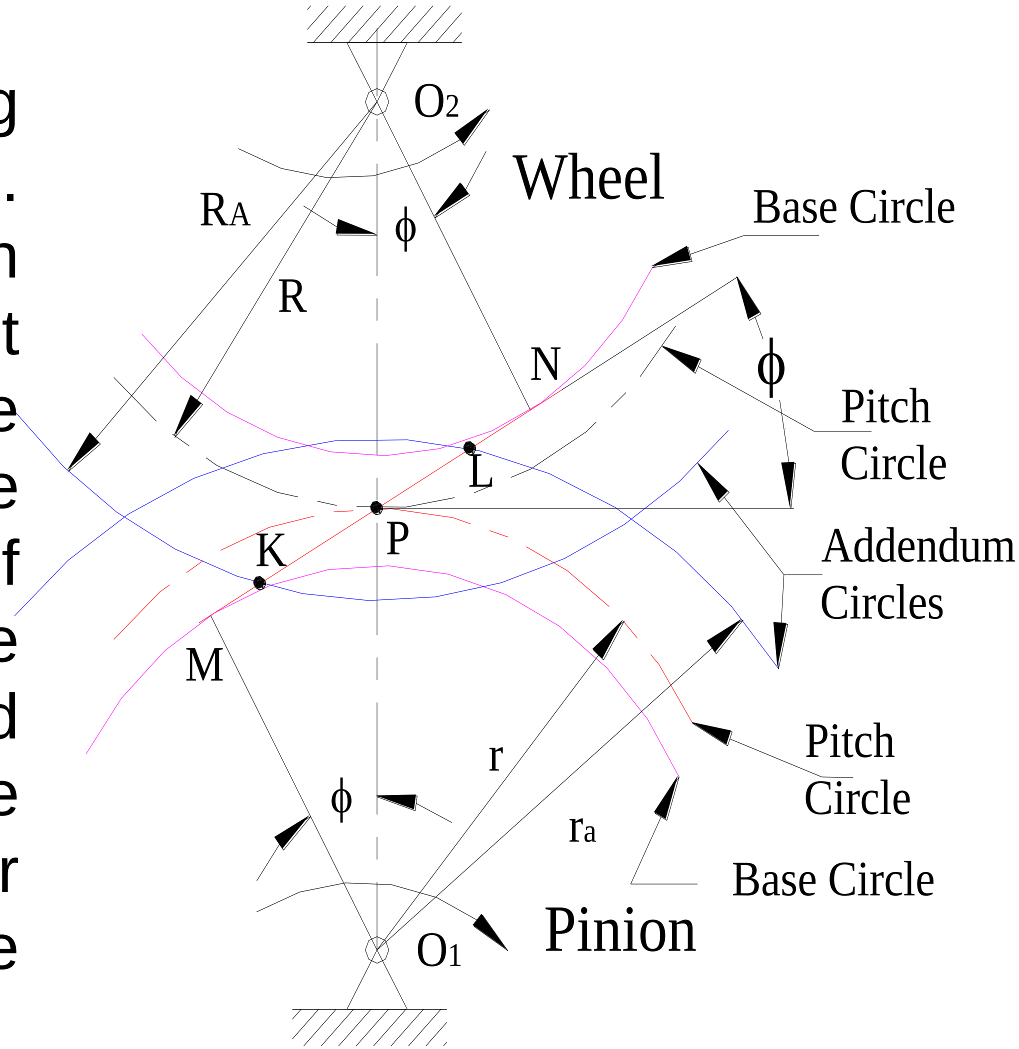
Length Of path of Contact:-

It is the length of common normal cut-off by the addendum circles of the wheel and the pinion



Path of contact

Consider a pinion driving wheel as shown in figure. When the pinion rotates in clockwise, the contact between a pair of involute teeth begins at *K* (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at *L* (outer end of the tooth face on the pinion or on the flank near the base circle of wheel).

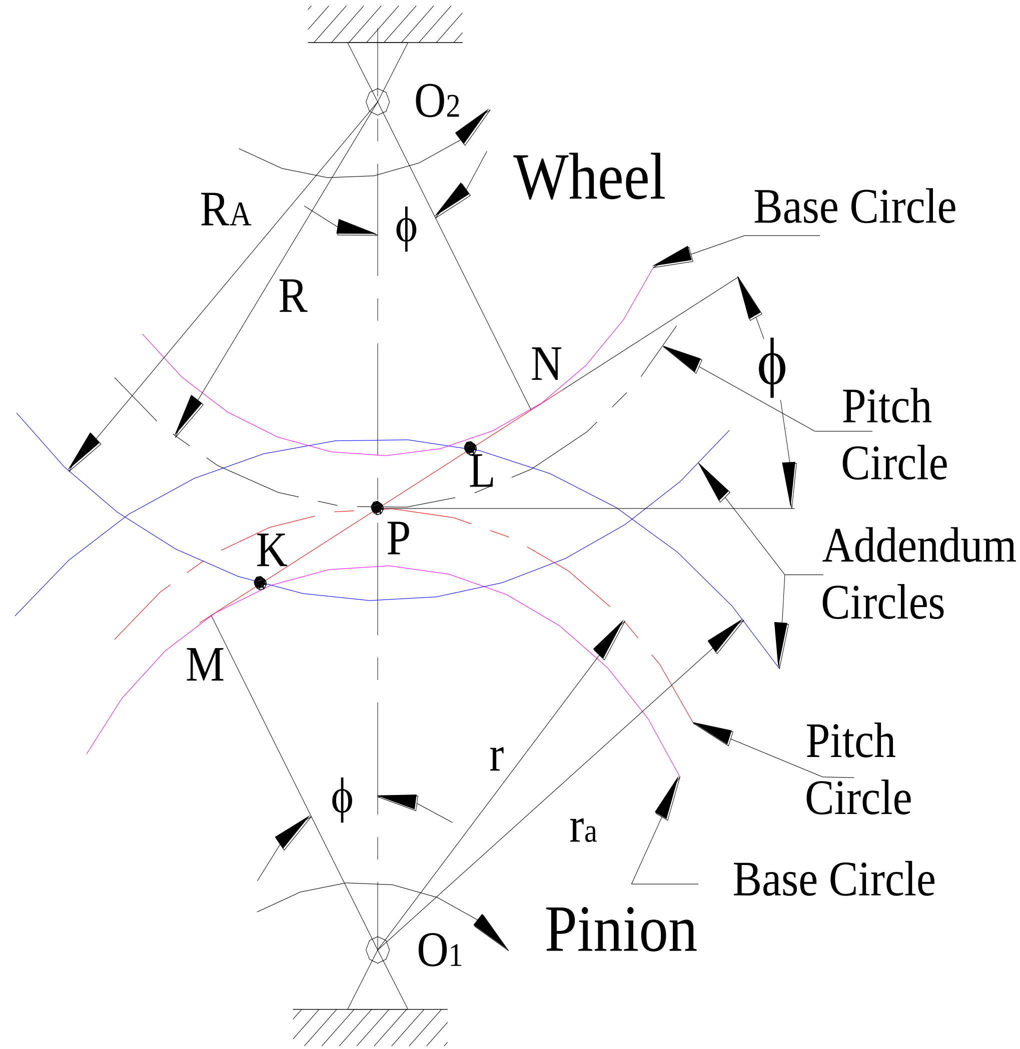


Path of contact

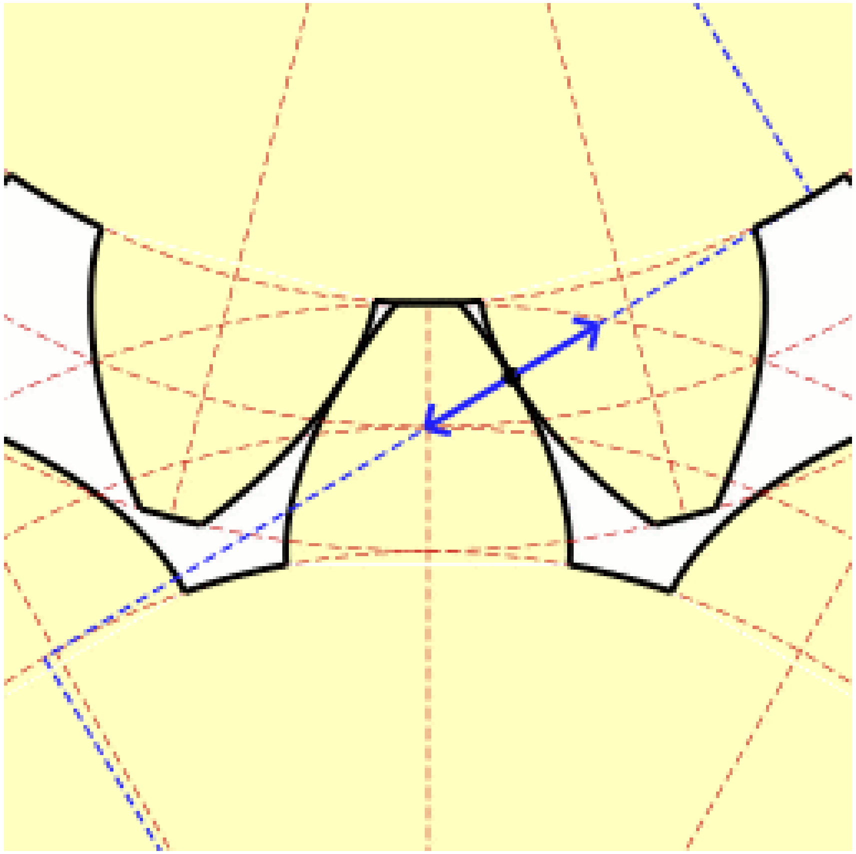
MN **-** common normal at the point of contacts and the common tangent to the base circles

K **-** Intersection of the addendum circle of wheel and the common tangent

L **-** Intersection of the addendum circle of pinion and common tangent

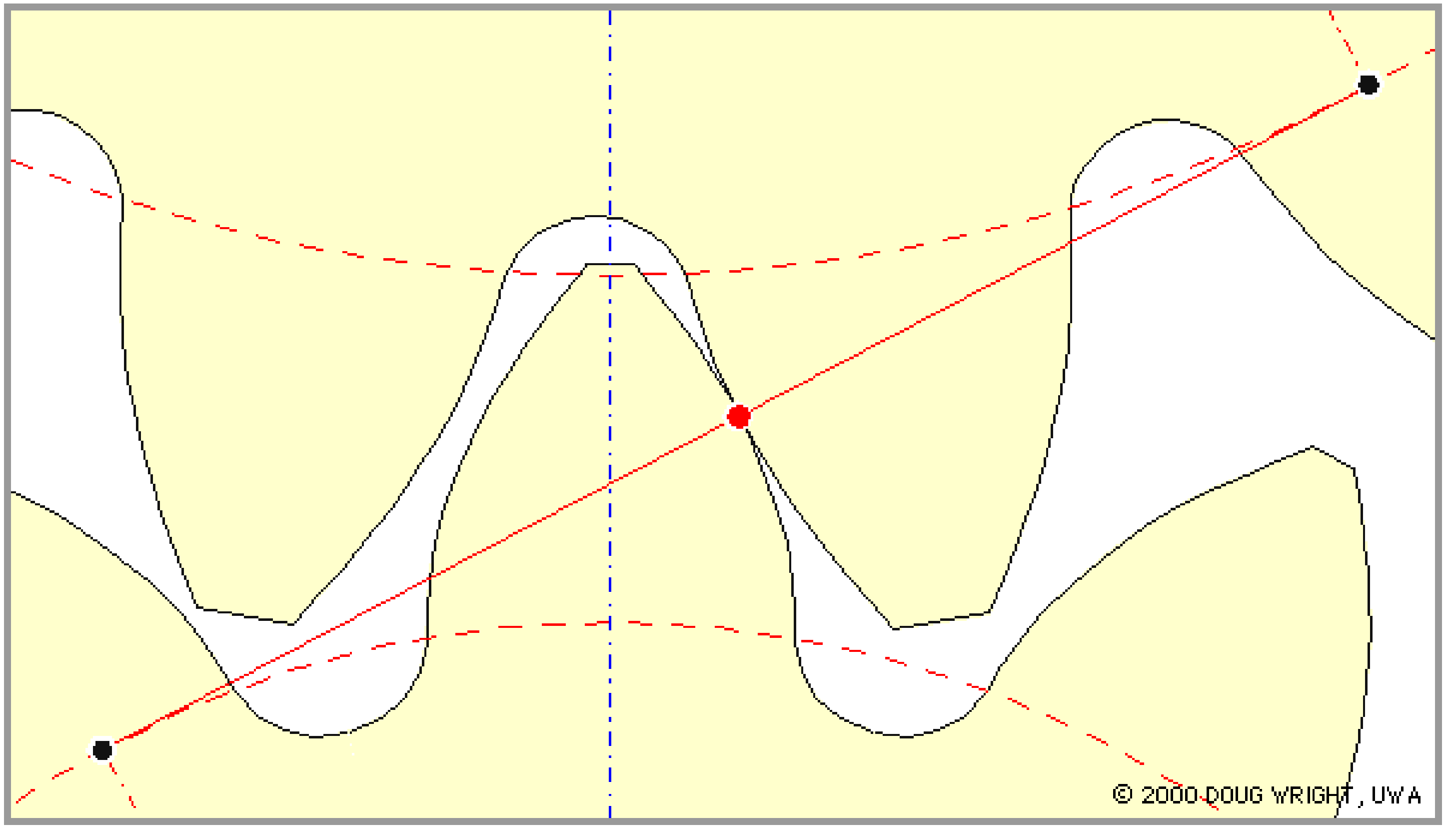


Path of contact



http://en.wikipedia.org/wiki/Image:Involute_wheel.gif

Path of contact



Path of contact

KL – Length of path of contact

KP - path of approach

PL - path of recess

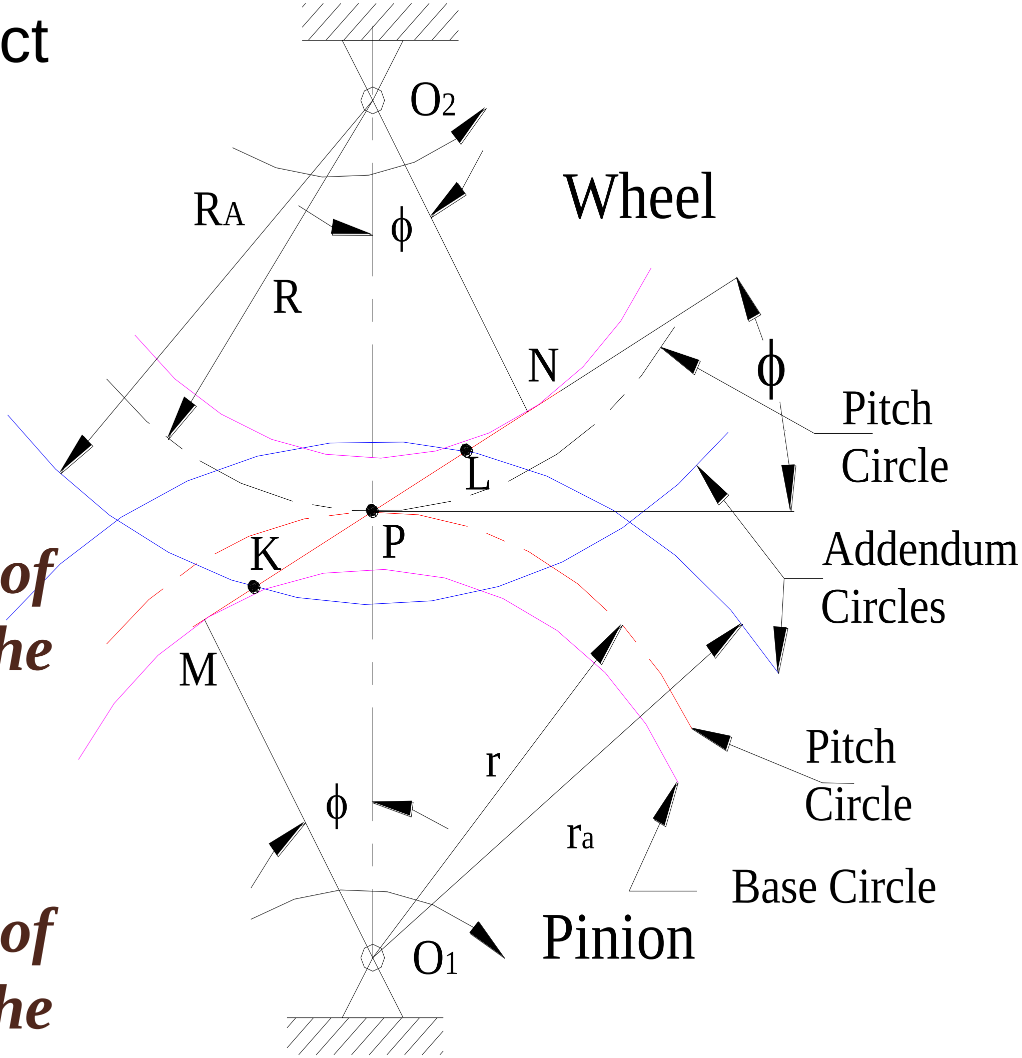
$$\boxed{KL = KP + PL}$$

Path of Approach :-

Portion of path of contact from pitch point to the End of engagement

Path of Recess :-

Portion of path of contact from pitch point to the End of engagement



Path of contact

$r_a = O_1L$

= Radius of addendum circle of pinion

&

$R_A = O_2K$

= Radius of addendum circle of wheel

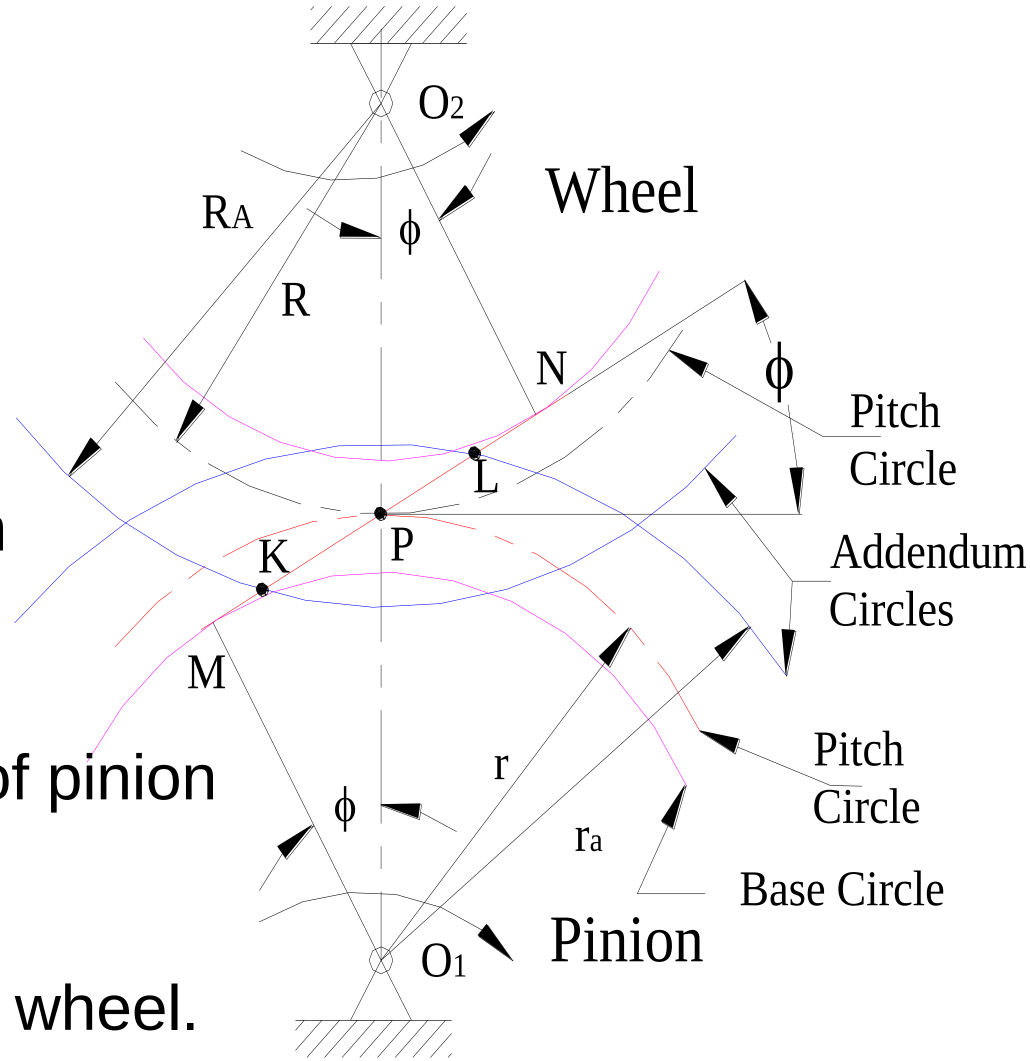
$r = O_1P$

= Radius of pitch circle of pinion

&

$R = O_2P$

= Radius of pitch cirle of wheel.



Path of contact

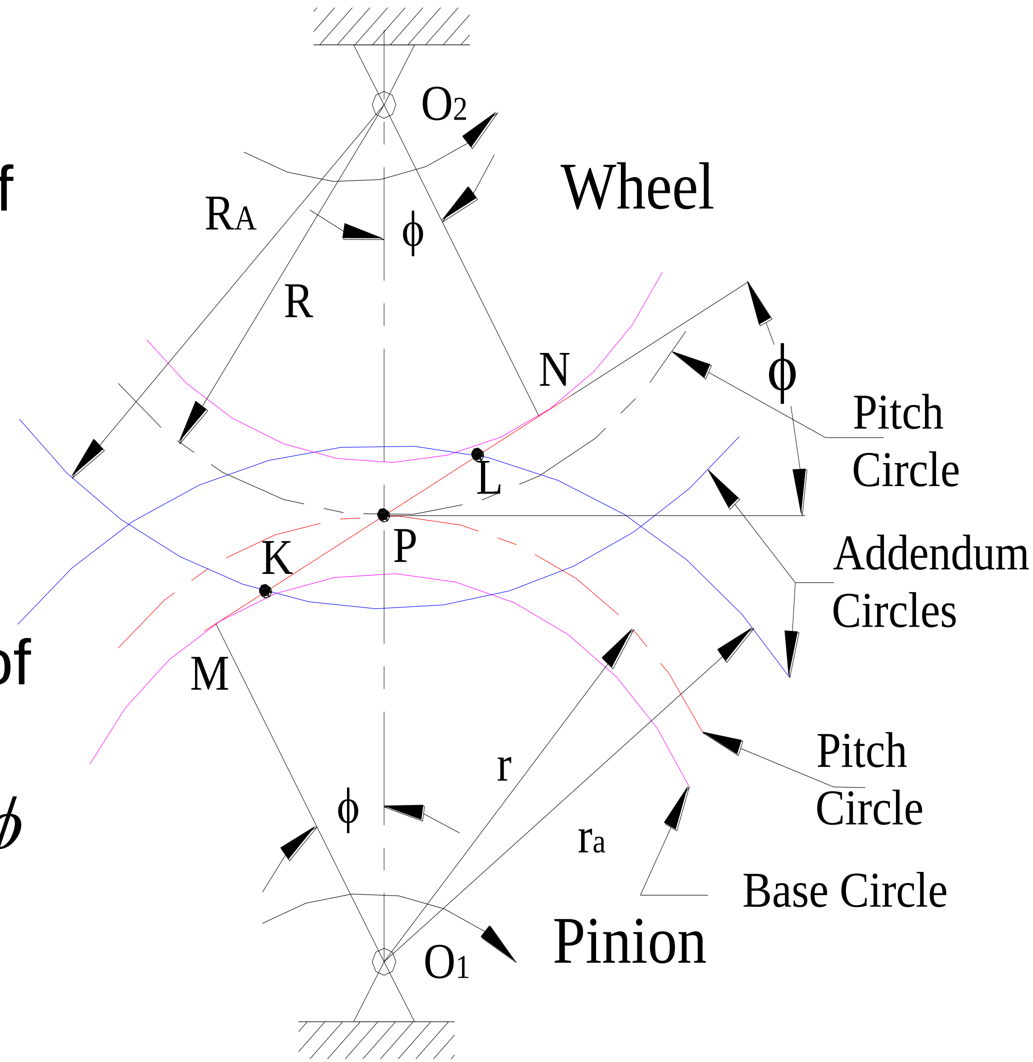
Radius of the base circle of pinion;

$O_1M = O_1P\cos\phi = r\cos\phi$

&

Radius of the base circle of wheel;

$O_2N = O_2P\cos\phi = R\cos\phi$



Path of contact

From right angle triangle O_2KN

$$KN = \sqrt{(O_2K)^2 - (O_2N)^2}$$

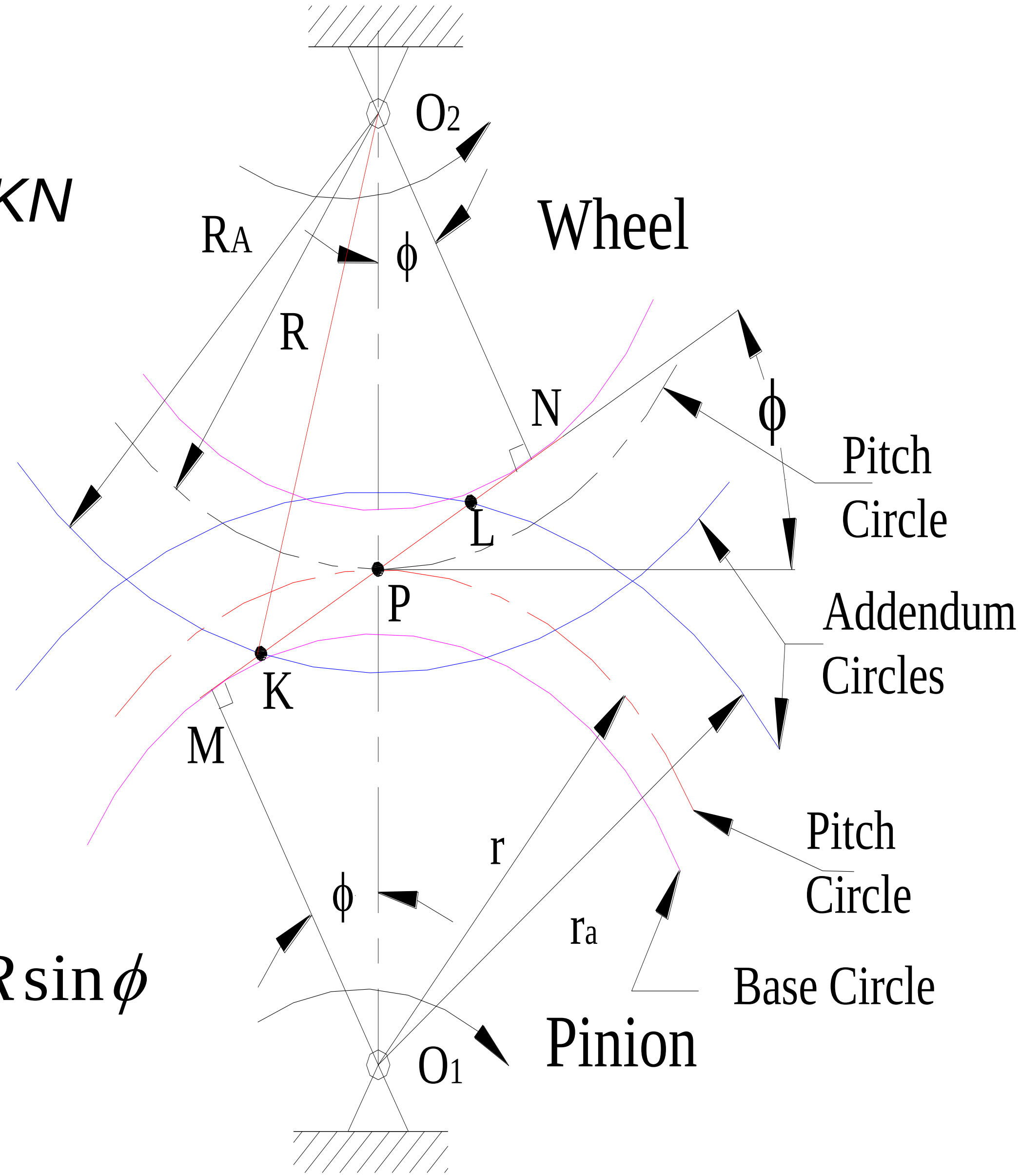
$$= \sqrt{(R_A)^2 - R^2\cos^2\phi}$$

$$PN = O_2P\sin\phi = R\sin\phi$$

Path of approach: *KP*

$$KP = KN - PN$$

$$= \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$



Path of contact

Similarly from right angle triangle O_1ML

$$ML = \sqrt{(O_1L)^2 - (O_1M)^2}$$

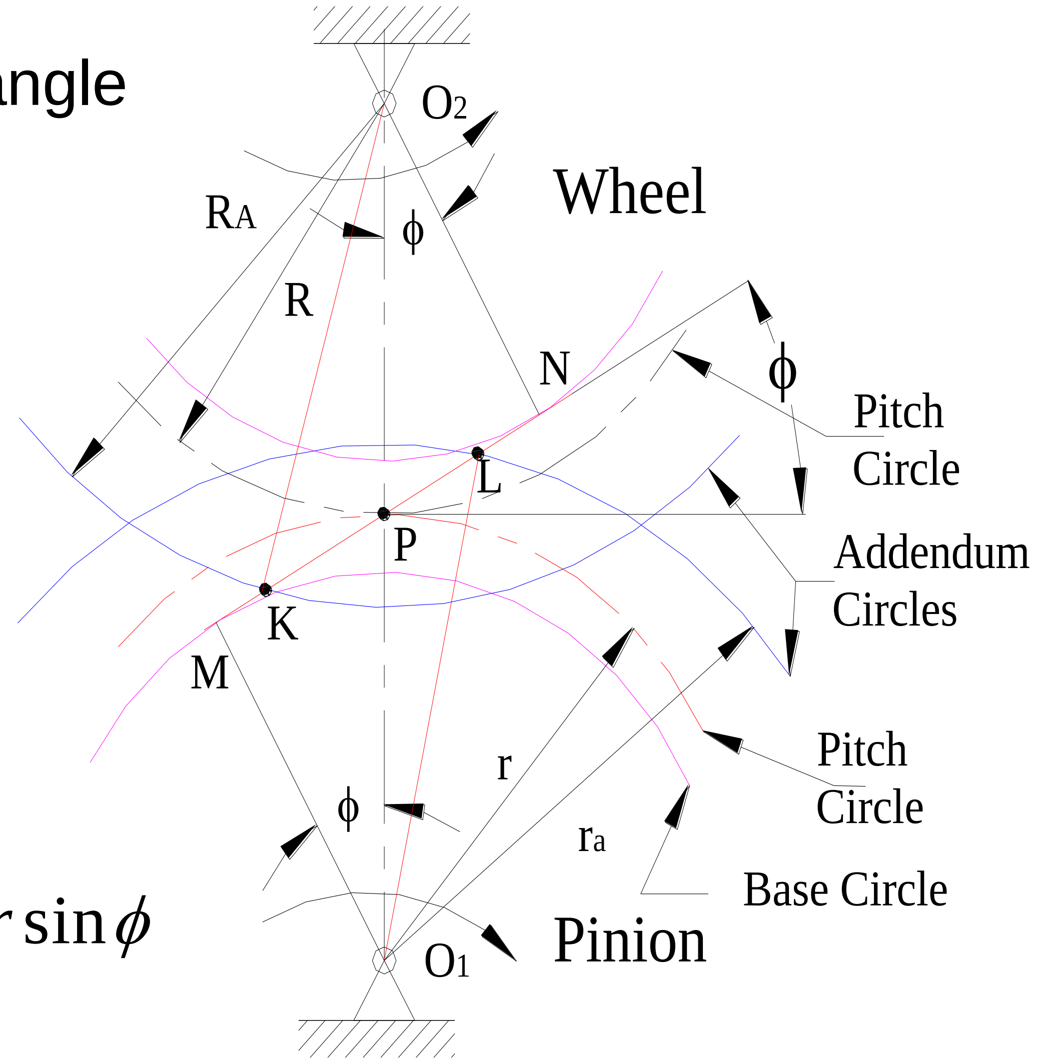
$$= \sqrt{(r_a)^2 - r^2\cos^2\phi}$$

$$MP = O_1P\sin\phi = r\sin\phi$$

Path of recess: *PL*

$$PL = ML - MP$$

$$= \sqrt{(r_a)^2 - r^2\cos^2\phi} - r\sin\phi$$



Path of contact

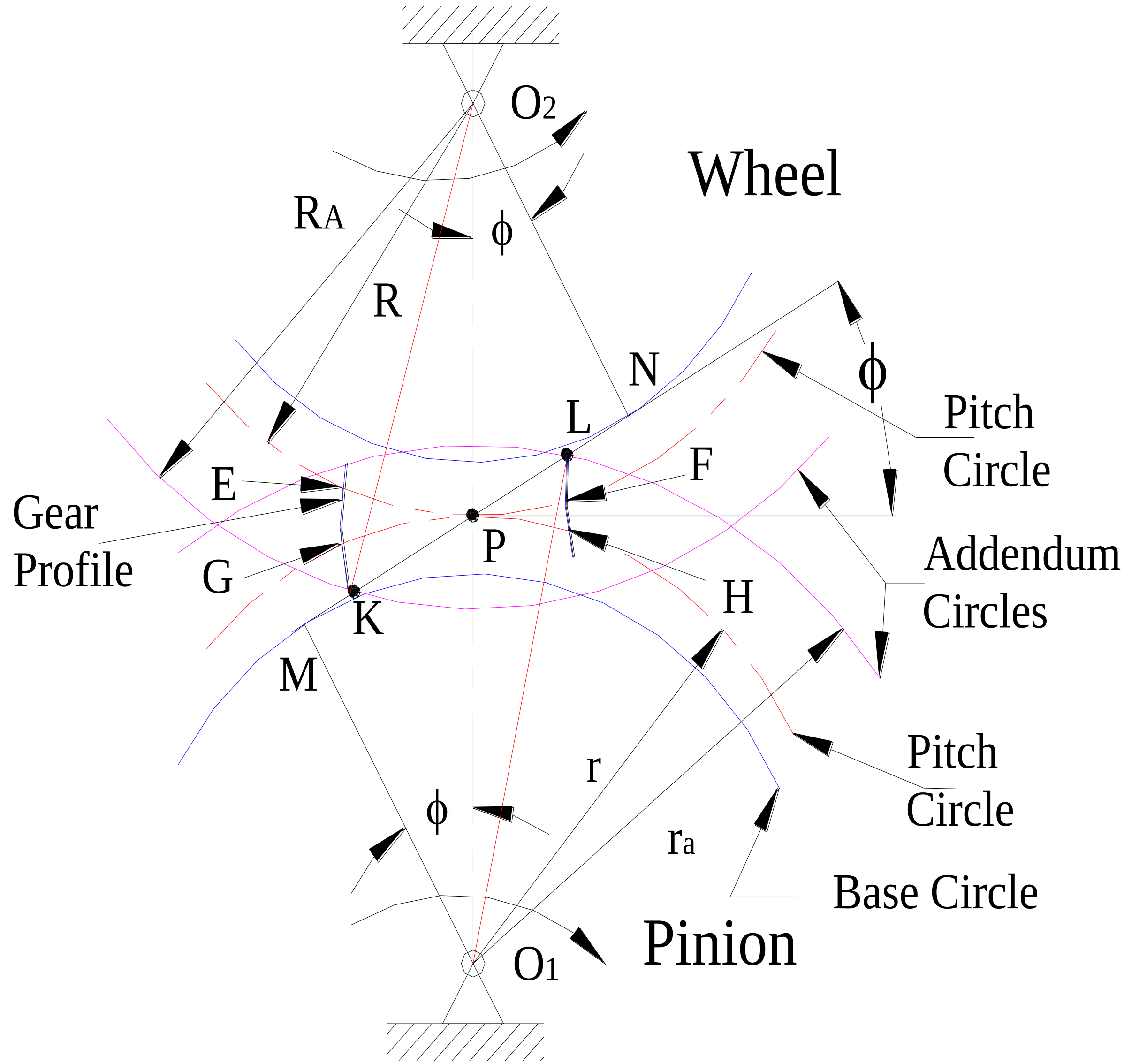
Length of path of contact = *KL*

$$KL = KP + PL$$

$$= \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_a)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

Arc of contact

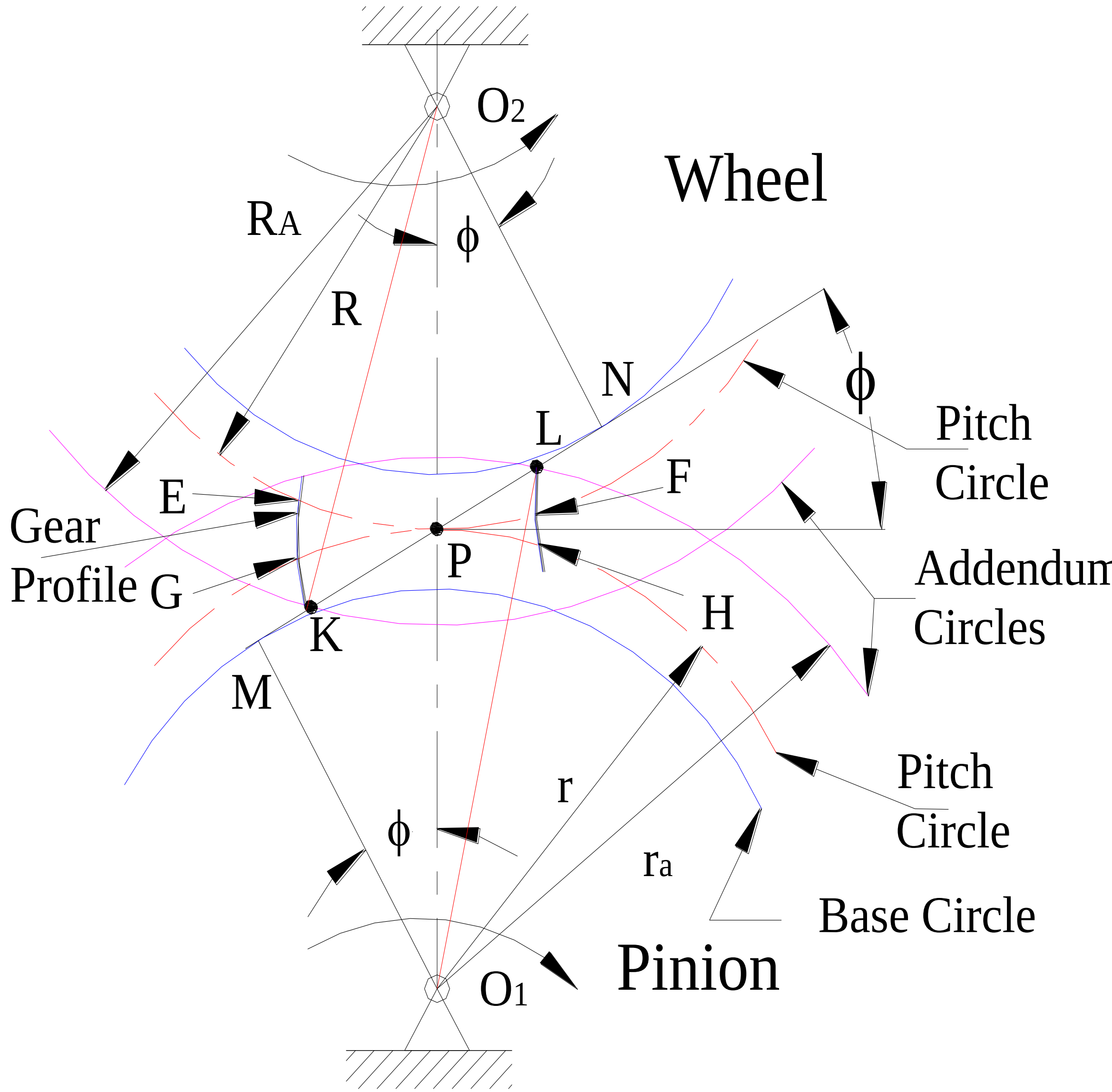
Arc of contact is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth. In Figure, the arc of contact is *EPF* or *GPH*.



Arc of contact

It is the path traced by a point on the pitch circle from the beginning to the end of engagement of a given pair of teeth.

arcEPF & arcGPH



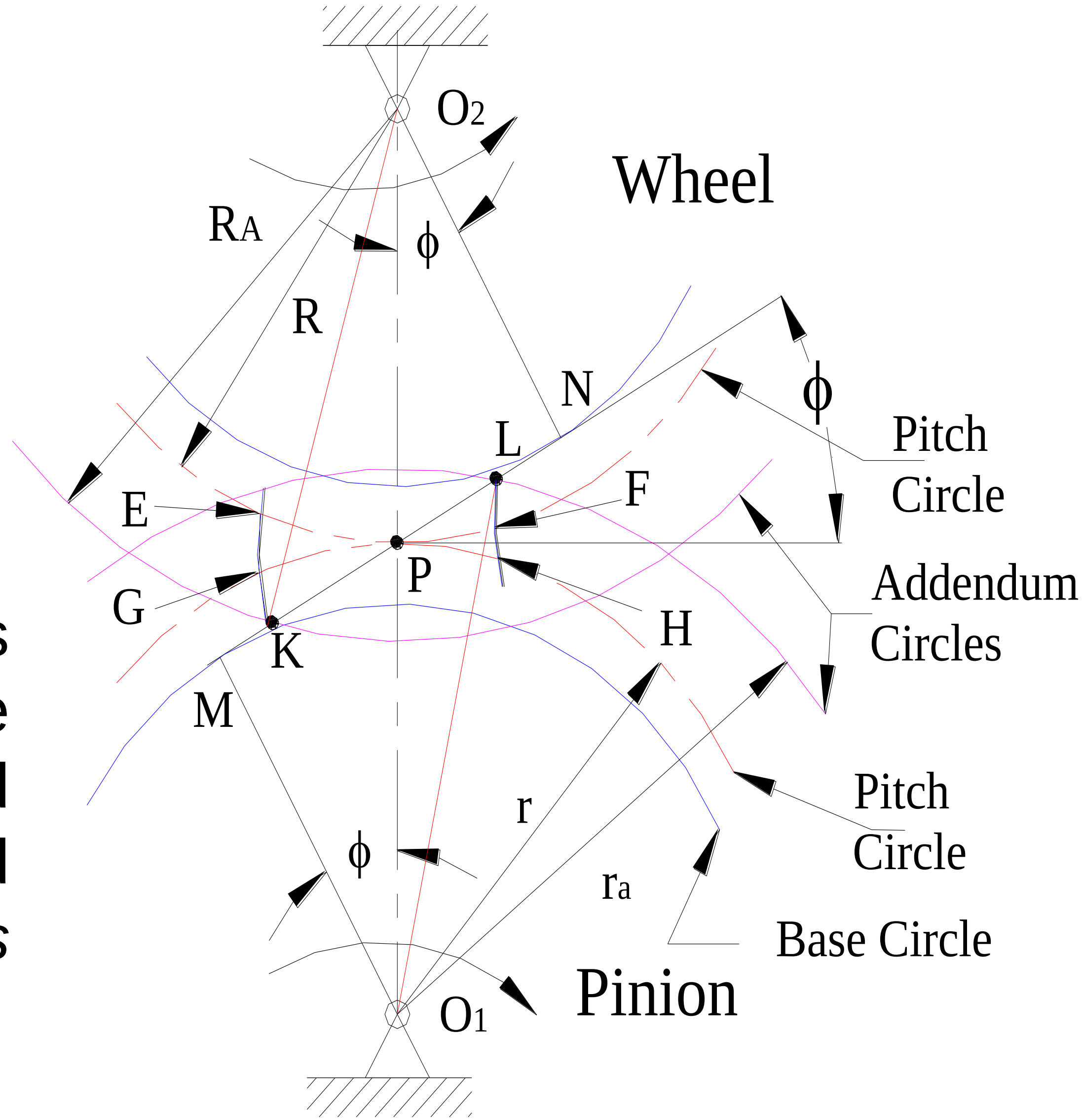
Arc of contact

GPH - arc of contact

GP - arc of approach

PH - arc of recess

The angles subtended by these arcs at O_1 are called *angle of approach* and *angle of recess* respectively.



Arc of contact

Length of arc of approach = arc *GP*

$$= \frac{Lenghtof\ pathof\ approach}{\cos\phi} = \frac{KP}{\cos\phi}$$

Length of arc of recess = arc *PH*

$$= \frac{Lenghtof\ pathof\ recess}{\cos\phi} = \frac{PL}{\cos\phi}$$

Length of arc contact = arc *GPH = arc GP + arc PH*

$$= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi} = \frac{KL}{\cos\phi} = \frac{Lengthof\ pathof\ contact}{\cos\phi}$$

Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the **ratio of the length of the arc of contact to the circular pitch**.

Mathematically,

$$Contatratio = \frac{Length\ of\ the\ arc\ of\ contact}{P_C}$$

Where:

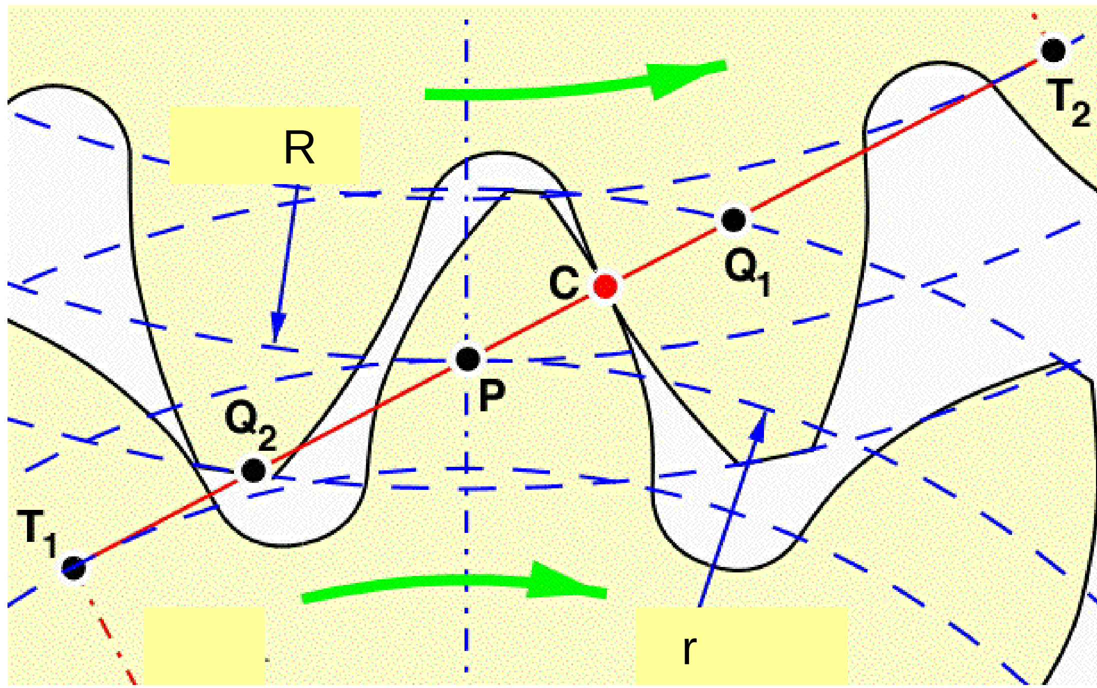
$$P_C = Circular\ pitch = \pi \times m$$

And

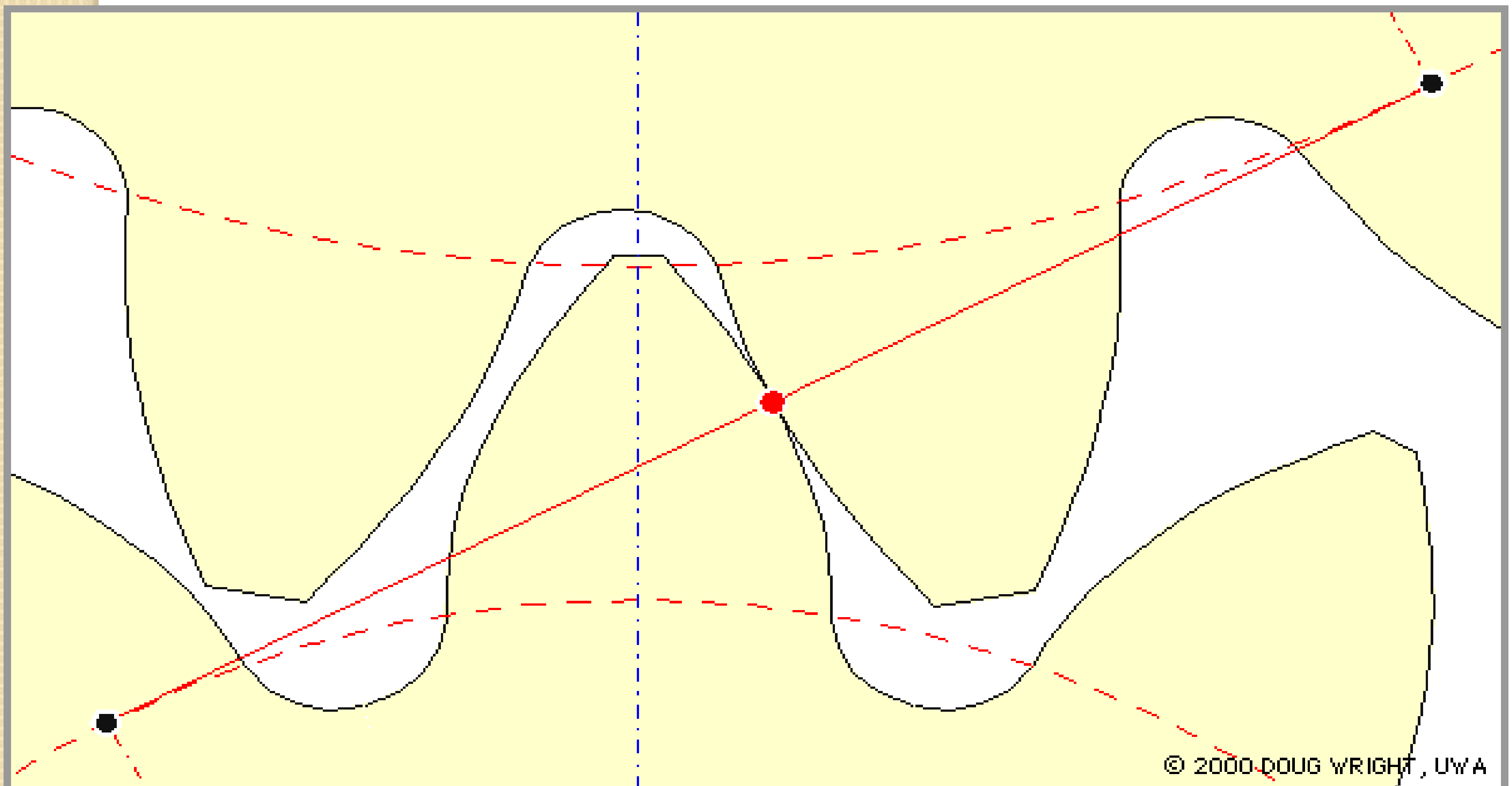
m = Module.

Number of Pairs of Teeth in Contact

Continuous motion transfer requires **two pairs of teeth in contact at the ends of the path of contact**, though there is only one pair in contact in the middle of the path, as in Figure.

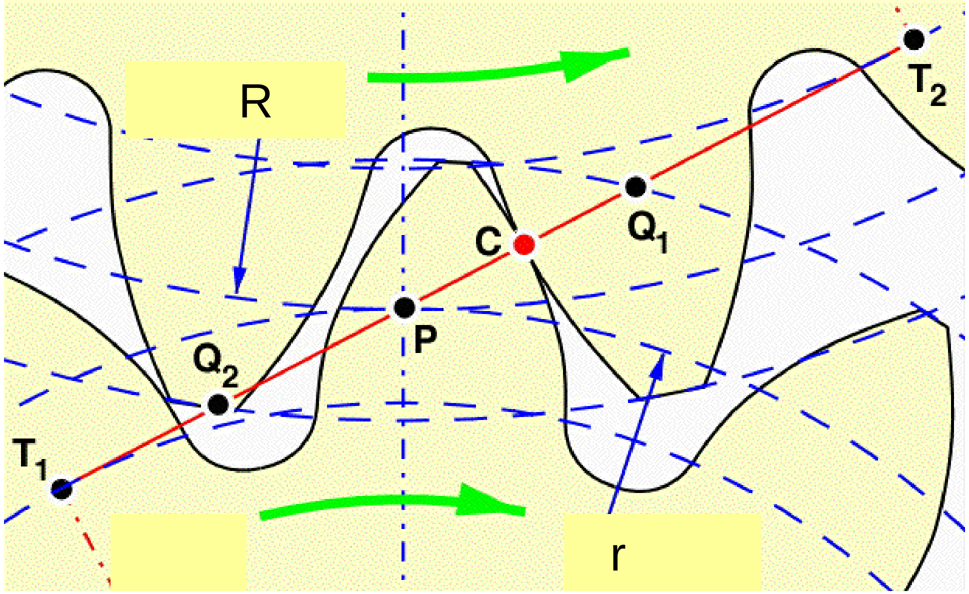


Number of Pairs of Teeth in Contact



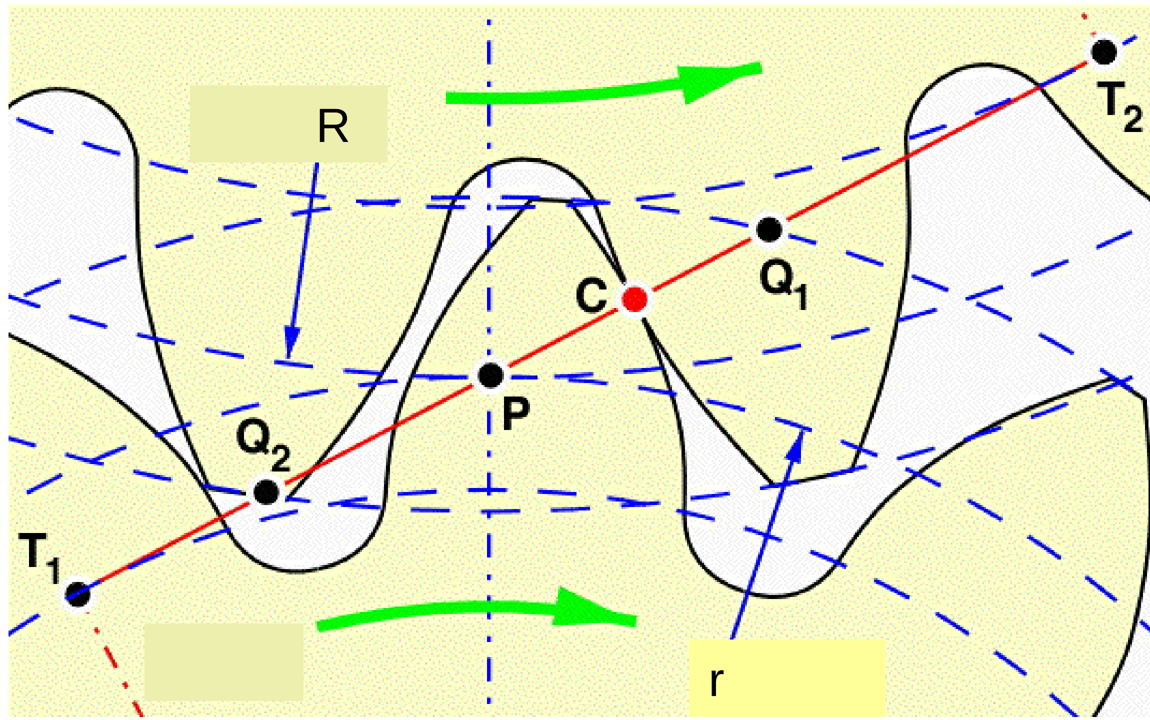
Number of Pairs of Teeth in Contact

The average number of teeth in contact is an important parameter - if it is too low due to the use of inappropriate profile shifts or to an excessive centre distance.The manufacturing inaccuracies may lead to loss of kinematic continuity - that is to impact, vibration and noise.



Number of Pairs of Teeth in Contact

The average number of teeth in contact is also a guide to load sharing between teeth; it is termed the ***contact ratio***

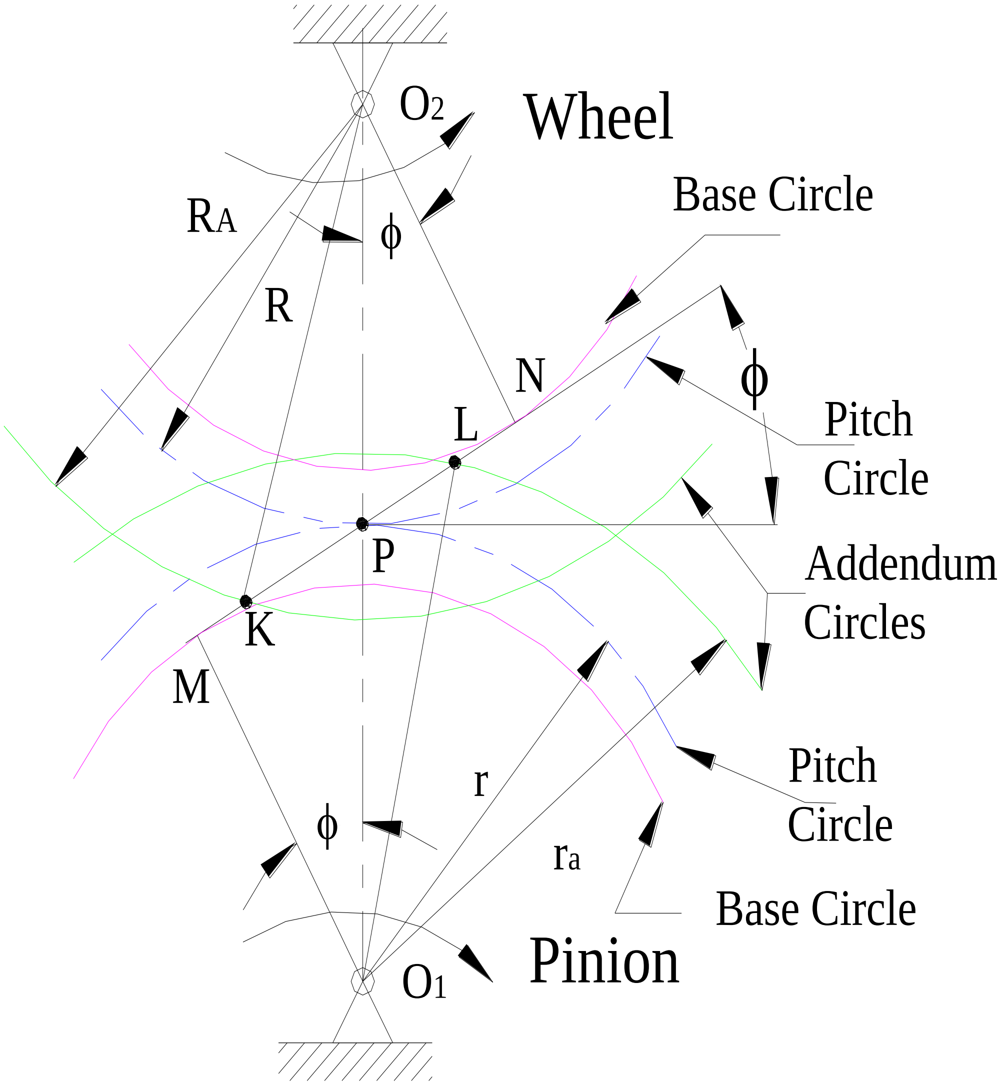


Interference in Involute Gears

O1andO2 – centers of pinion and a gear in mesh

MN - common tangent to the base circle

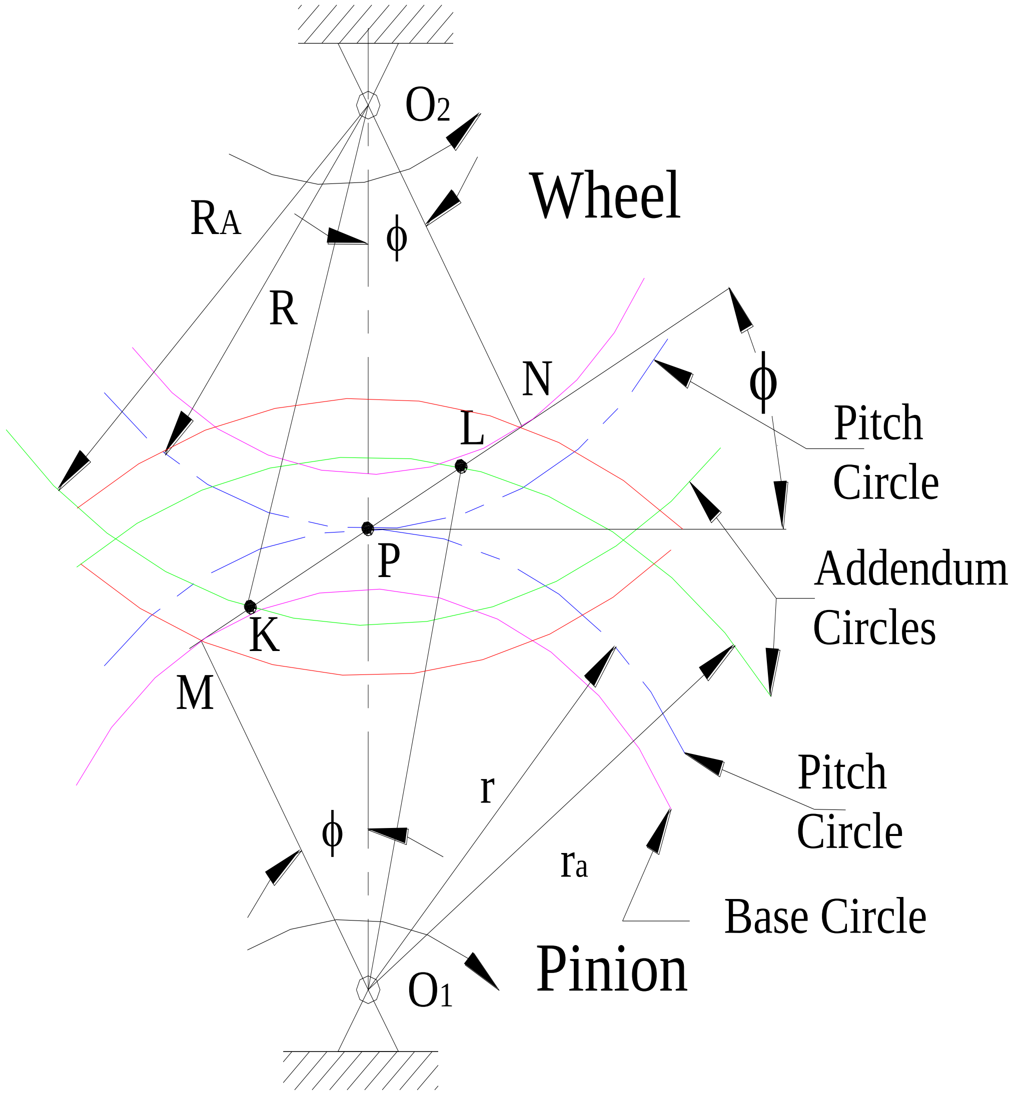
KL-path of contact between two mating teeth.



Interference in Involute Gears

Consider, the radius of the addendum circle of pinion is increased to O_1N, the point of contact *L* will moves from *L* to *N*.

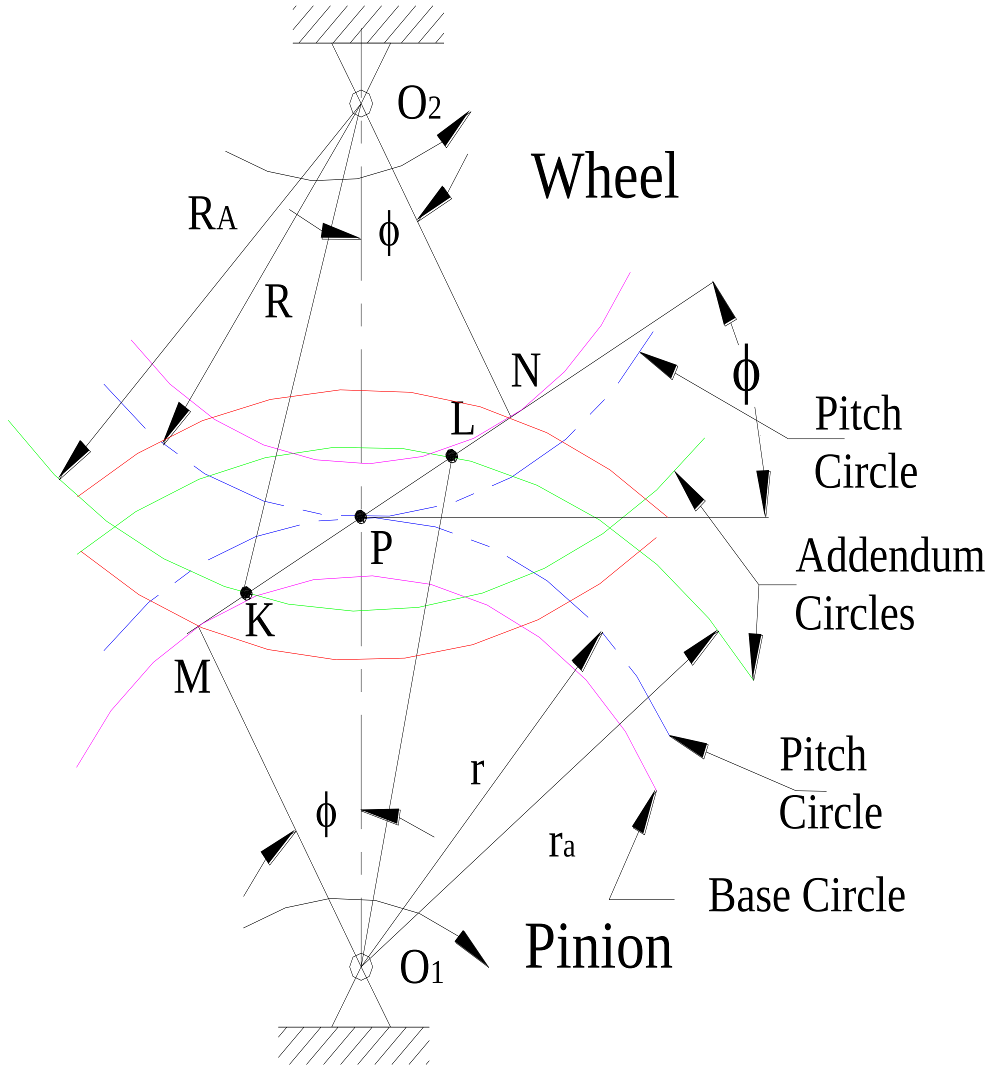
If this radius is further increased, the point of contact *L* will be inside of base circle of wheel and not on the involute profile of the pinion.



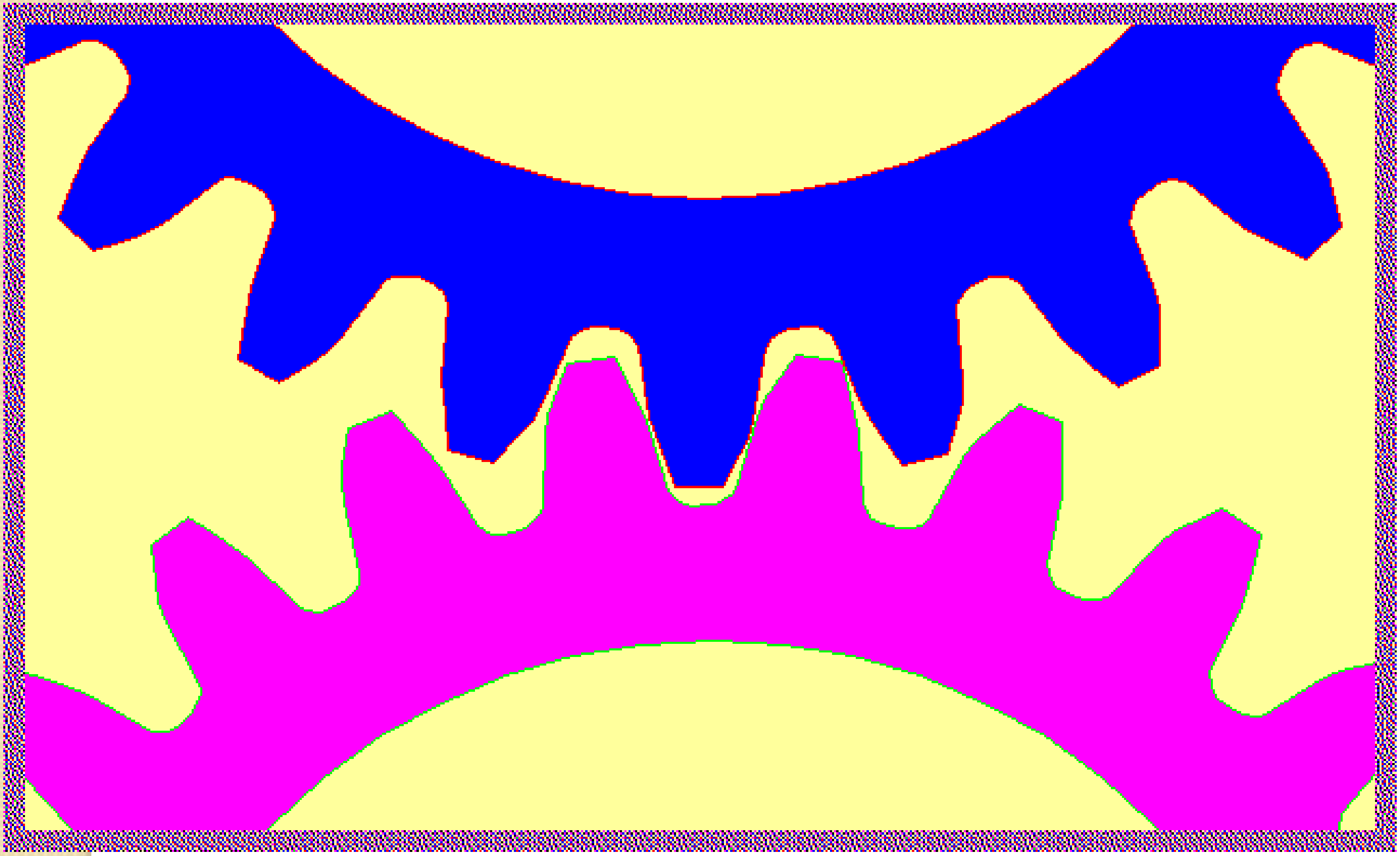
Interference in Involute Gears

The tooth tip of the pinion will then undercut the tooth on the wheel at the root and damages part of the involute profile. This effect is known as interference…….,

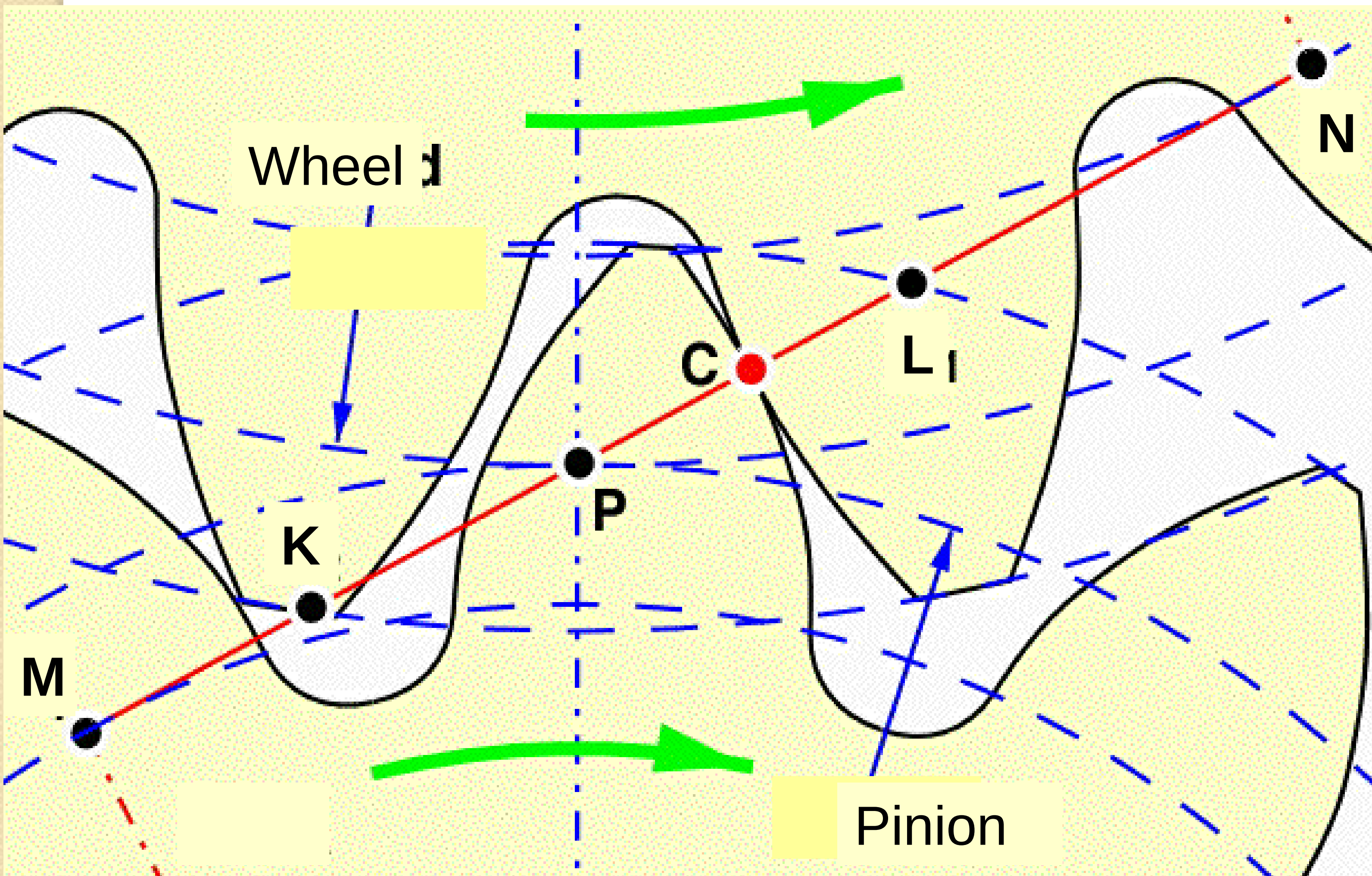
……..and occurs when the teeth are being cut and weakens the tooth at its root.



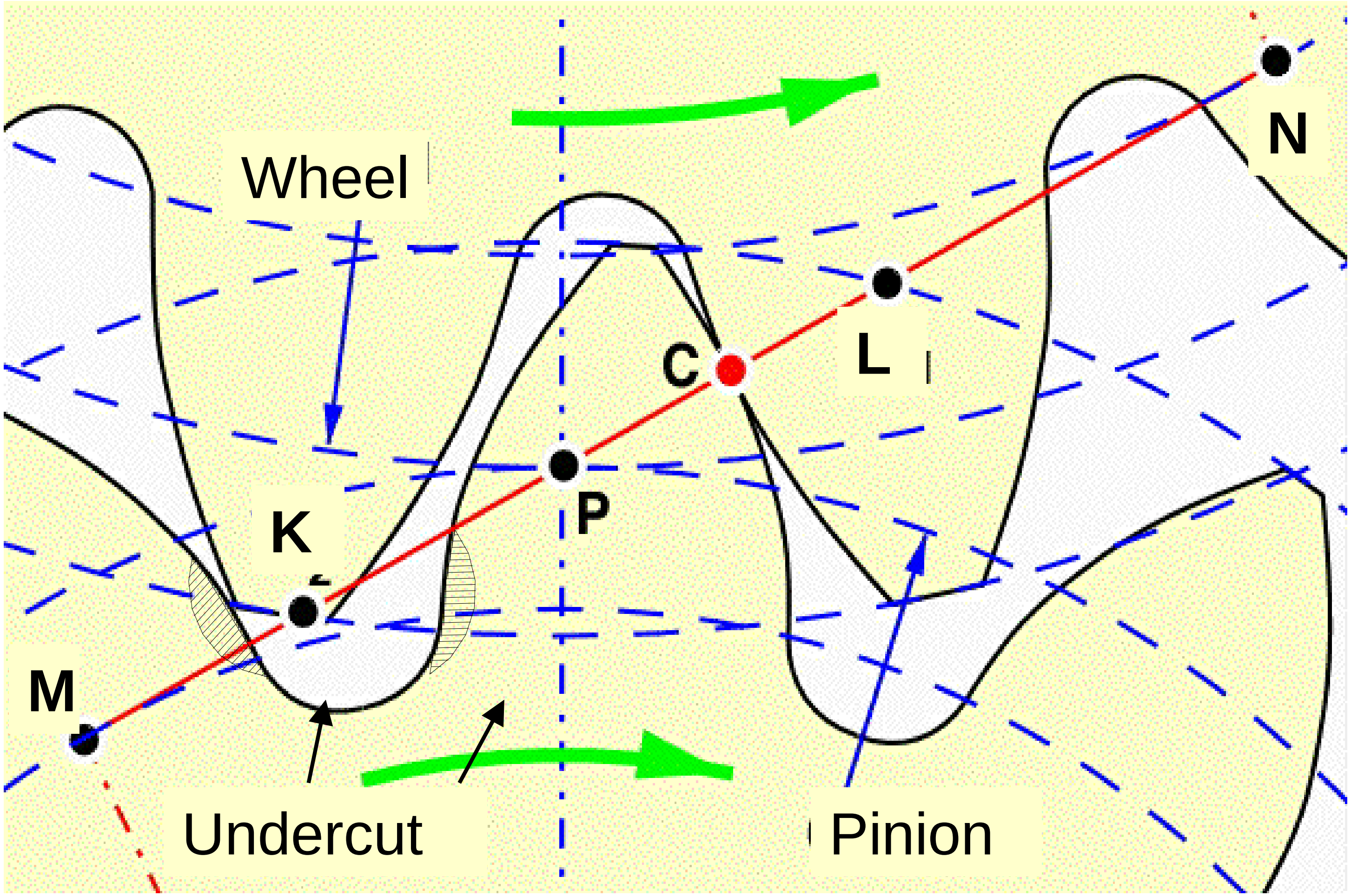
Interference in Involute Gears



Interference in Involute Gears

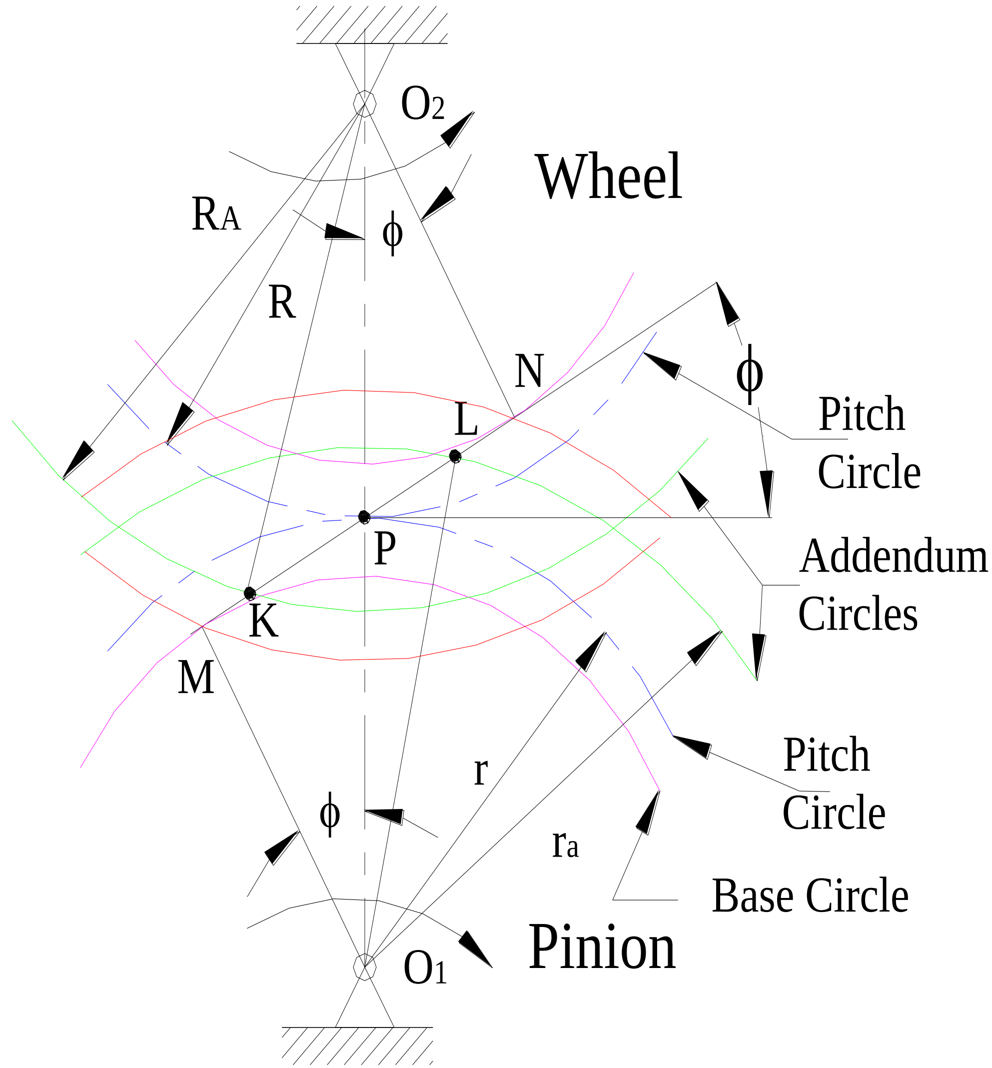


Interference in Involute Gears



Interference in Involute Gears

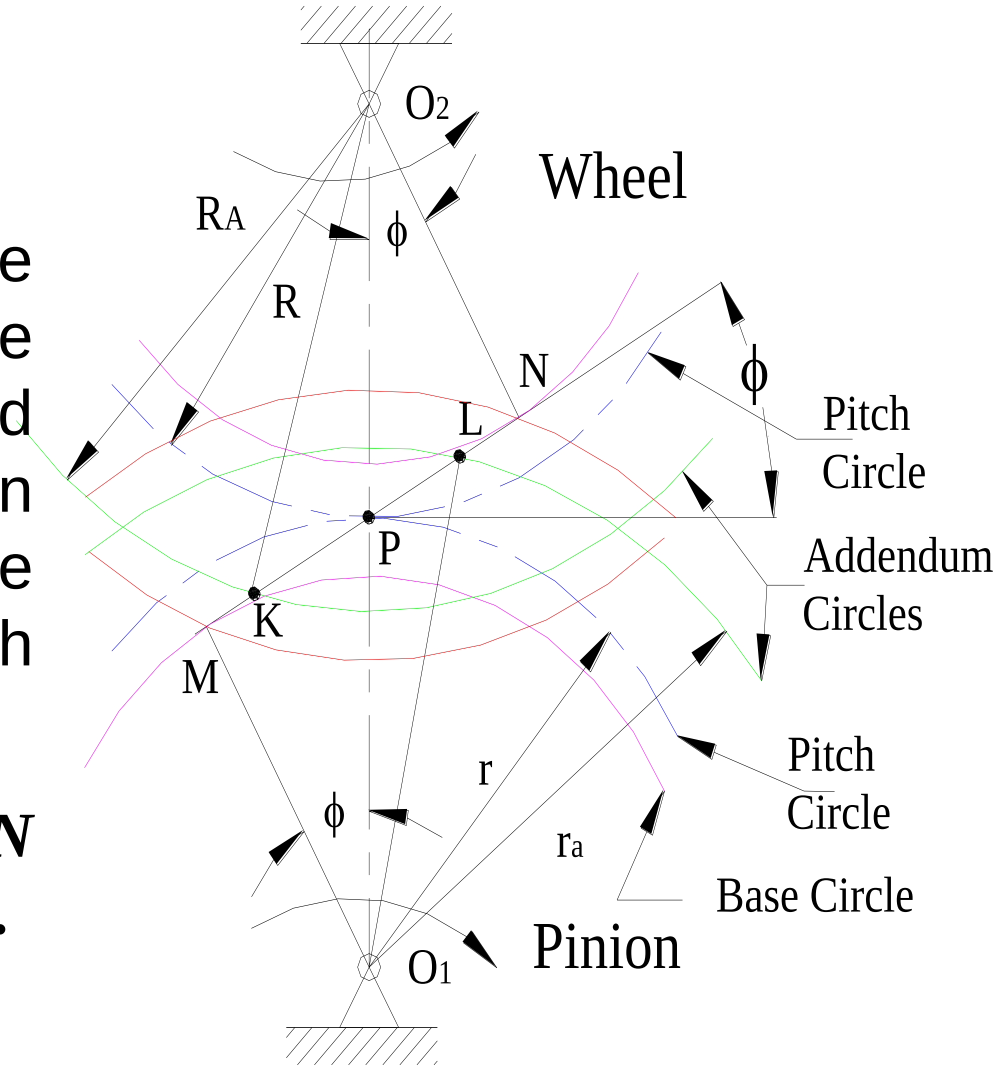
The phenomenon, when the tip of tooth undercuts the root on its mating gear is known as interference.



Interference in Involute Gears

Similarly, if the radius of the addendum circles of the wheel increases beyond O_2M, then the tip of tooth on wheel will cause interference with the tooth on pinion.

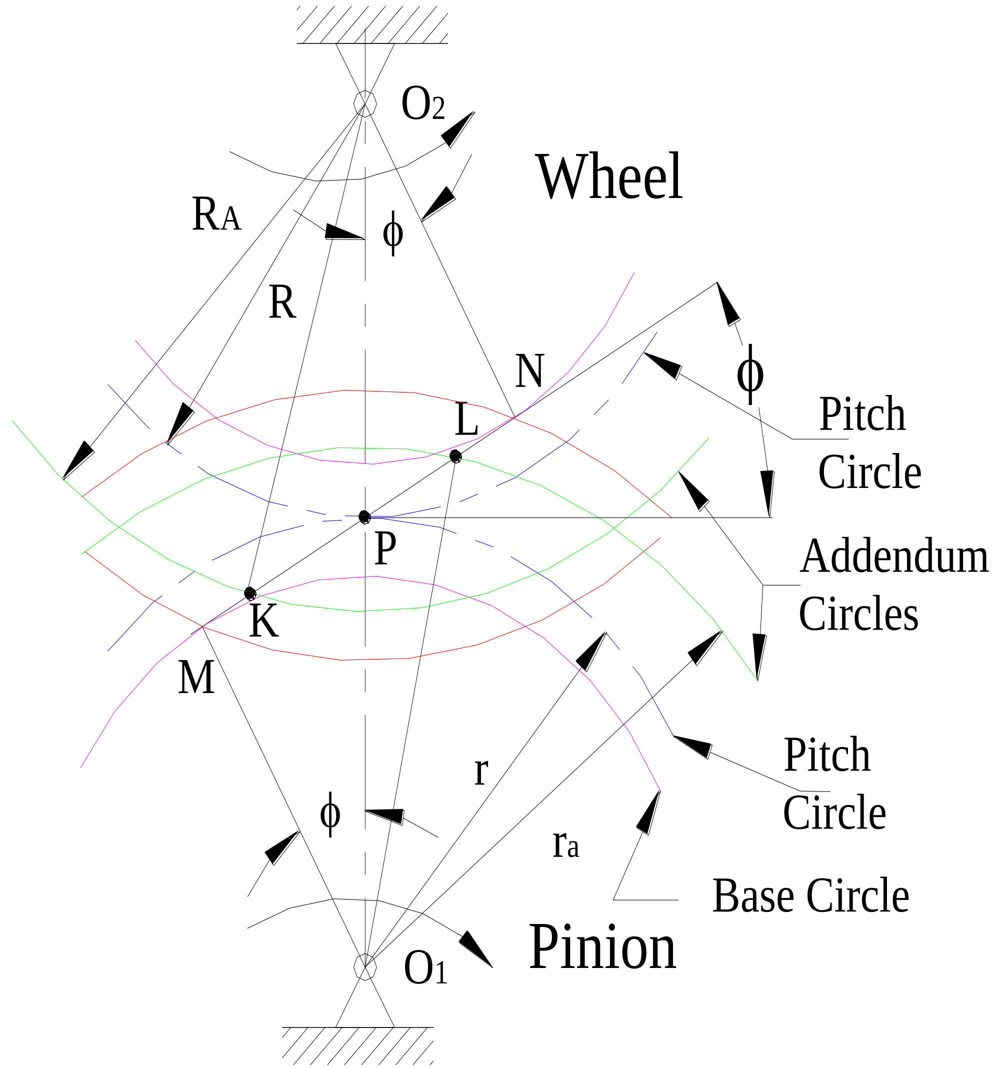
The points M and N are called interference points.



Interference in Involute Gears

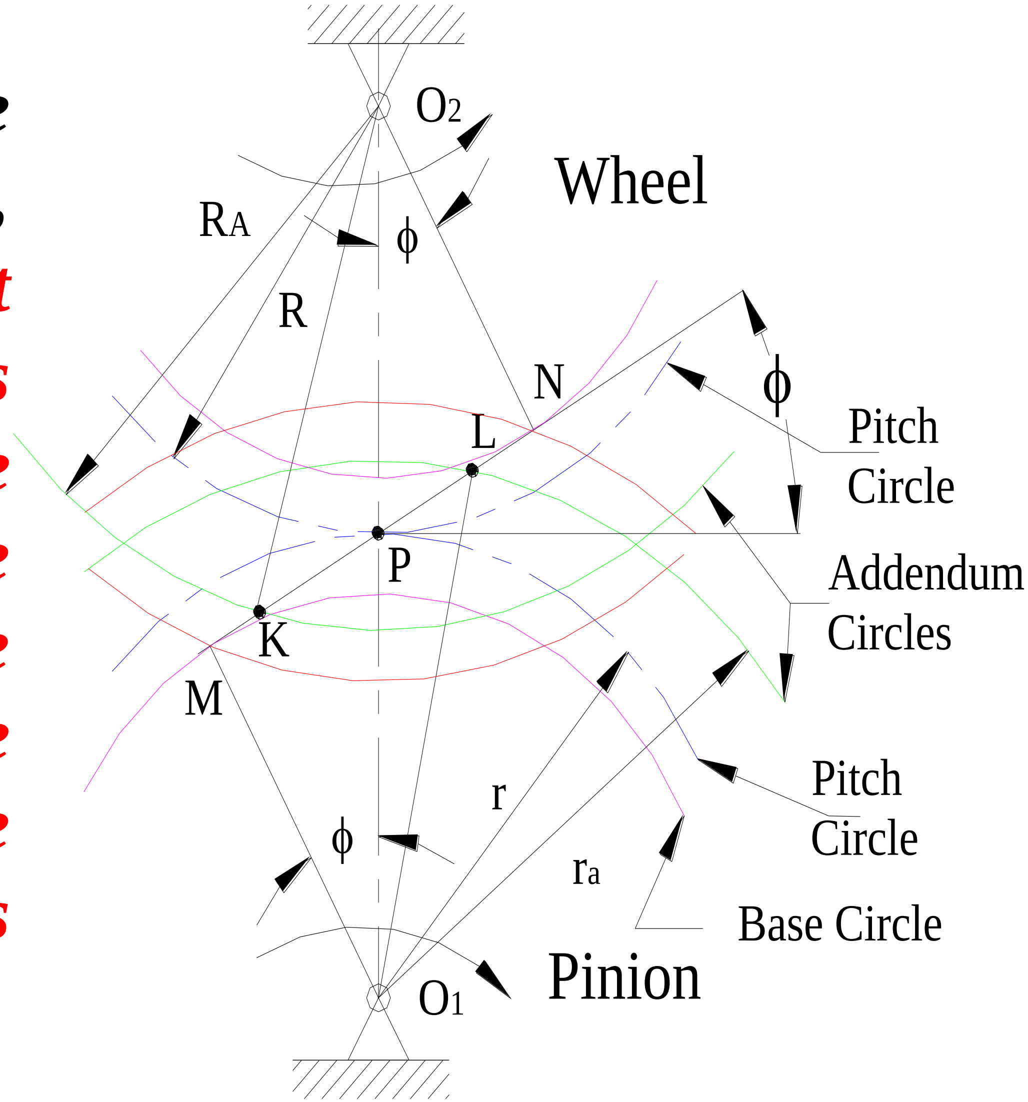
Interference may be avoided if the path of the contact does not extend beyond interference points.

The ***limiting value*** of the radius of the addendum circle of the pinion is $\boldsymbol{O_1N}$ and of the wheel is $\boldsymbol{O_2M}$.



Interference in Involute Gears

The interference may only be prevented, if the point of contact between the two teeth is always on the involute profiles and if the addendum circles of the two mating gears cut the common tangent to the base circles at the points of tangency.



Interference in Involute Gears

When interference is just prevented, the maximum length of path of contact is MN.

$$Maximum\ path of\ approach = MP = r\sin\phi$$

$$Maximum\ path of\ recess = PN = R\sin\phi$$

$$Maximum\ length of\ path of\ contact = MN$$

$$MN = MP + PN = (r+R)\sin\phi$$

$$Maximum\ length of\ arc of\ contact:$$

$$= \frac{(r+R)\sin\phi}{\cos\phi} = (r+R)\tan\phi$$

Methods to avoid Interference

1. Height of the teeth may be reduced.

2. Under cut of the radial flank of the pinion.

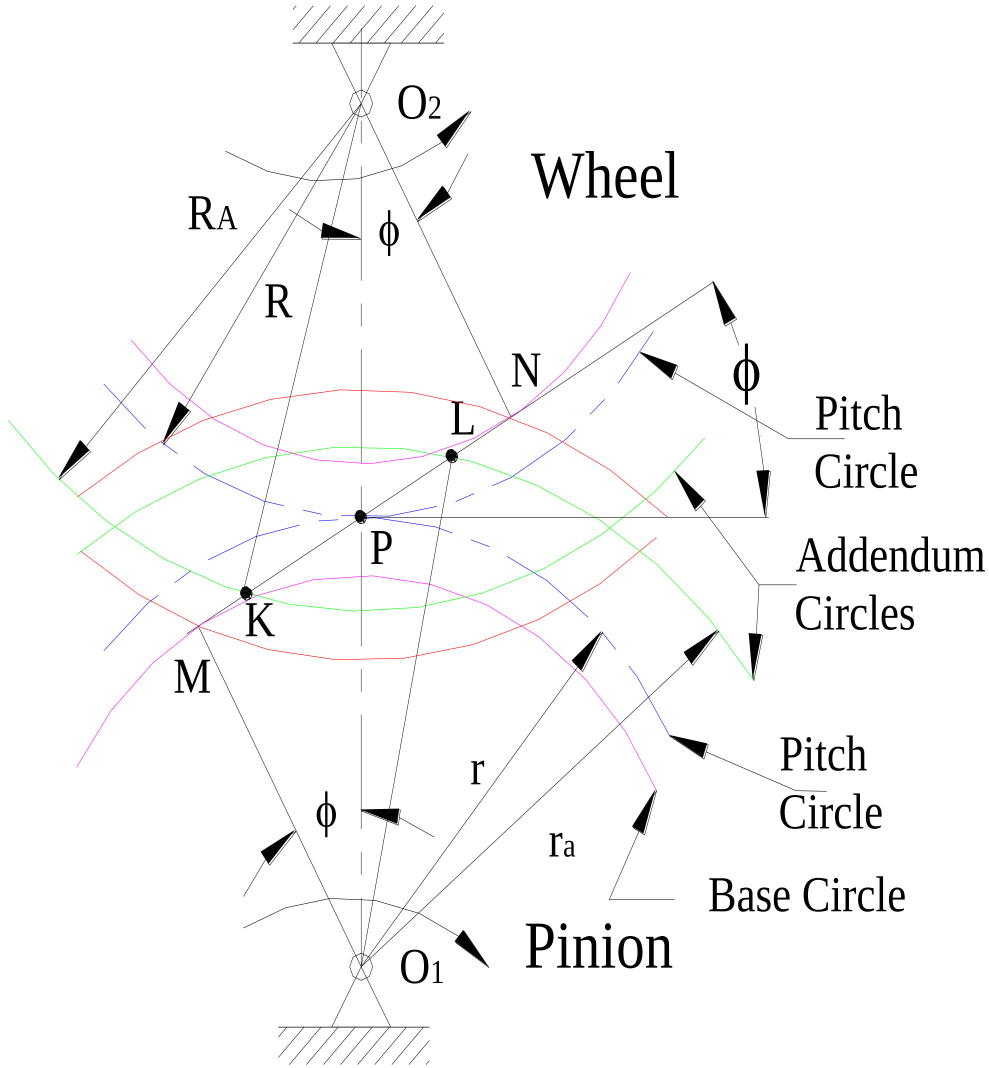
3. Centre distance may be increased. It leads to increase in pressure angle.

4. By tooth correction, the pressure angle, centre distance and base circles remain unchanged, but tooth thickness of gear will be greater than the pinion tooth thickness.

Minimum number of teeth on the pinion avoid Interference

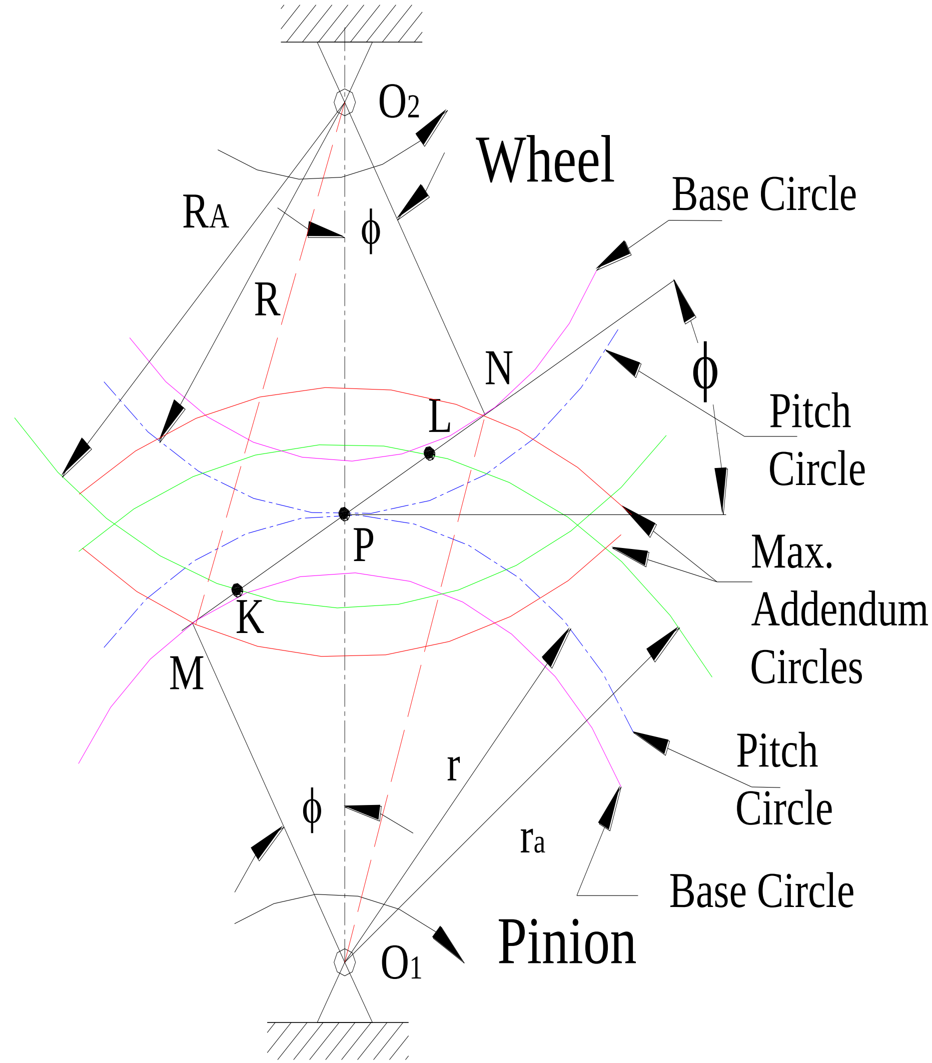
The pinion turns clockwise and drives the gear as shown in Figure.

Points M and N are called interference points. i.e., if the contact takes place beyond M and N, interference will occur.



Minimum number of teeth on the pinion avoid Interference

The **limiting value** of addendum circle radius of pinion is **O_1N** and the limiting value of addendum circle radius of gear is **O_2M**. Considering the critical addendum circle radius of gear, the limiting number of teeth on gear can be calculated.



Minimum number of teeth on the pinion avoid Interference

Let

Φ = pressure angle

R = pitch circle radius of gear

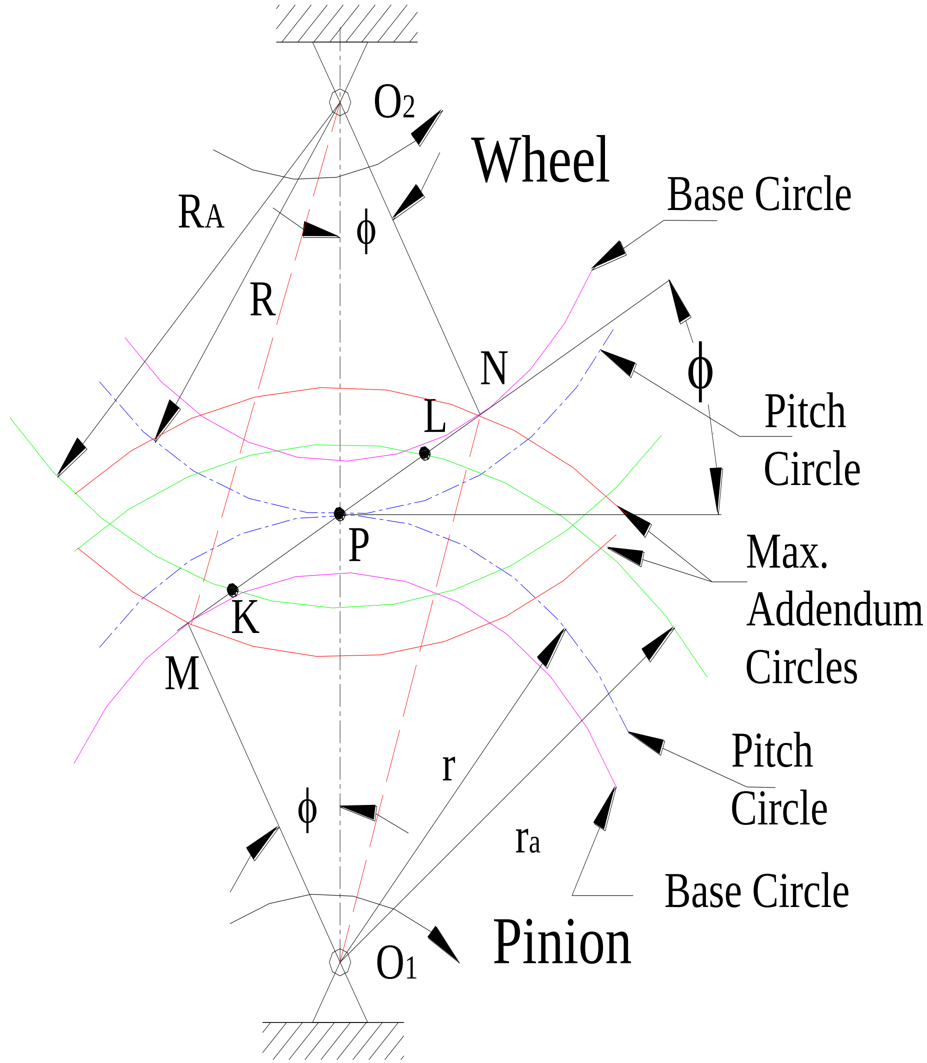
= ½*mT*

r = pitch circle radius of pinion

= ½*mt*

T & t = number of teeth on gear & pinion

m = module



Minimum number of teeth on the pinion avoid Interference

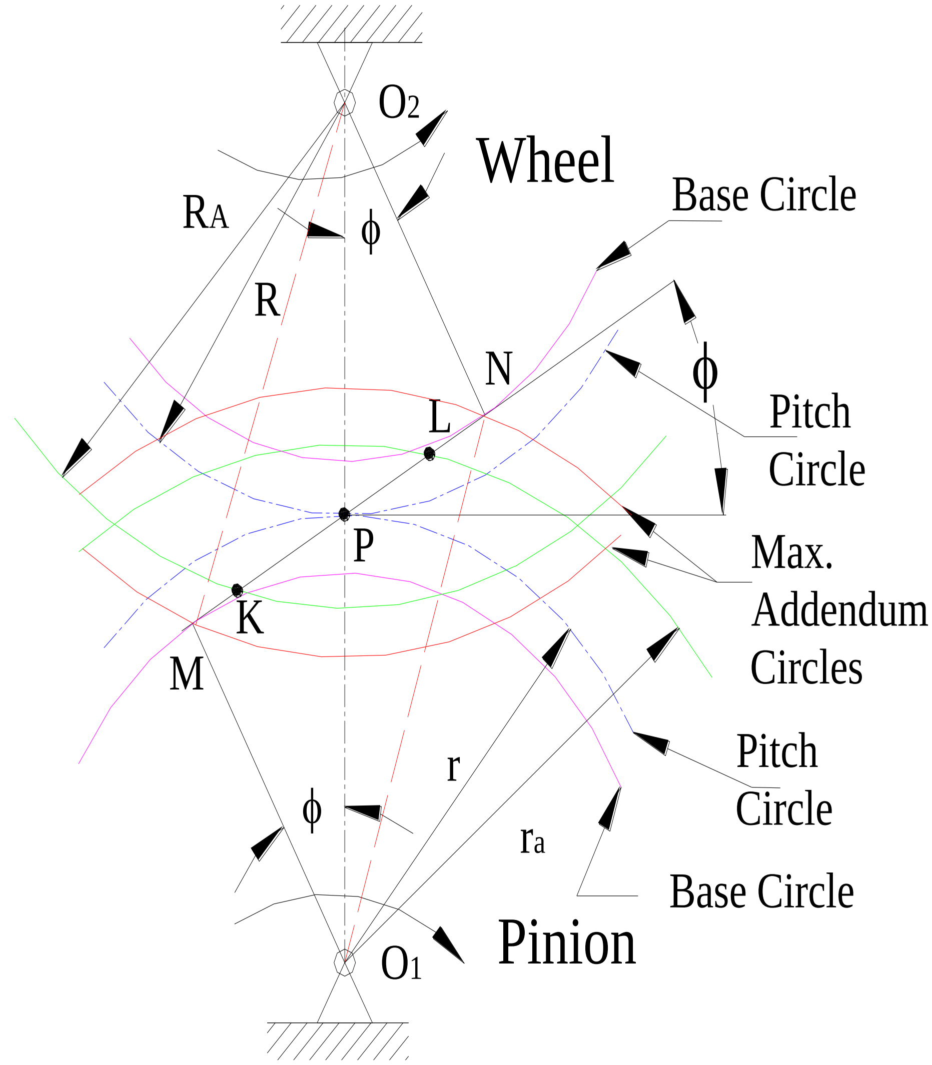
a_w = Addendum constant of gear (or) wheel

a_p = Addendum constant of pinion

a_w. m = Addendum of gear

a_p. m = Addendum of pinion

G = Gear ratio = *T/t*



Minimum number of teeth on the pinion avoid Interference

From triangle O_1NP, Applying cosine rule

$$O_1N^2 = O_1P^2 + NP^2 - 2 \times O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2\,r\,R\,\sin\phi\,\cos(90+\phi)$$

$$= r^2 + R^2 \sin^2\phi + 2\,r\,R \sin^2\phi$$

$$= r^2\left[1 + \frac{R^2 \sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$$(\because PN = O_2P\sin\phi = R\sin\phi)$$

Minimum number of teeth on the pinion avoid Interference

Limiting radius of the pinion addendum circle:

$$O_1N = r\sqrt{\left[1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi\right]} = \frac{mt}{2}\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}$$

Minimum number of teeth on the pinion avoid Interference

Addendum of the pinion = O_1N - O_1P

$$a_p m = \frac{mt}{2}\sqrt{\left[1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi\right]} - \frac{mt}{2}$$

$$= \frac{mt}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

Minimum number of teeth on the pinion avoid Interference

Addendum of the pinion = O_1N - O_1P

$$a_p = \frac{t}{2}\left[\sqrt{\left(1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi\right)}-1\right]$$

$$t = \frac{2a_p}{\left[\sqrt{\left(1+G\left(G+2\right)\sin^2\phi\right)}-1\right]}$$

The equation gives minimum number of teeth required on the pinion to avoid interference.

Minimum number of teeth on the pinion avoid Interference

If the number of teeth on pinion and gear is same:

G=1

$$t = \left[\frac{2a_p}{\sqrt{\left(1+3\sin^2\phi\right)}-1}\right]$$

Minimum number of teeth on the pinion avoid Interference

1. $14\frac{1}{2}^{O}$ Composite system = 12
2. $14\frac{1}{2}^{O}$ Full depth involute system = 32
3. 20^{O} Full depth involute system = 18
4. 20^{O} Stub involute system = 14

Minimum number of teeth on the wheel to avoid Interference

From triangle O_2MP, applying cosine rule :

$$O_1M^2 = O_2P^2 + PM^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2 \sin^2\phi - 2\,Rr\,\sin\phi\,\cos(90+\phi)$$

$$= R^2 + r^2 \sin^2\phi + 2\,Rr\sin^2\phi$$

$$= R^2\left[1 + \frac{r^2 \sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right] = R^2\left[1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\phi\right]$$

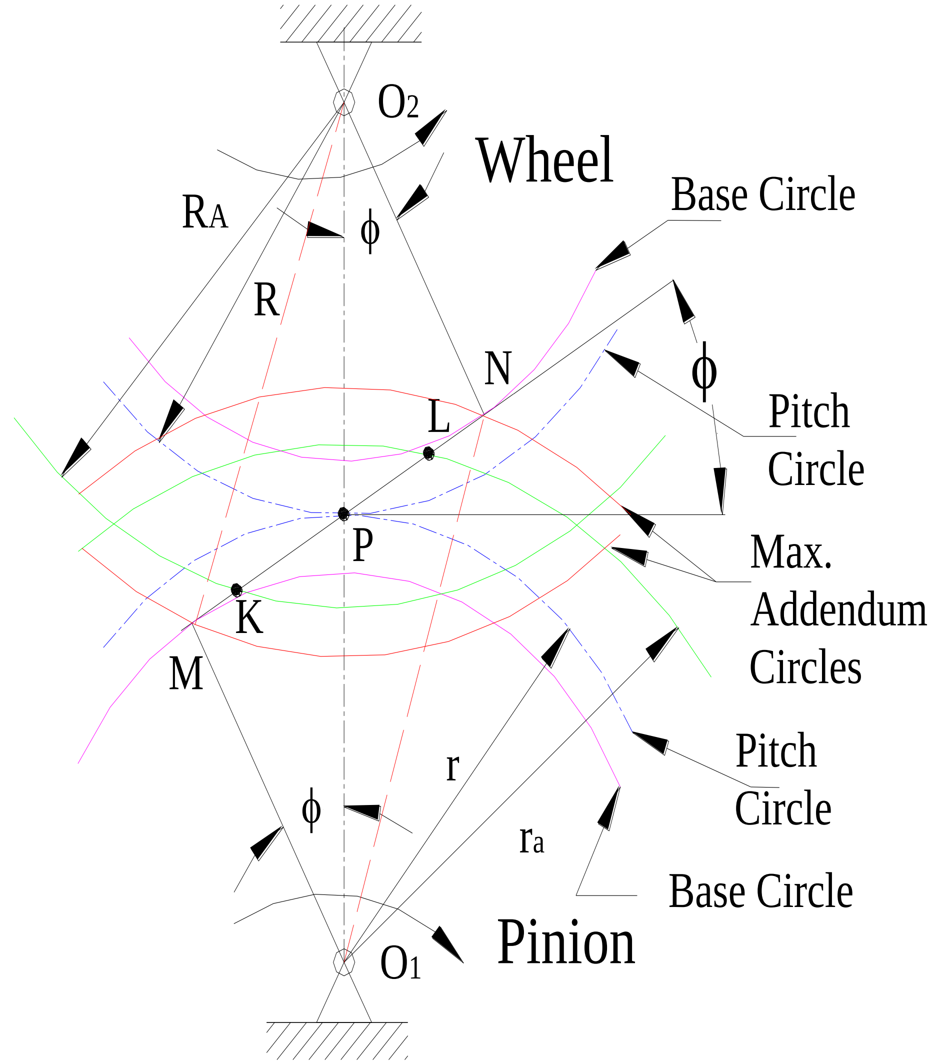
$$(\because PM = O_1P\sin\phi = r\sin\phi)$$

Minimum number of teeth on the wheel to avoid Interference

The limiting radius of wheel addendum circle:

$$O_2M = R\left[\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi}\right]$$

$$= \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi}\right]$$



Minimum number of teeth on the wheel to avoid Interference

Addendum of the pinion = $O_2 M - O_2 P$

$$a_w m = \frac{mT}{2}\left[\sqrt{\left(1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi\right)}-1\right]$$

$$a_w = \frac{T}{2}\left[\sqrt{\left(1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi\right)}-1\right]$$

Minimum number of teeth on the wheel to avoid Interference

$$T = \frac{2a_W}{\left[\sqrt{\left(1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi\right)}-1\right]}$$

$$T = \frac{2a_W}{\left[\sqrt{\left(1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi\right)}-1\right]}$$

The equation gives minimum number of teeth required on the wheel to avoid interference.

Minimum number of teeth on the wheel to avoid Interference

Multiplying by t/T, the equation gives minimum number of teeth required on the pinion to avoid interference.

$$T * \frac{t}{T} = \frac{2a_W * \frac{t}{T}}{\left[\sqrt{\left(1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi\right)}-1\right]}$$

$$t = \frac{2a_W}{\left[G\sqrt{\left(1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi\right)}-1\right]}$$

Minimum number of teeth on the pinion avoid Interference

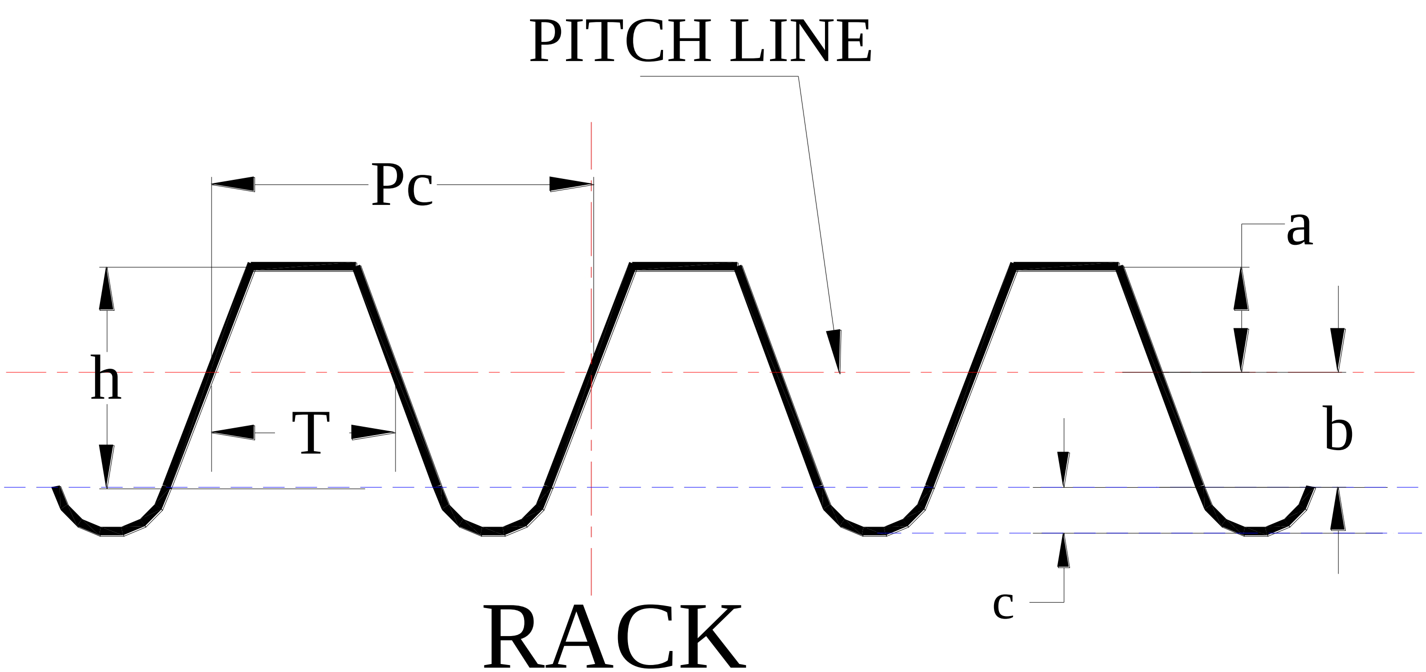
If the number of teeth on pinion and gear is same:

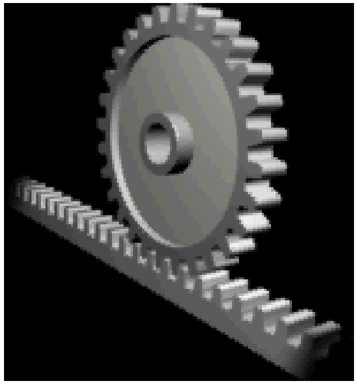
G=1

$$T = \left[\frac{2a_w}{\sqrt{\left(1+3\sin^2\phi\right)}-1}\right]$$

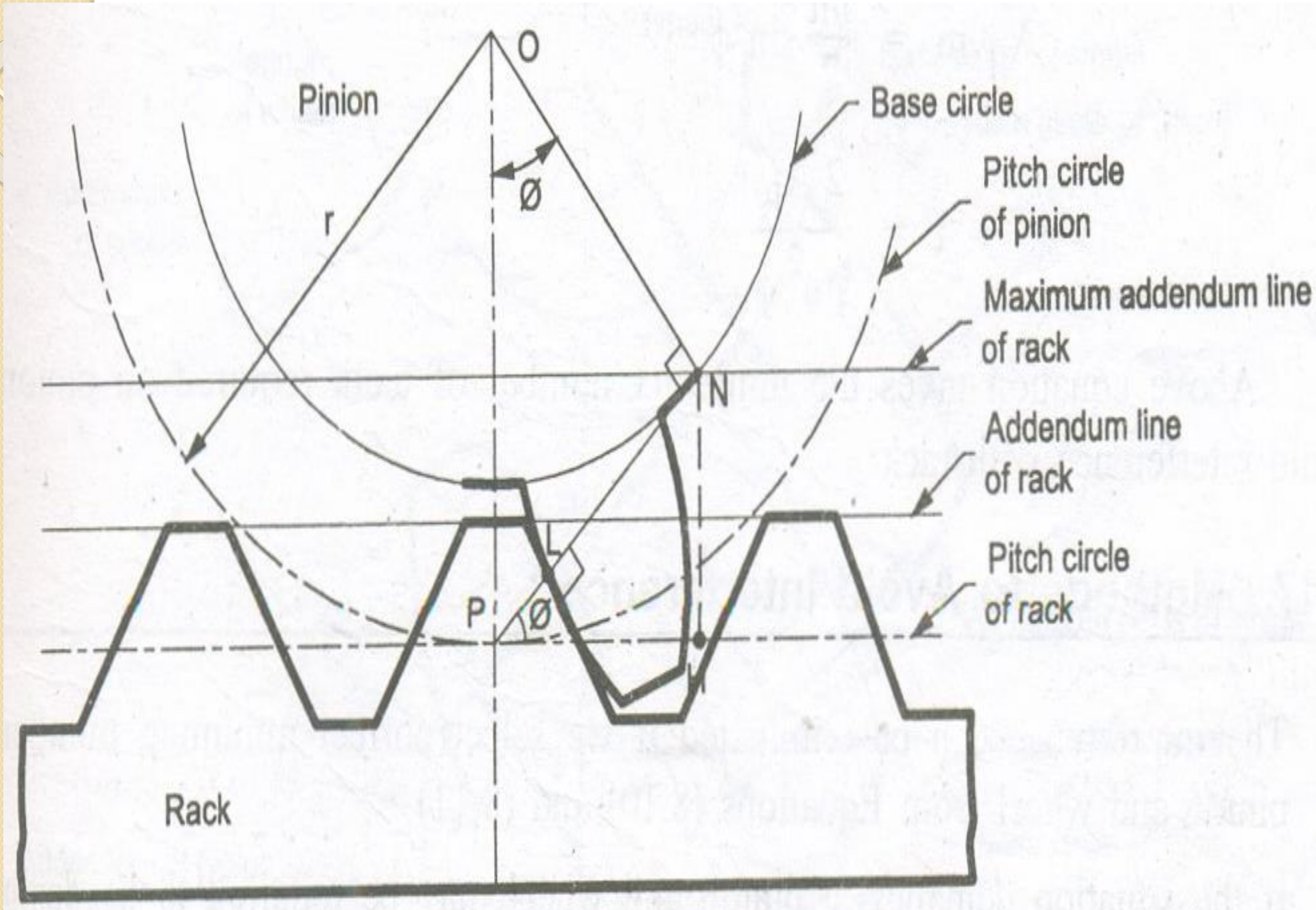
Rack and Pinion

The rack is part of toothed wheel of infinite diameter. The base circle diameter and profile of the involute teeth are straight lines.





Minimum number of teeth on the pinion to avoid Interference with Rack



Minimum number of teeth on the pinion to avoid Interference with Rack

Let

Φ = pressure angle

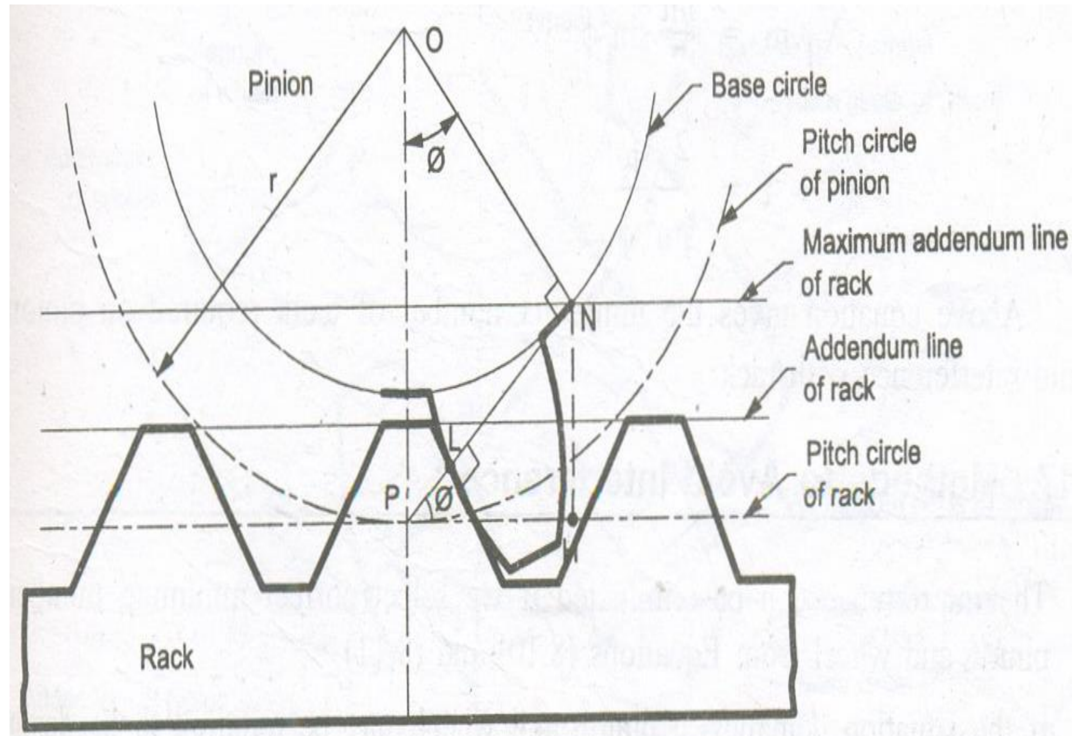
r = pitch circle radius of pinion

= ½*mt*

t = number of teeth on pinion

m = module

A_r = Addendum coefficient of rack



Minimum number of teeth on the pinion to avoid Interference with Rack

Let,

$$A_R * m = NH = PN\sin\phi$$

$$But,.....(PN = OP\sin\phi)$$

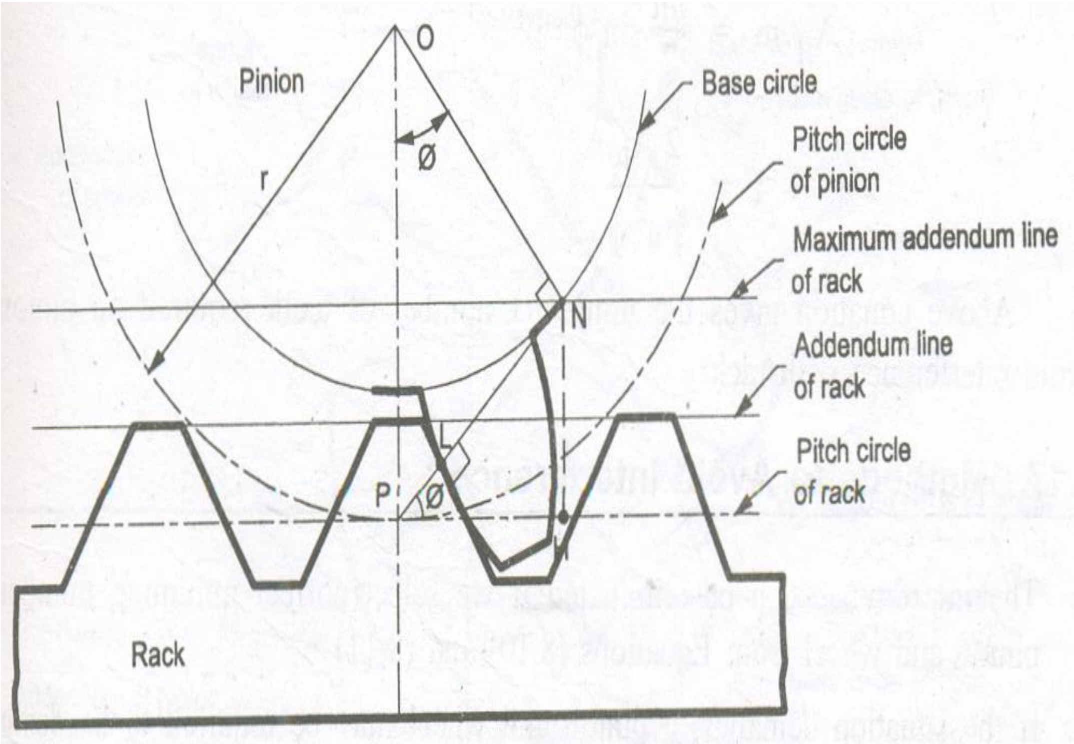
$$\therefore A_R * m = (OP\sin\phi)\sin\phi$$

$$\therefore A_R * m = OP\sin^2\phi$$

$$\therefore A_R * m = r\sin^2\phi$$

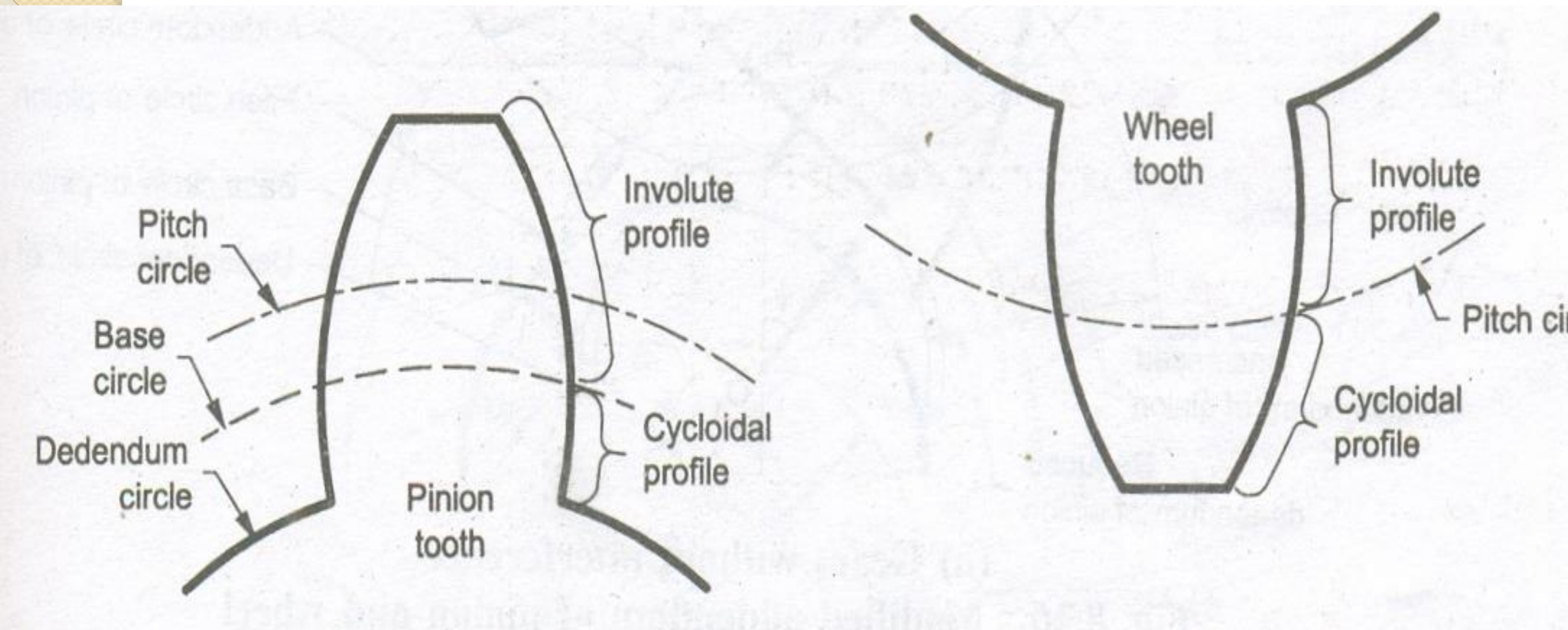
$$\therefore A_R * m = \frac{mt}{2}\sin^2\phi$$

$$t = \frac{2A_R}{\sin^2\phi}$$



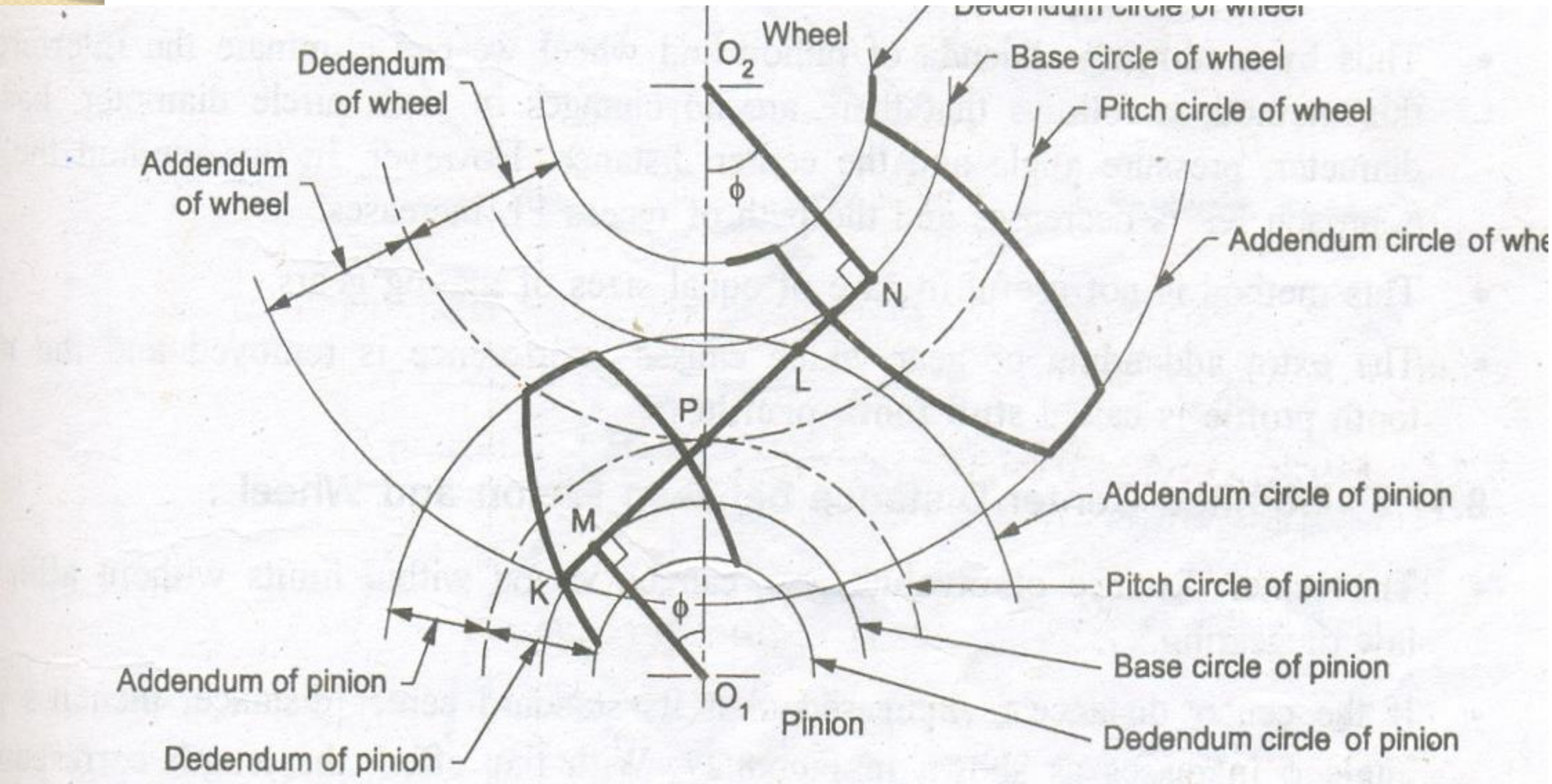
Methods To Avoid Interference

I) Modified Tooth Profile



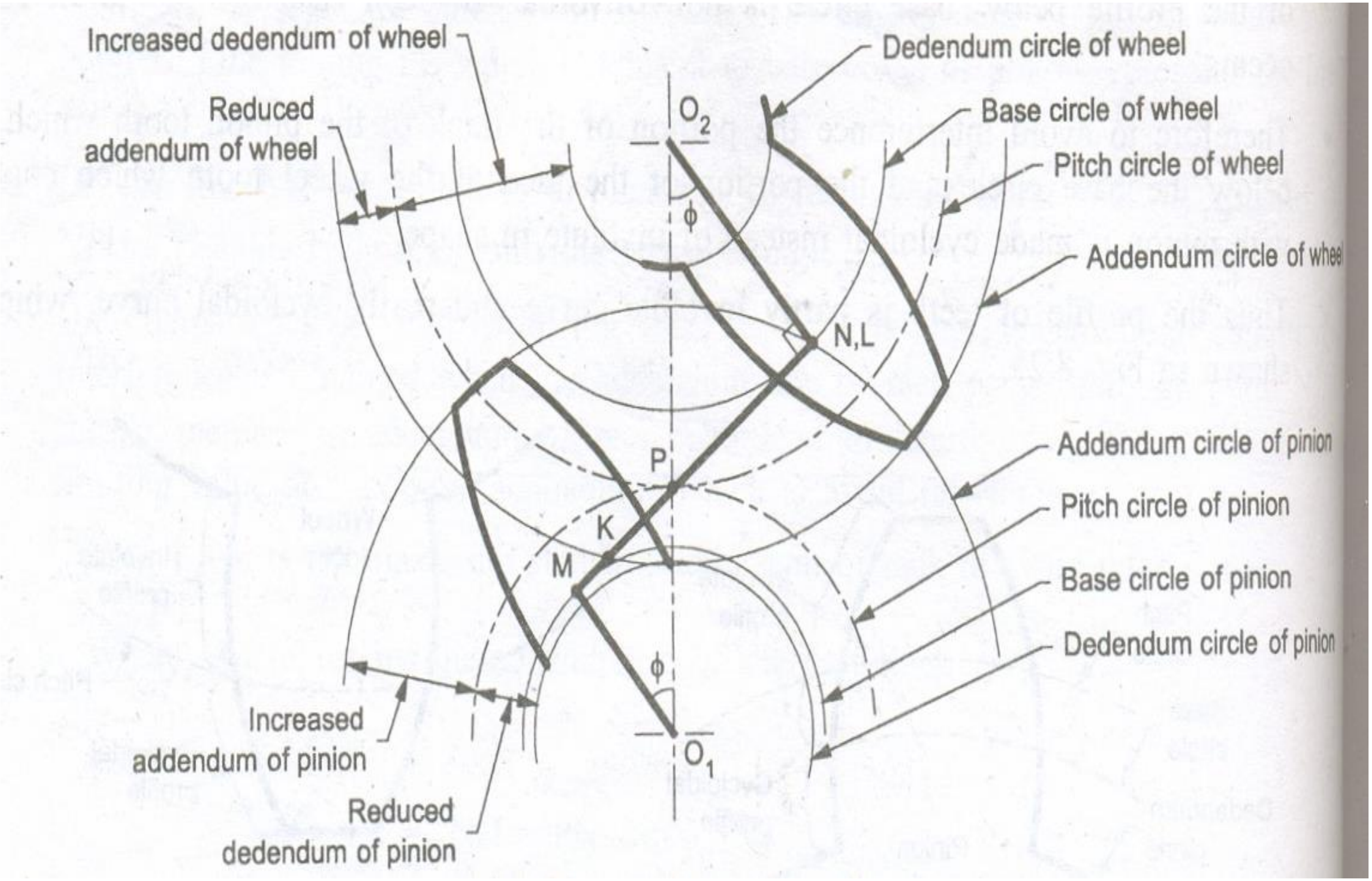
Methods To Avoid Interference

II) Modified Addendum Of Pinion & Wheel



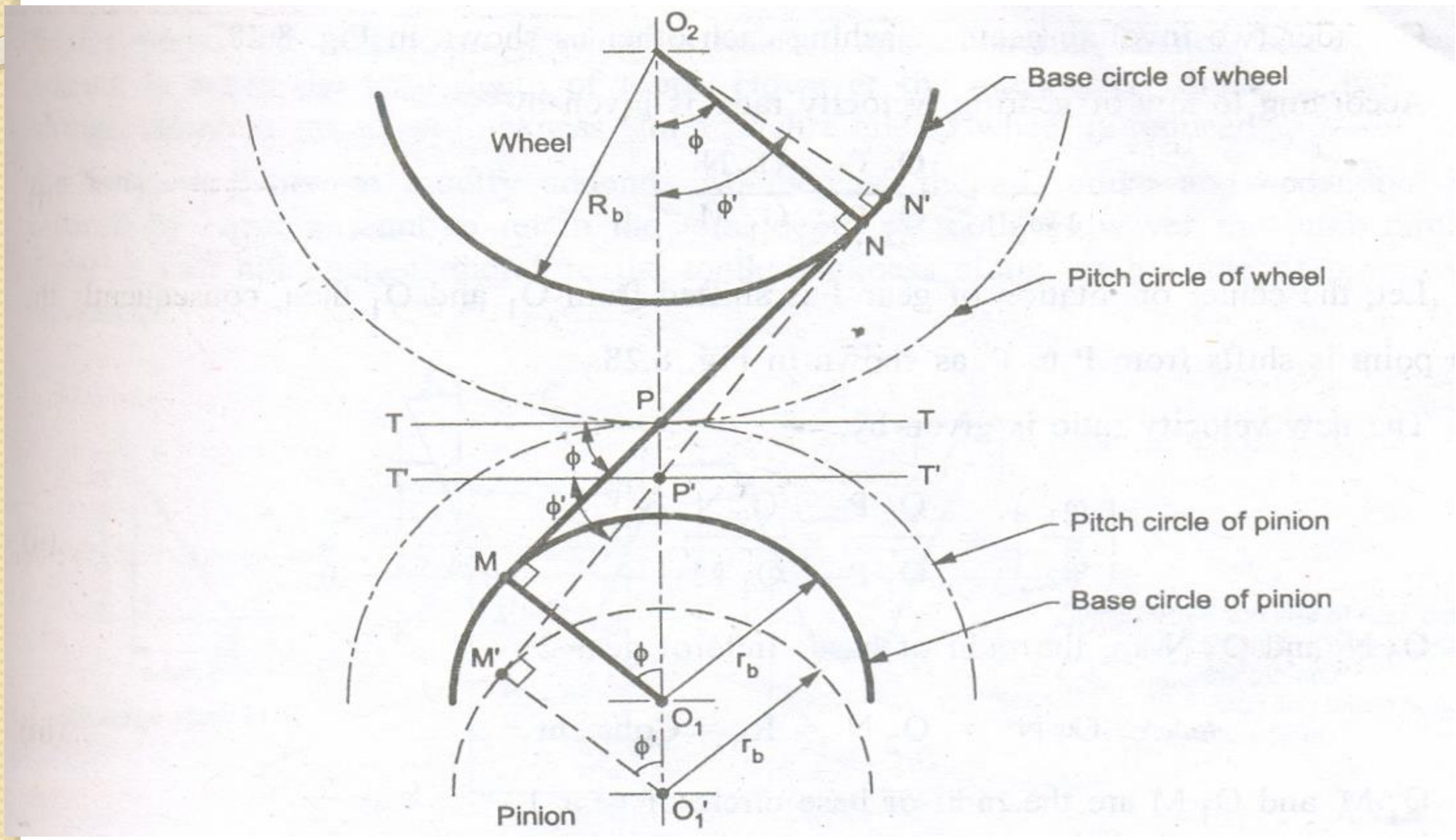
Methods To Avoid Interference

II) Modified Addendum Of Pinion & Wheel

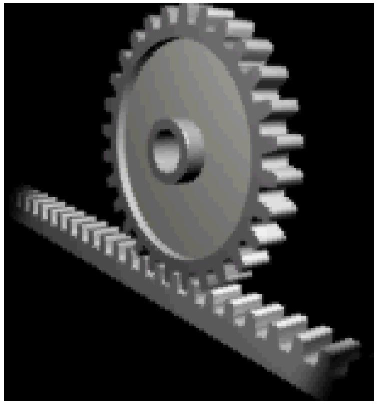


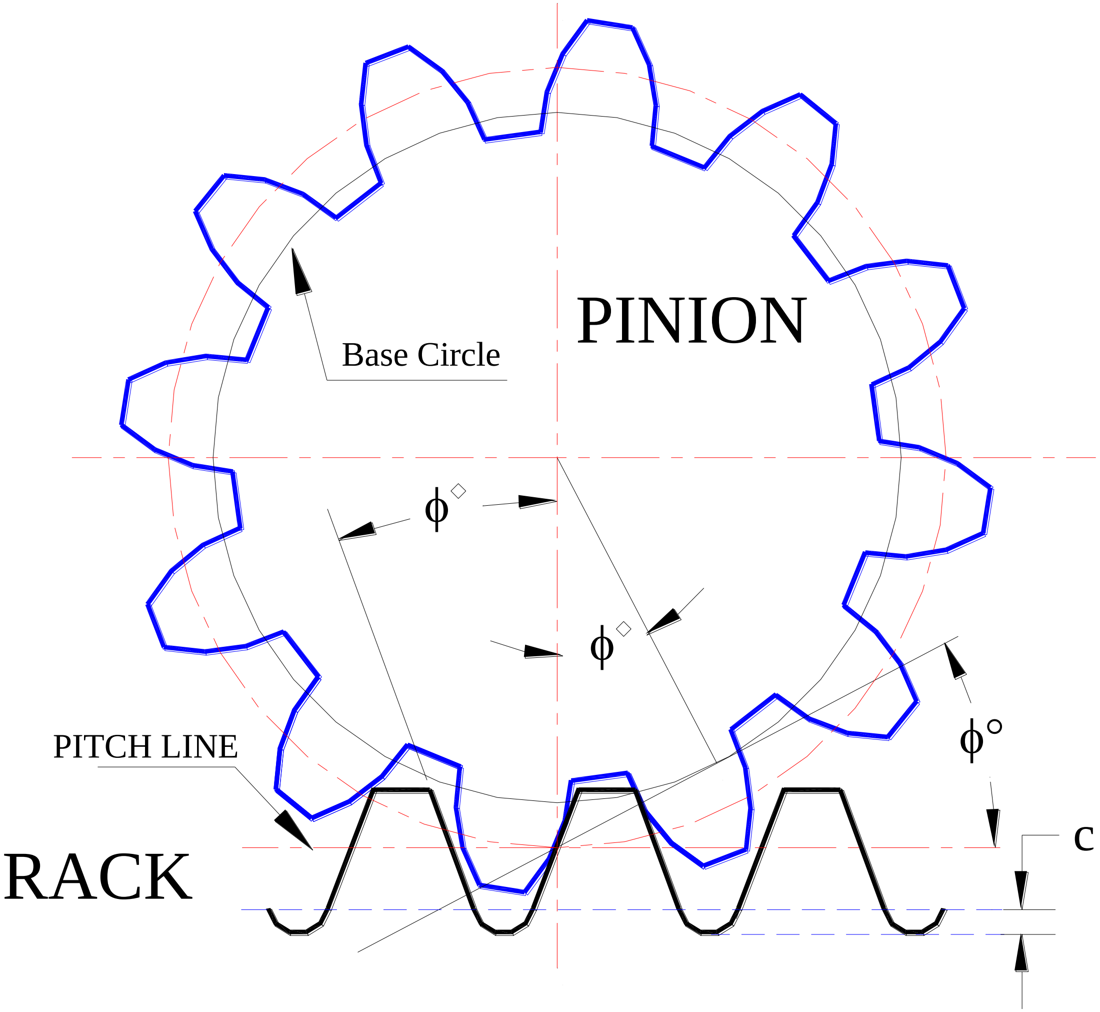
Methods To Avoid Interference

III) Modified Centre Distance Between Pinion & Wheel



Length of path of contact for Rack and Pinion





Length of path of contact for Rack and Pinion

Let

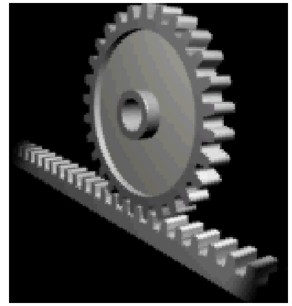
r = Pitch circle radius of the pinion=O_1P

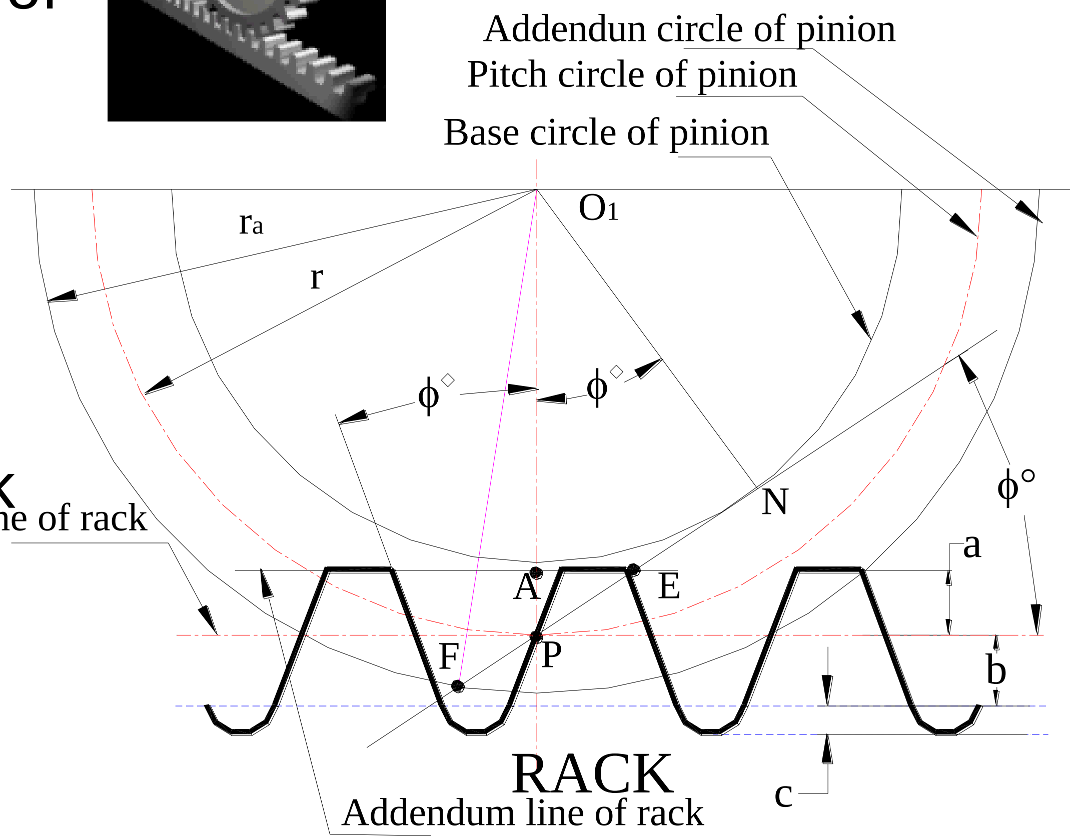
Φ= Pressure angle

$r_{a.}$= Addendum radius of the pinion

a = Addendum of rack

EF=Length of path of contact





Length of path of contact for Rack and Pinion

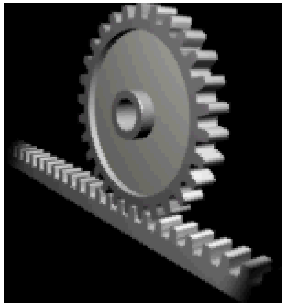
EF = Path of approach EP + Path of recess PF

$$\sin\phi = \frac{AP}{EP} = \frac{a}{EP} \qquad (1)$$

$$Path\ of\ \ approach = EP = \frac{a}{\sin\phi} \qquad (2)$$

$$Path\ of\ \ recess = PF = NF - NP \qquad (3)$$

$From\ triangle O_1NP:$



$$NP = O_1P\sin\phi = r\sin\phi$$

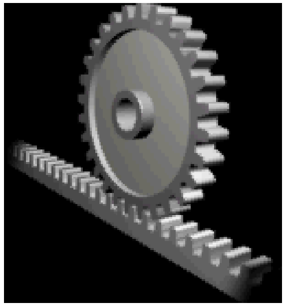
$$O_1N = O_1P\cos\phi = r\cos\phi$$

Length of path of contact for Rack and Pinion

From triangle O_1NF:

$$NF = \left(O_1F^2 - O_1N^2\right)^{\frac{1}{2}} = \left(r_a^2 - r^2\cos^2\phi\right)^{\frac{1}{2}}$$

$$Substituting\ NP\ and\ NF\ values\ in\ the\ equation\ (3)$$



$$Path\ of\ racess = PF = \left(r_a^2 - r^2\cos^2\phi\right)^{\frac{1}{2}} - r\sin\phi$$

$$\therefore Path\ of\ length\ of\ contact = EF = EP + PF$$

$$= \frac{a}{\sin\phi} + \left(r_a^2 - r^2\cos^2\phi\right)^{\frac{1}{2}} - r\sin\phi$$

Summary

Length of path of contact in spur gear

Arc of contact in spur gear

Contact ratio in spur gear

Number of pair of teeth in contact in spur gears

Length of path of contact in Rack and pinion

Exercise 1

Two spur wheels have 24 and 30 teeth with a standard addendum of 1 module. The pressure angle is 20°. Calculate the path of contact and arc of contact.

Solution:

Data: t =24; T=30; addendum = 1m and ϕ=20°

$$Pitch\ circle\ radius\ of\ the\ pinion = r = \frac{mt}{2} = \frac{m \times 24}{2} = 12m$$

$$Pitch\ circle\ radius\ of\ the\ gear = R = \frac{mT}{2} = \frac{m \times 30}{2} = 15m$$

Exercise 1- Continued

$$Addendum\ circle\ radius\ of\ the\ pinion = r_a = r + m$$

$$r_a = 12m + m = 13m$$

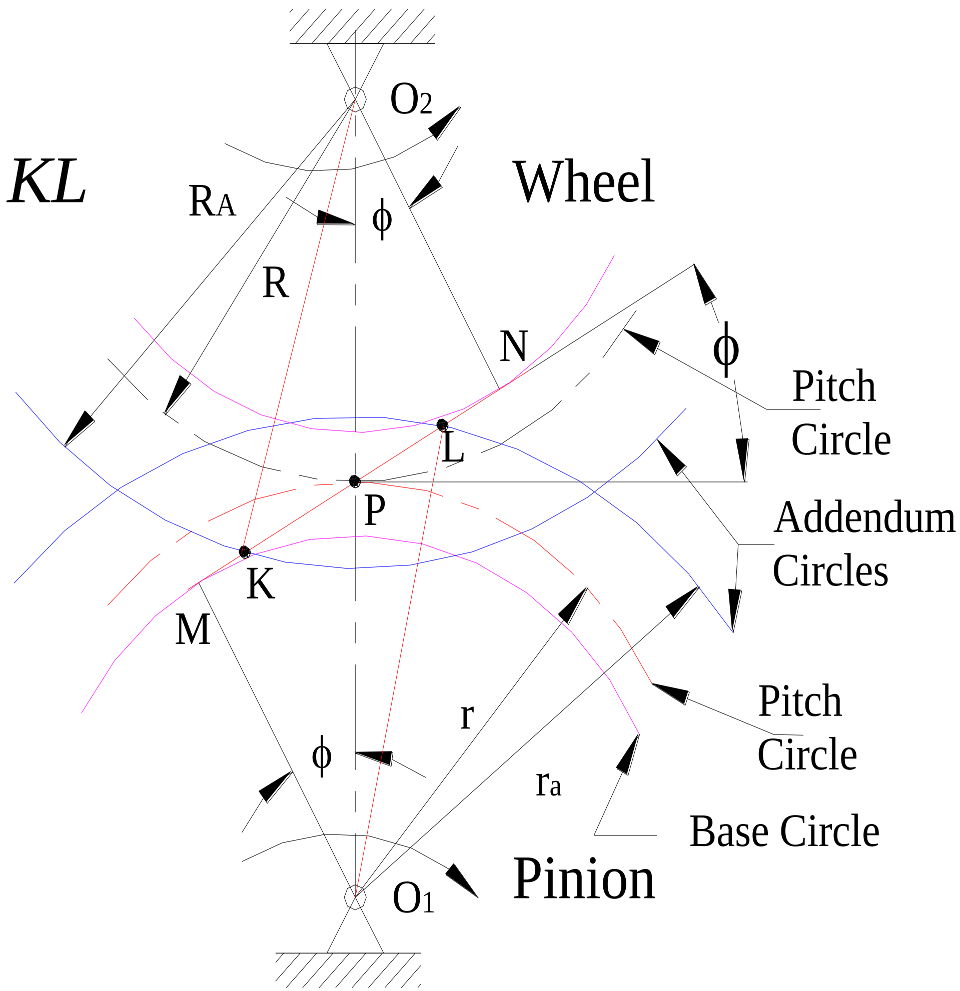
$$Addendum\ circle\ radius\ of\ the\ gear \quad = R_A = R + m$$

$$R_A \quad = 15m + m = 16m$$

Exercise 1- Continued

$$Length\ of\ \ path\ of\ \ contact = KL$$
$$= KP + PL$$

$$KL = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_a)^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$



Exercise 1- Continued

$$Length\ of\ \ path\ of\ \ contact = KL = KP + PL$$

$$= \sqrt{\left(R_A\right)^2 - R^2\cos^2\phi} + \sqrt{\left(r_a\right)^2 - r^2\cos^2\phi} - \left(R + r\right)\sin\phi$$

$$= \sqrt{\left(16m\right)^2 - \left(15m\right)^2\cos^2 20} + \sqrt{\left(13m\right)^2 - \left(12m\right)^2\cos^2 20}$$

$$- \left(15m + 12m\right)\sin 20$$

$$= 4.802m$$

$$Length\ of\ \ arc\ of\ \ contact = \frac{Length\ of\ \ path\ of\ \ contact}{\cos\phi}$$

$$= \frac{4.802m}{\cos 20} = 5.11m, mm$$

Exercise 2

Two gears in mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 teeth while the pinion has 23 teeth. If the addenda on pinion and gear wheel are equal to one module (1*m*), find

1. The number of pairs of teeth in contact and
2. The angle of action of the pinion and the gear wheel.

Solution:

Data: t =23; T=57; addendum = 1m= $8mm$ and ϕ=20°

Exercise 2 - continued

$$Pitch\ circle\ radius\ of\ the\ pinion = r = \frac{mt}{2} = \frac{8 \times 23}{2} = 92mm$$

$$Pitch\ circle\ radius\ of\ the\ gear = R = \frac{mT}{2} = \frac{8 \times 57}{2} = 228mm$$

$$Addendum\ circle\ radius\ of\ the\ pinion = r_a = r + addendum$$

$$r_a = 92 + 8 = 100mm$$

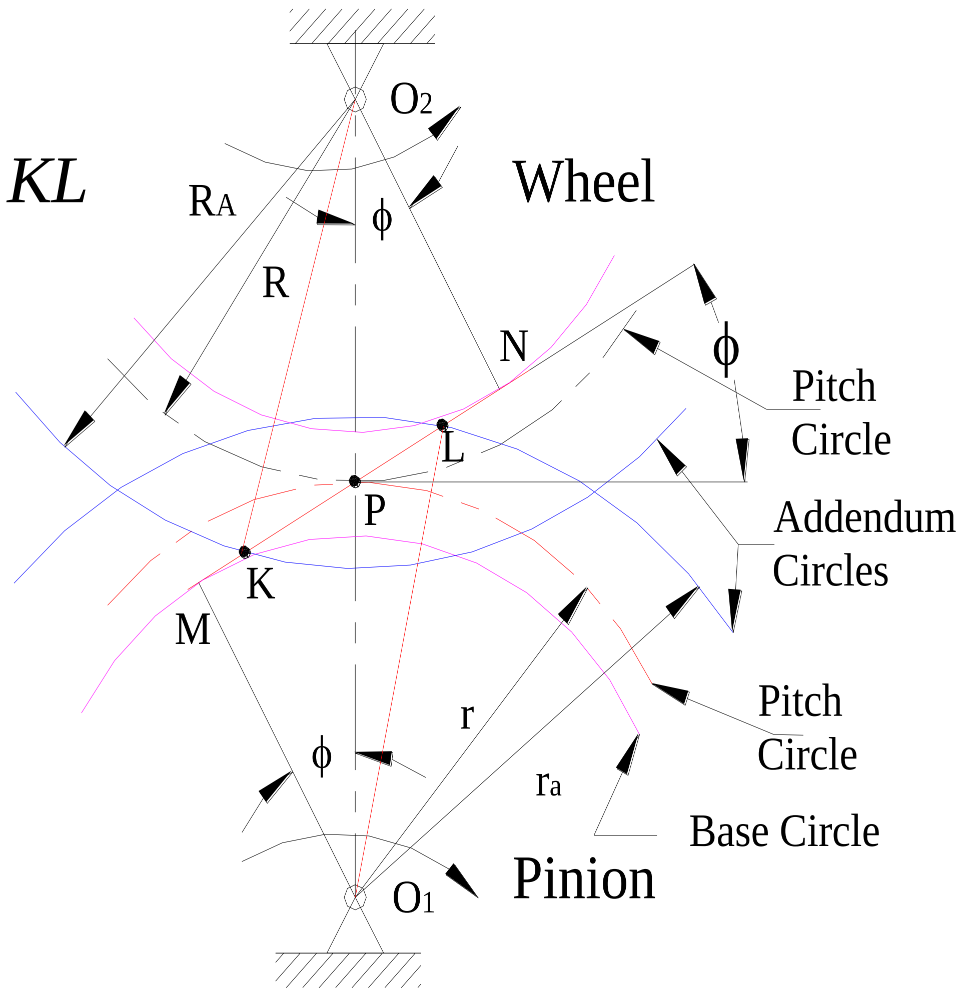
$$Addendum\ circle\ radius\ of\ the\ gear = R_A = R + addendum$$

$$R_A = 228 + 8 = 236mm$$

Exercise 2 - continued

$$Length\,of\;\;path\,of\;\;contact = KL$$
$$= KP + PL$$

$$KL = \sqrt{(R_A)^2 - R^2\cos^2\phi}$$
$$+\sqrt{(r_a)^2 - r^2\cos^2\phi}$$
$$-(R+r)\sin\phi$$



Exercise 2 - continued

$$Length\ of\ \ path\ of\ \ contact = KL = KP + PL$$

$$= \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_a)^2 - r^2\cos^2\phi} - (R + r)\sin\phi$$

$$= \sqrt{(236)^2 - (228)^2\cos^2 20} + \sqrt{(100)^2 - (92)^2\cos^2 20}$$

$$- (228 + 92)\sin 20$$

$$= 39.76mm$$

$$Length\ of\ \ arc\ of\ \ contact = \frac{Length\ of\ \ path\ of\ \ contact}{\cos\phi}$$

$$= \frac{39.76}{\cos 20} = 42.31mm$$

Exercise 2 - continued

$$Number\ of\ \ pairs\ of\ \ teeth\ in\ contact = \frac{Length\ of\ \ arc\ of\ \ contact}{circular\ \ pitch}$$

$$= \frac{Length\ of\ \ arc\ of\ \ contact}{p_c} = \frac{42.31}{\pi\, m} = 1.684 \approx 2$$

$$Angle\ of\ \ action\ of\ \ gear\ wheel = \frac{Length\ of\ \ arc\ of\ \ contact}{2\pi \times R} \times 360^o$$

$$= \frac{42.31}{2\pi \times 228} \times 360 = 10.637^o\ = 10^o\ 38'16''$$

$$Angle\ of\ \ action\ of\ \ pinion = \frac{Length\ of\ \ arc\ of\ \ contact}{2\pi \times r} \times 360^o$$

$$= \frac{42.31}{2\pi \times 92} \times 360 = 26.36^o\ = 26^o\ 21'47''$$

Exercise 2 - continued

$$\frac{Angle\ of\ \ action\ of\ \ pinion}{Angle\ of\ \ action\ of\ \ gear} = \frac{26.36}{10.637} = 2.478\ \ and$$

$$\frac{T}{t} = \frac{57}{23} = 2.478$$

Exercise 3

The following data refers to two mating involute gears of 20° pressure angle. Number of teeth on pinion is 20. Gear ratio = 2, speed of pinion is 250 rpm, module = 12 mm. If the addendum on each wheel is such that the path of approach and the path of recess on each side are half of the maximum permissible length, find the maximum velocity of sliding during approach and recess and the length of arc of contact.

Solution:

Data: t =20; G=2; m= 12mm; n=250rpm and ϕ=20°

Exercise 3 - continued

$$Pitch\ circle\ radius\ of\ the\ pinion = r = \frac{mt}{2} = \frac{12\times 20}{2} = 120mm$$

$$Gear\ ratio = \frac{T}{t} = 2 \quad \Rightarrow \quad \frac{T}{20} = 2$$

$$Number\ of\ teeth\ on\ gear = T = 2\times 20 = 40$$

$$Pitch\ circle\ radius\ of\ the\ gear \quad = R = \frac{mT}{2} = \frac{12\times 40}{2} = 240mm$$

Exercise 3 - continued

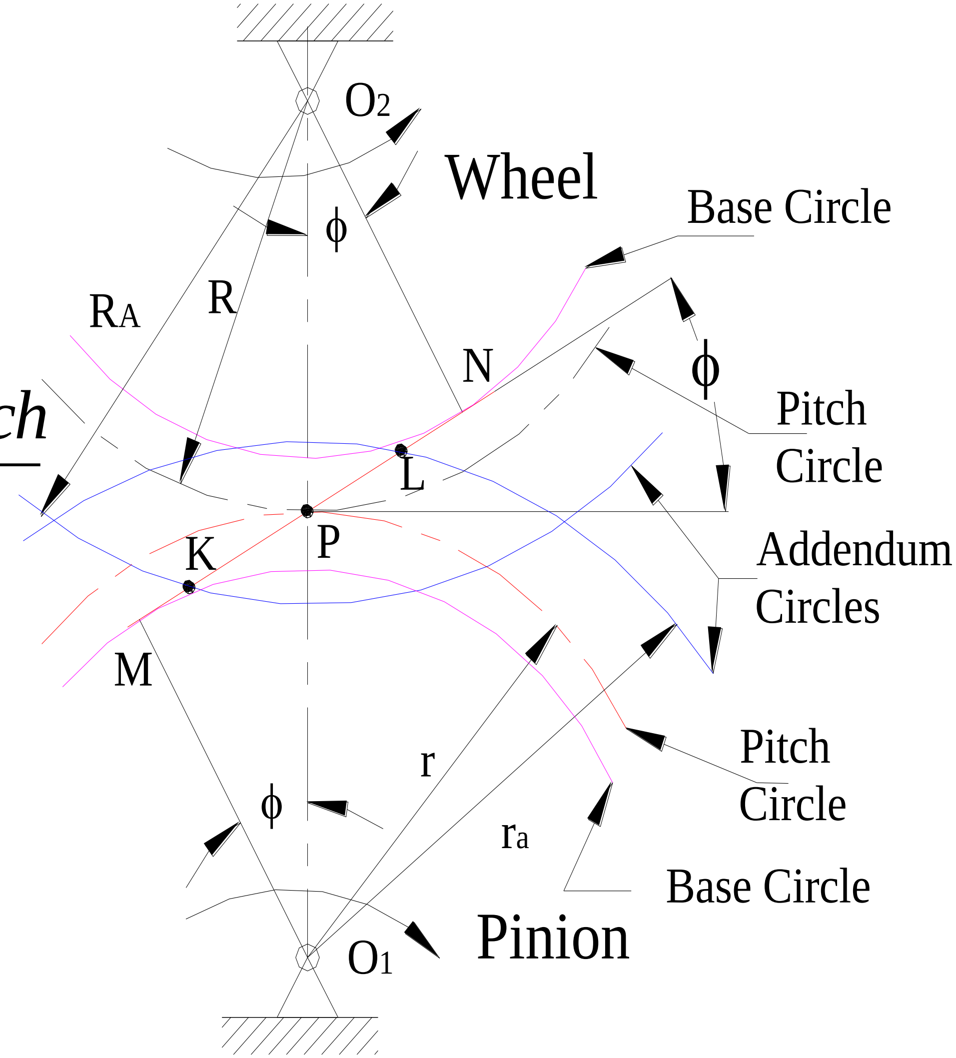
From the given condition:

$$\text{Re}\,quired\ path\ of\ \ approach:$$

$$=\frac{Maximum\ path\ of\ \ approach}{2}$$

$$=\frac{r\times\sin\phi}{2}$$

$$=\frac{120\times\sin 20}{2}=20.52mm$$



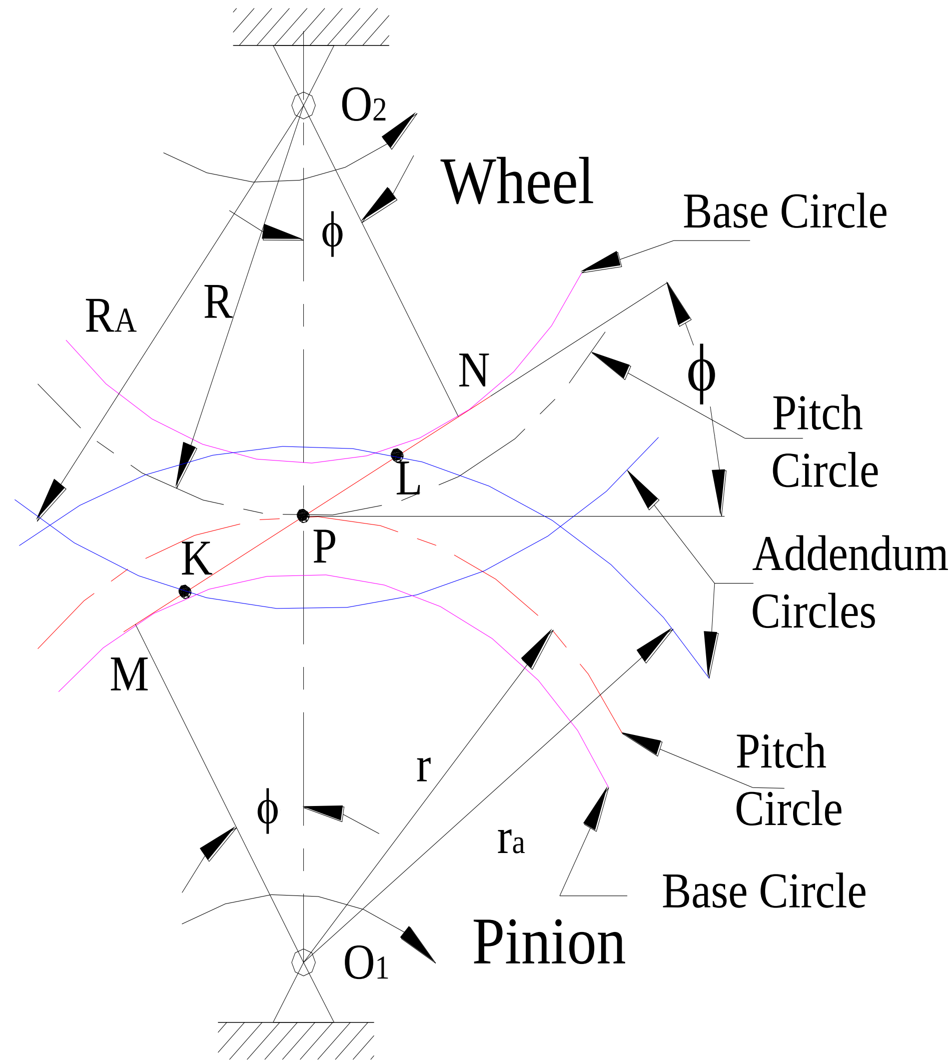
Exercise 3 - continued

$$\text{Required path of recess:}$$

$$= \frac{\text{Maximum path of recess}}{2}$$

$$= \frac{R \times \sin\phi}{2}$$

$$= \frac{240 \times \sin 20}{2} = 41.04mm$$



Exercise 3 - continued

Path of approach: *KP*

$$KP = KN - PN$$

$$= \sqrt{(R_A)^2 - R^2 \cos^2 \phi} - R \sin \phi$$

$$20.52 = \sqrt{(R_A)^2 - (240)^2 \cos^2 20} - 240 \times \sin 20$$

$$\therefore \quad R_A = 247.77 mm = addendum\, radius\, of\ the\ gear$$

$$Addendum\, of\ the\ gear = R_A - R = 247.77 - 240 = 7.77 mm$$

Exercise 3 - continued

Path of recess: *PL*

$$PL = ML - MP$$

$$= \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

$$41.04 = \sqrt{(r_A)^2 - (120)^2\cos^2 20} - 120\times\sin 20$$

$$\therefore \quad r_A = 139.46mm = addendum\, radius\, of\ the\ pinion$$

$$Addendum\, of\ the\ pinion = r_A - r = 139.46 - 120 = 19.46mm$$

Path of contact = Path of approach + Path of recess

= 20.52 + 41.04 = 61.56 mm

Exercise 3 - continued

Path of recess: *PL*

$$PL = ML - MP$$

$$= \sqrt{(r_A)^2 - r^2 \cos^2\phi} - r\sin\phi$$

$$41.04 = \sqrt{(r_A)^2 - (120)^2 \cos^2 20} - 120 \times \sin 20$$

$$\therefore \quad r_A = 139.46mm = addendum\, radius\, of\ the\ pinion$$

$$Addendum\, of\ the\ pinion = r_A - r = 139.46 - 120 = 19.46mm$$

Path of contact = Path of approach + Path of recess

= 20.52 + 41.04 = 61.56 mm

Exercise 3 - continued

$$Arc\ of\ contact = \frac{Path\ of\ contact}{\cos\phi} = \frac{61.56}{\cos 20} = 65.51mm$$

$$Angular\ speed\ of\ the\ pinion = \omega_p = \frac{2\pi \times n}{60}$$

$$= \frac{2\pi \times 250}{60} = 26.16 rad/\sec$$

$$Angular\ speed\ of\ the\ gear = \omega_G = \frac{2\pi \times N}{60} = \frac{2\pi \times n}{60 \times G}$$

$$= \frac{2\pi \times 250}{60 \times 2} = 13.08 rad/\sec$$

Exercise 3 - continued

$$Maximum\ velocity\ of\ sliding\ during\ approach:$$

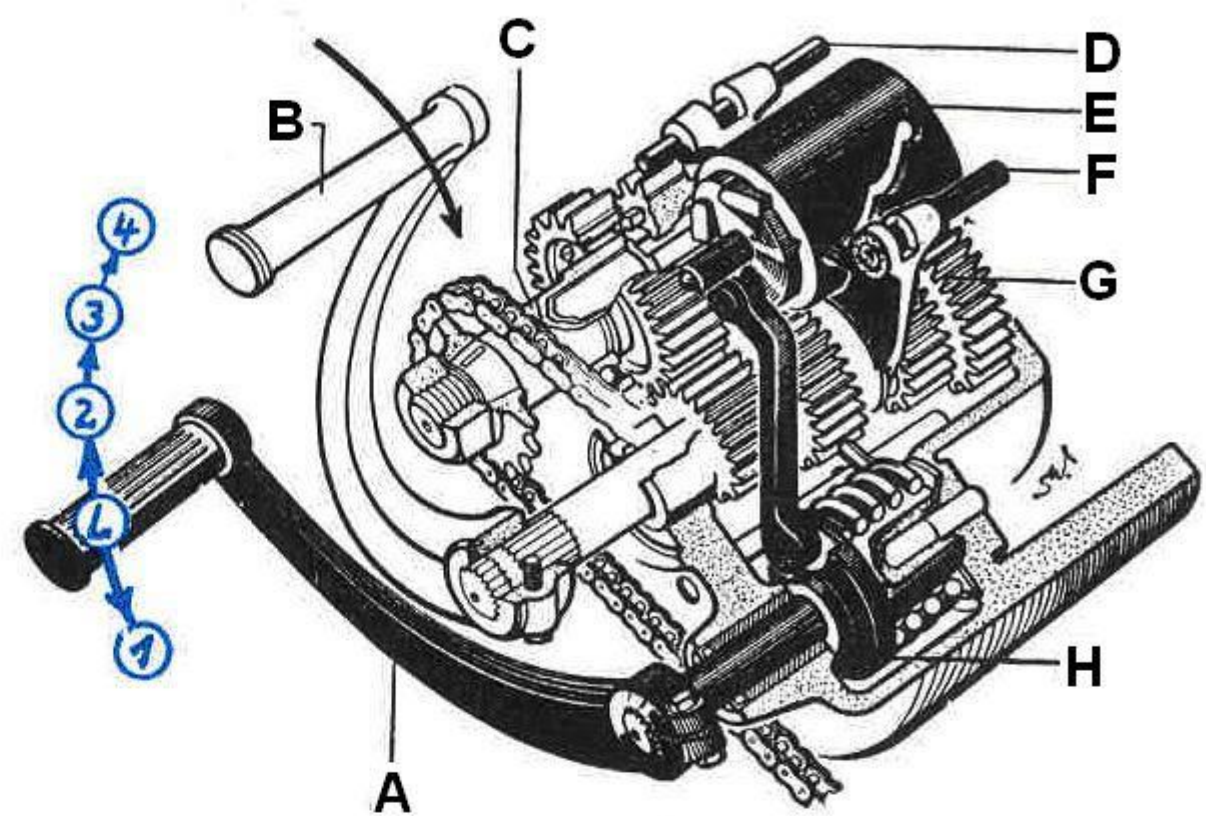
$$=\left(\omega_P+\omega_G\right)\times Path\ of\ \ approach$$

$$=\left(26.16+13.08\right)\times 20.52=805.2mm/\sec$$

$$Maximum\ velocity\ of\ sliding\ during\ recess:$$

$$=\left(\omega_P+\omega_G\right)\times Path\ of\ \ recess$$

$$=\left(26.16+13.08\right)\times 41.04=1610.4mm/\sec$$

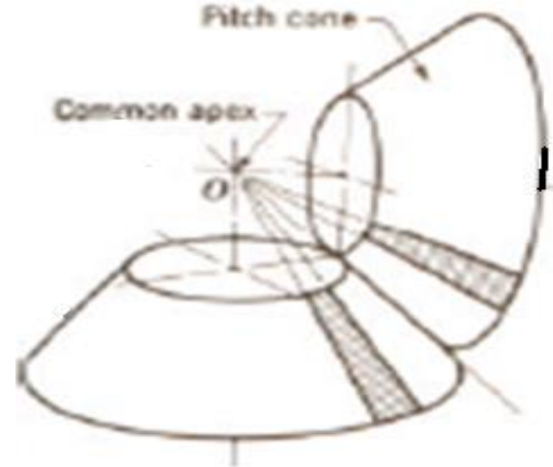


Foot pedal gear control by shift cam assembly

A Gear shift pedal
B Kick starter
C main and clutch shaft
D and F shift forks
E shift cam
G Start and counter shaft
H cam shaft with linkage for shift cam

Gearbox of a motor cycle using spur gears

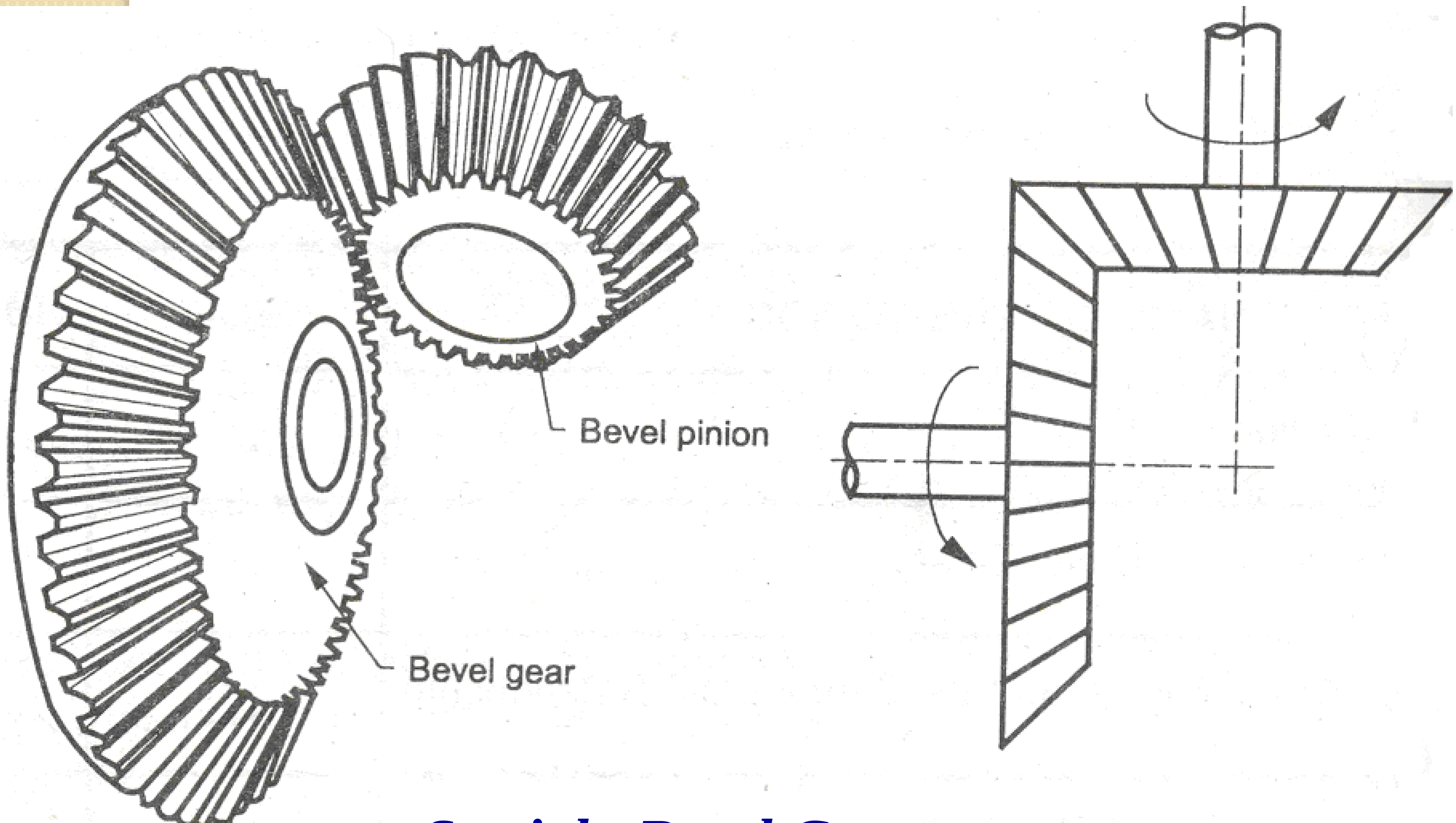
Bevel Gears



- Used for intersecting shafts (90^0) in same plane

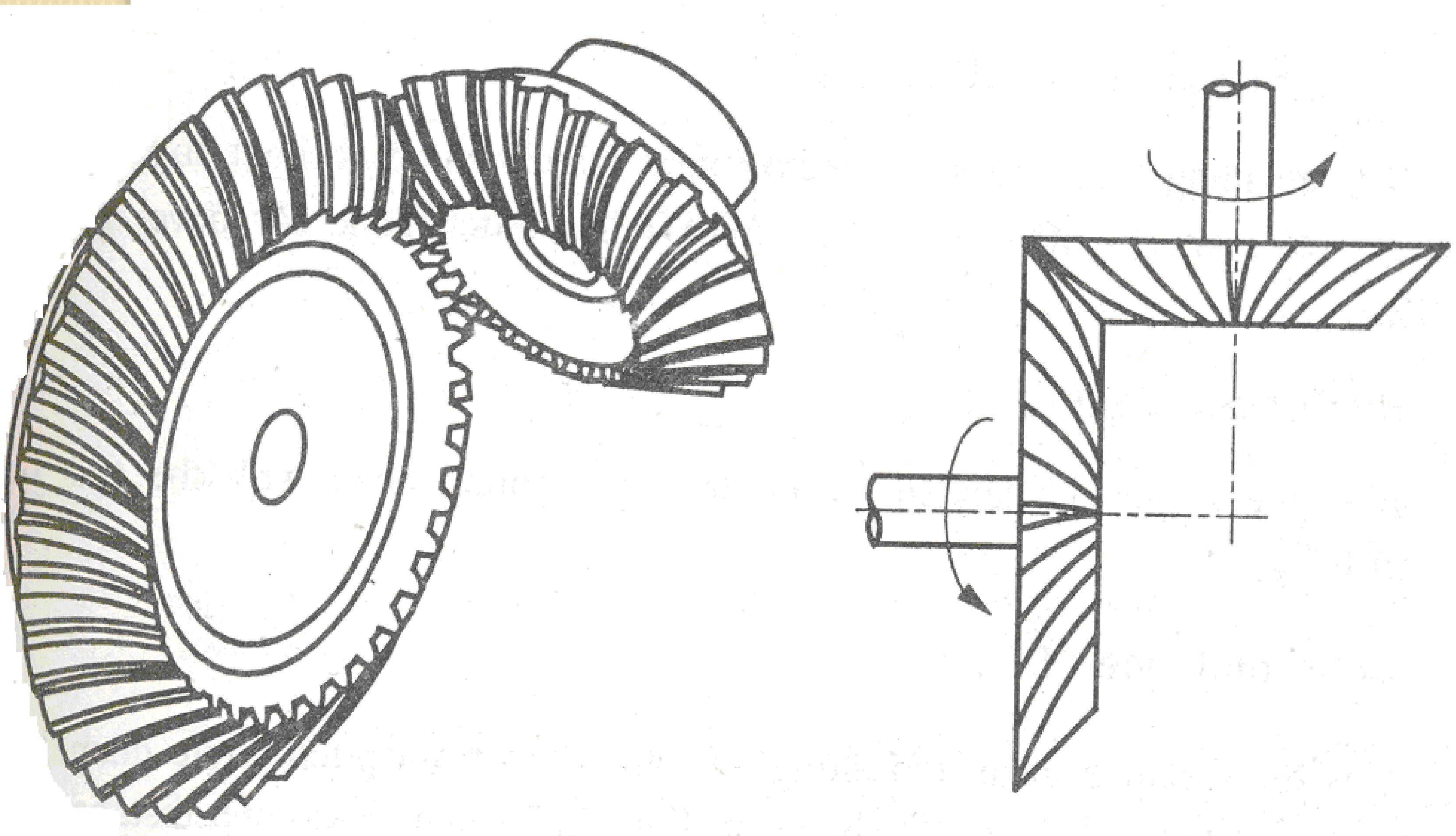
- ***Straight*** Bevel Gear / ***Spiral*** Bevel Gear
- For one to one ratio
- Used to change direction

Bevel Gears



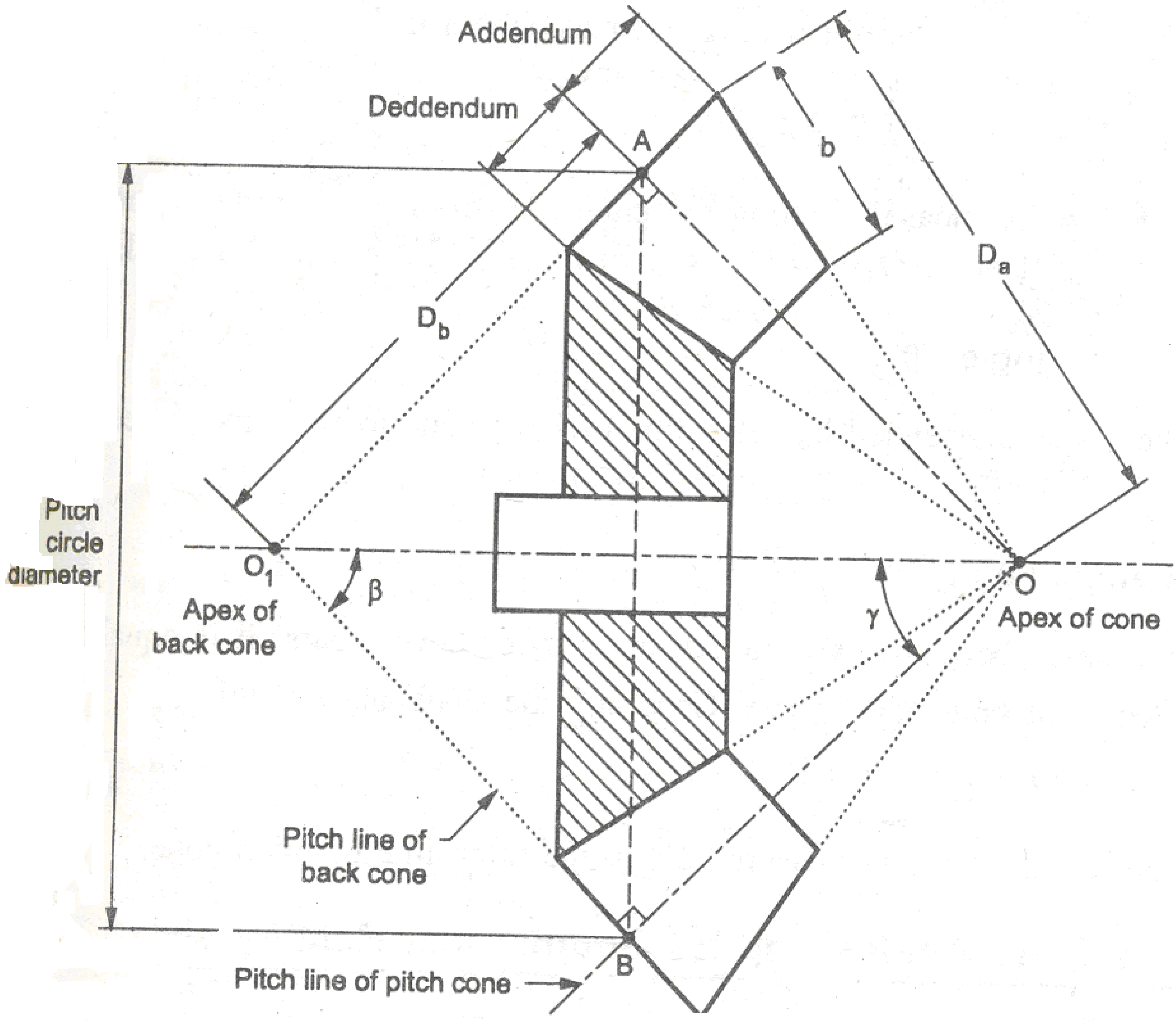
Straight Bevel Gear

Bevel Gears



Spiral Bevel Gear

Terminology Of Bevel Gears



Terminology Of Bevel Gears

- **Pitch cones :**

  It is an imaginary cones in a bevel gear which rolls without slipping when they are in peripheral contact with each other. **(ΔOAB)**

- **Pitch cone angle (γ) :**

  It is the angle subtended by pitch line of pitch cone with the axis of gear.

- **Cone distance (Da) :**

  It is the distance measured along the pitch line from PC diameter to the apex of cone.

  i.e. distance **AO** or **BO**

- **Back cone :**

  It is an imaginary cone and its elements are perpendicular to the elements of pitch cone.

  (ΔO_1 AB)

Terminology Of Bevel Gears

- **Back cone Distance (D_b) :**

It is the distance measured along the pitch line from PCD to the apex of back cone .

i.e. distance **AO_l** or **BO_l**

- **Back cone angle (β)** :

It is the angle subtended by pitch line of back cone to the axis of gear.

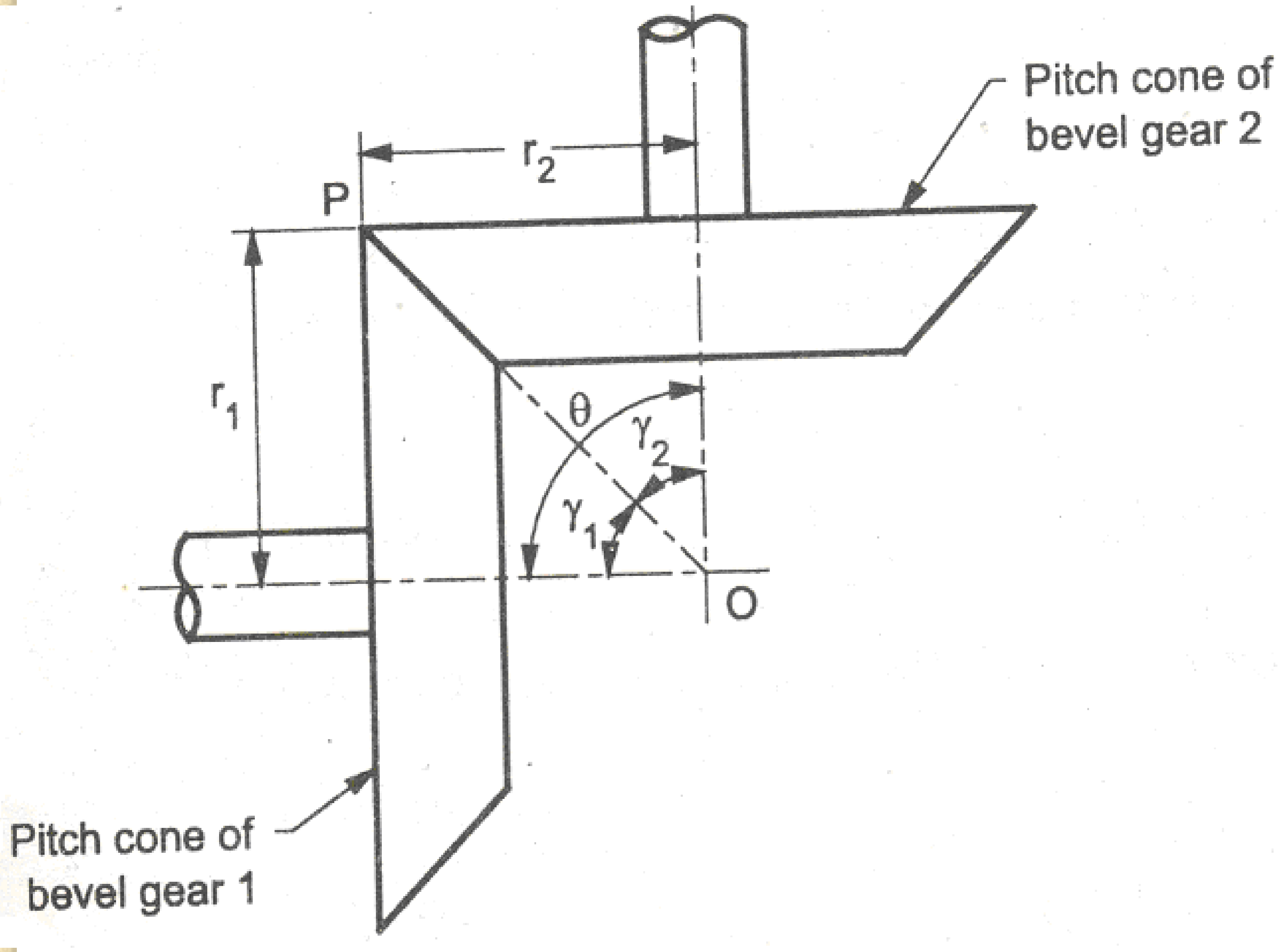
- **Shaft angle (θ) :**

It is the angle between two intersecthg axis of bevel gears. It is equal to sum of the pitch cone angles of both bevel gears. (90^0)

- **Face width (b) :**

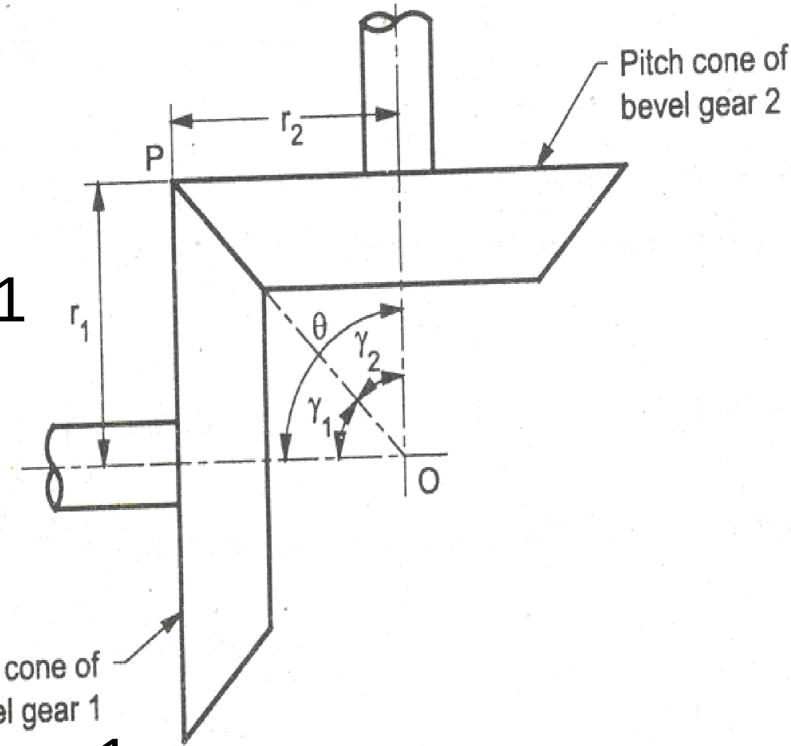
It is the length of tooth measured along the pitch line of pitch cone.

Pitch Cone Angles & Its Geometrical Relations



Pitch Cone Angles & Its Geometrical Relations

Let,



$\boldsymbol{\theta}$ = Shaft angle $= \boldsymbol{\gamma_1 + \gamma_2}$

γ_1 = Pitch cone angle of bevel gear 1

$\boldsymbol{\gamma}_1$ = Pitch cone angle bevel gear 2

$\mathbf{T_1}$ = No. of teeth on bevel gear 1

$\mathbf{T_2}$ = No. of teeth on bevel gear 2

$\boldsymbol{\omega_1}$ = Transverse module for bevel gear 1

$\boldsymbol{\omega}_2$ = Transverse module for bevel gear 2

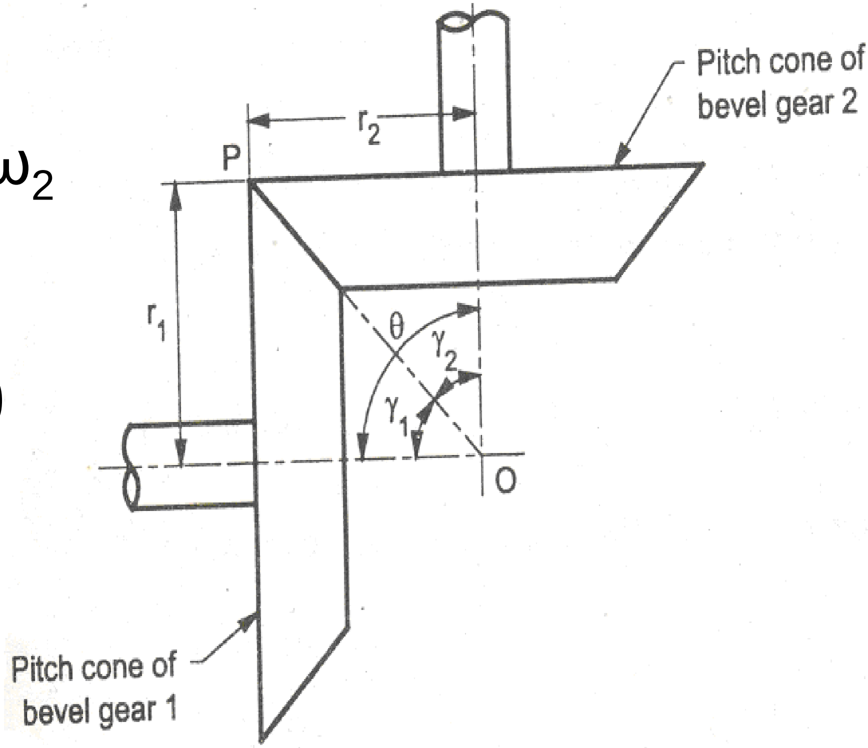
$\mathbf{r_1}$ = PC radius for bevel gear 1

$\mathbf{r_2}$ = PC radius for bevel gear 2

Pitch Cone Angles & Its Geometrical Relations

G = Gear ratio = $T_2/T_1 = r_2/r_1 = \omega_1/\omega_2$

Frm fig;



$$OP = \frac{r_1}{\sin\gamma_1} = \frac{r_2}{\sin\gamma_2} \ldots\ldots\ldots(1)$$

$$\therefore \frac{\sin\gamma_1}{\sin\gamma_2} = \frac{r_1}{r_2}$$

$$\therefore \sin\gamma_1 = \frac{r_1}{r_2}\cdot\sin\gamma_2$$

$$\therefore \sin\gamma_1 = \frac{r_1}{r_2}\cdot\sin(\theta-\gamma_1)\ldots\ldots\ldots\ldots..[\theta = \gamma_1+\gamma_2]$$

$$\therefore \sin\gamma_1 = \frac{r_1}{r_2}\cdot\sin\theta\cos\gamma_1 - \cos\theta\sin\gamma_1$$

Pitch Cone Angles & Its Geometrical Relations

Dividing above eq. by $\cos\gamma_1$;

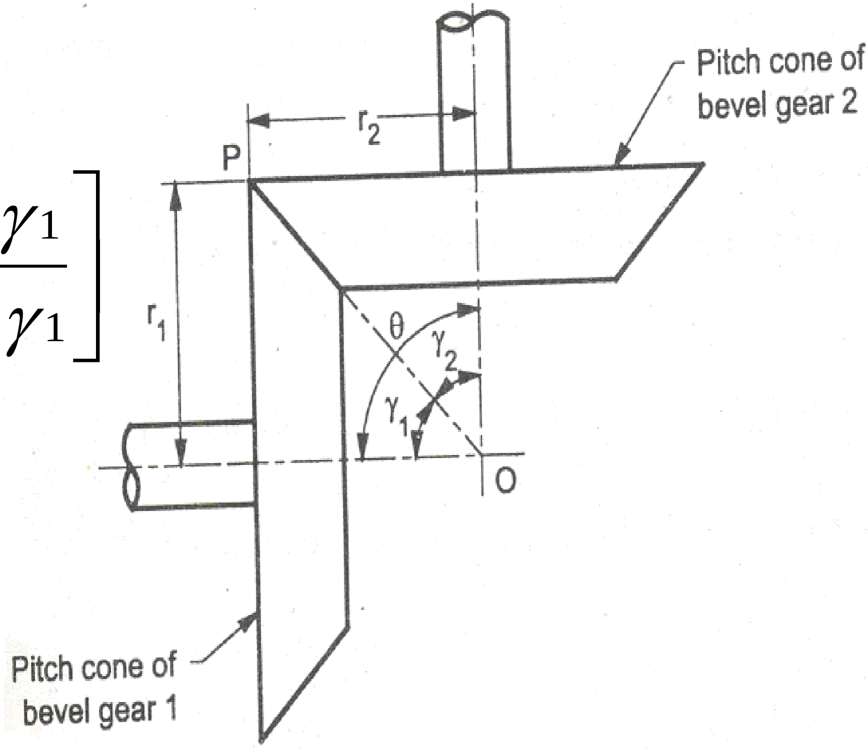
$$\therefore \frac{\sin\gamma_1}{\cos\gamma_1} = \frac{r_1}{r_2}\left[\sin\theta - \cos\theta \cdot \frac{\sin\gamma_1}{\cos\gamma_1}\right]$$

$$\tan\gamma_1 = \frac{r_1}{r_2}\left[\sin\theta - \cos\theta \cdot \tan\gamma_1\right]$$

$$\frac{r_2}{r_1}\tan\gamma_1 = \left[\sin\theta - \cos\theta \cdot \tan\gamma_1\right]$$

$$\frac{r_2}{r_1}\tan\gamma_1 + \cos\theta \cdot \tan\gamma_1 = \sin\theta$$

$$\therefore \tan\gamma_1\left[\frac{r_2}{r_1} + \cos\theta\right] = \sin\theta$$



Pitch Cone Angles & Its Geometrical Relations

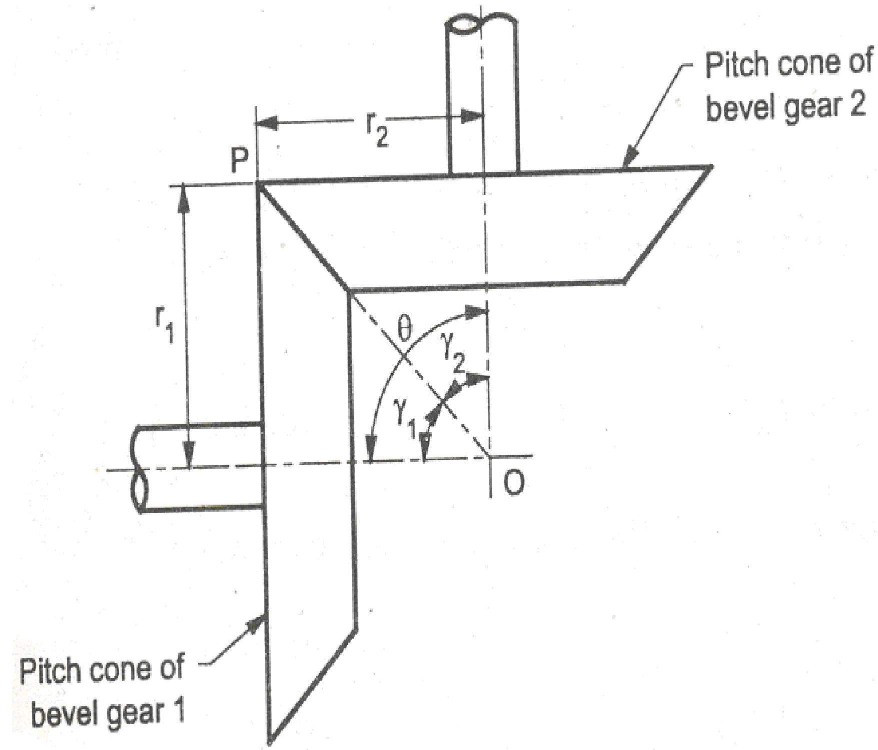
OR

$$\tan\gamma_1 = \frac{\sin\theta}{\left[\frac{r_2}{r_1}+\cos\theta\right]}$$

$$\boxed{\tan\gamma_1 = \frac{\sin\theta}{\left[\frac{\omega_1}{\omega_2}+\cos\theta\right]}}$$

Similarly,

$$\boxed{\tan\gamma_2 = \frac{\sin\theta}{\left[\frac{\omega_2}{\omega_1}+\cos\theta\right]}}$$



Eq. to find out Pitch Cone Angles of bevel pinion & gear

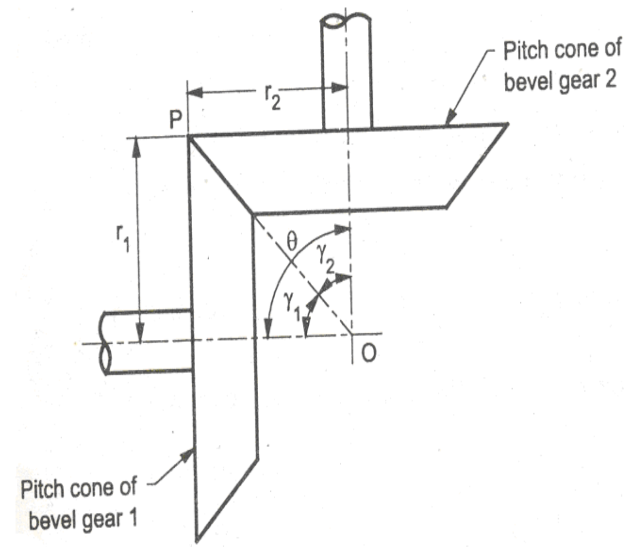
Pitch Cone Angles & Its Geometrical Relations

When Shaft angle θ=90^0

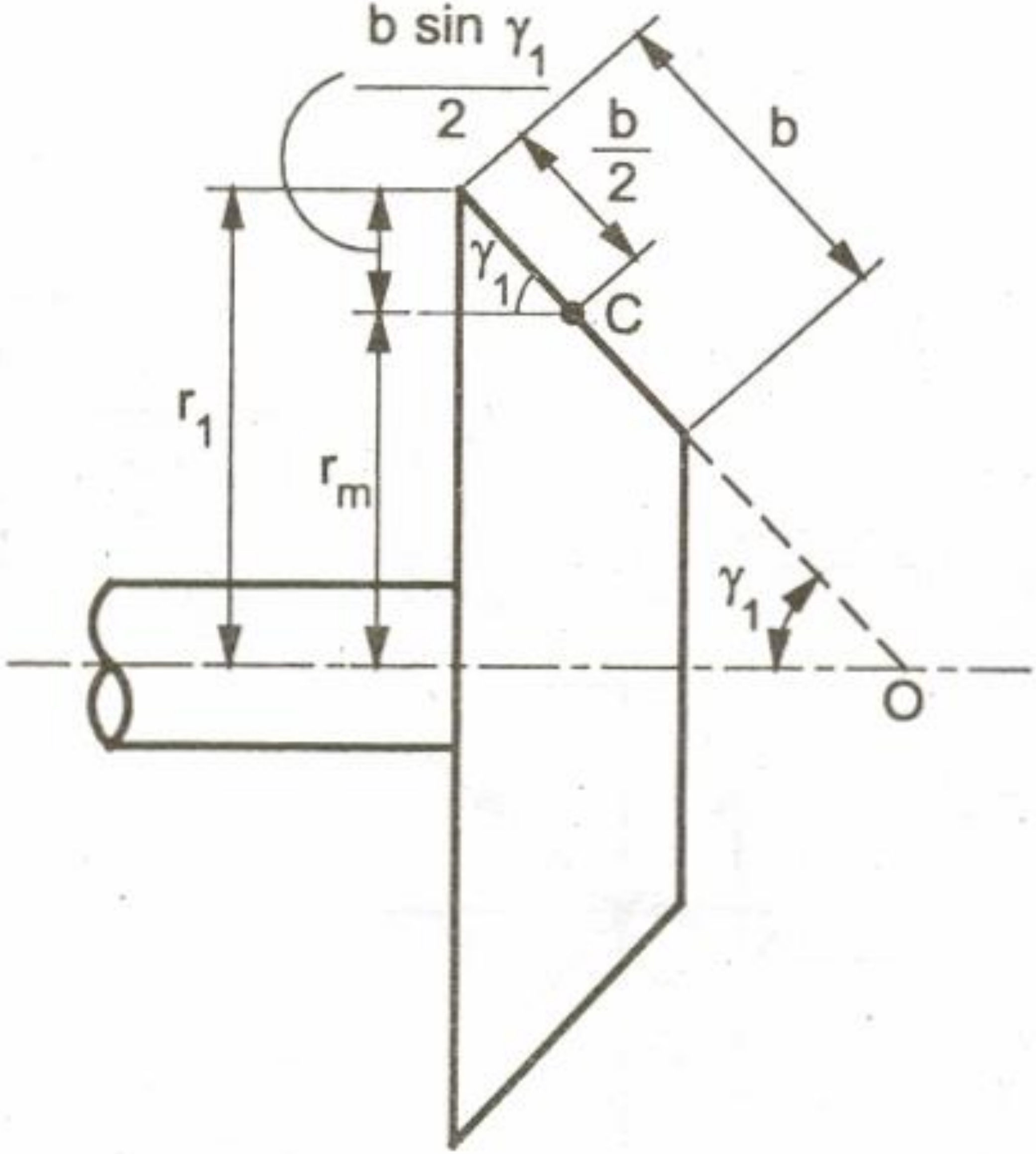
$$\tan\gamma_1 = \frac{\sin 90^0}{\left[\frac{\omega_1}{\omega_2} + \cos 90^0\right]} = \frac{\omega_2}{\omega_1} = \frac{T_1}{T_2} = \frac{1}{G}$$

And

$$\tan\gamma_2 = \frac{\sin 90^0}{\left[\frac{\omega_2}{\omega_1} + \cos 90^0\right]} = \frac{\omega_1}{\omega_1} = \frac{T_2}{T_1} = G$$

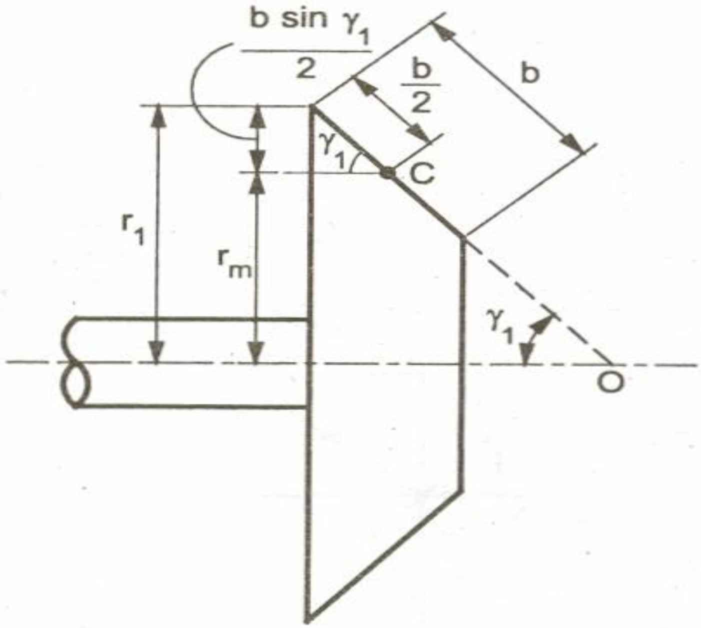


Force Analysis Of Bevel Gears



Force Analysis Of Bevel Gears

Considering pitch cone of bevel gear 1



The resultant force F is acting at the point of contact C on the tooth which is at the mid-point along the face of tooth

Let,

Φ = *Pressure angle*

γ_1 = Pitch cone angle of bevel gear 1

r_m = PC radius for bevel gear at the mid-point of face width

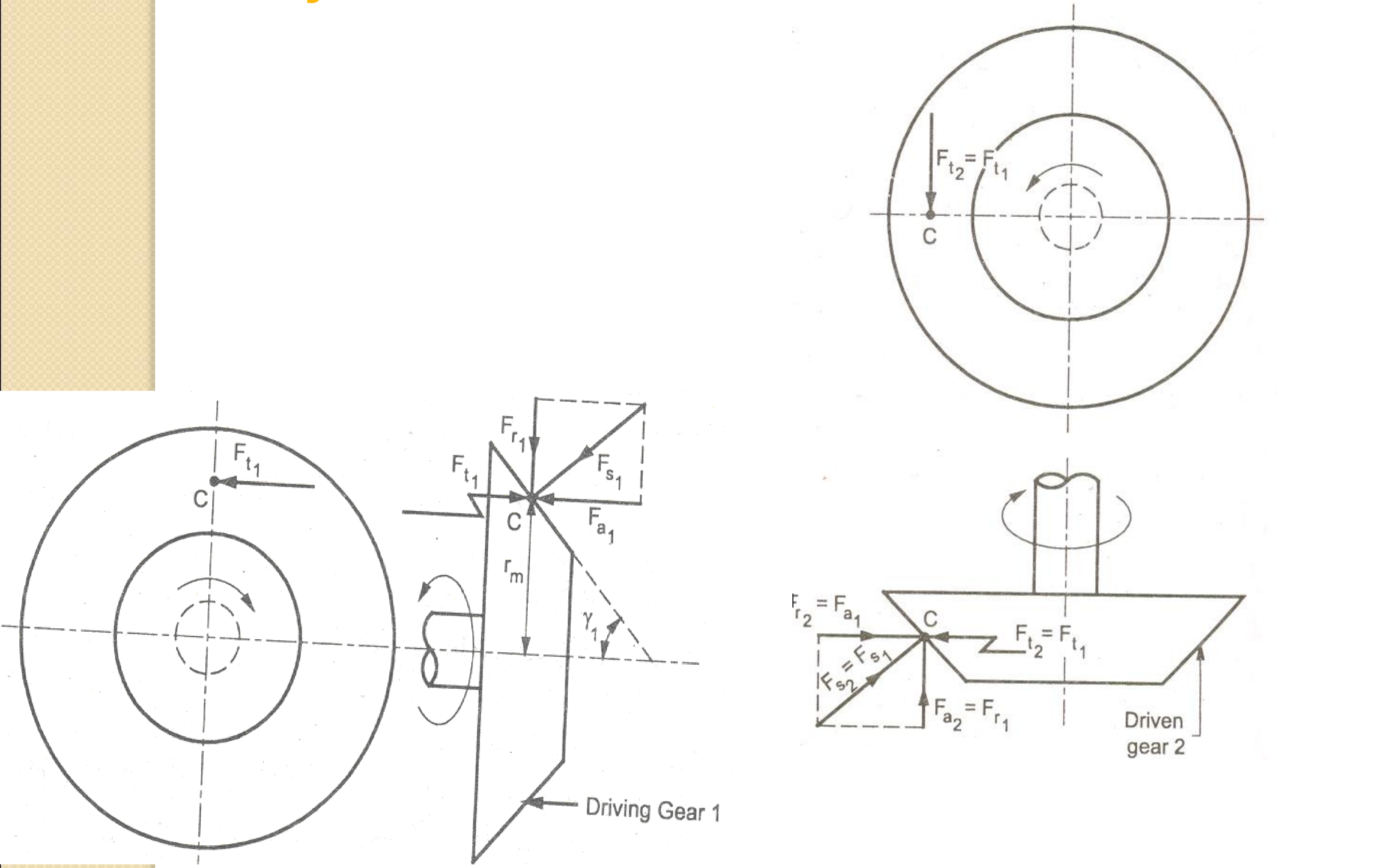
r_1 = PC radius for bevel gear 1

b = Face width

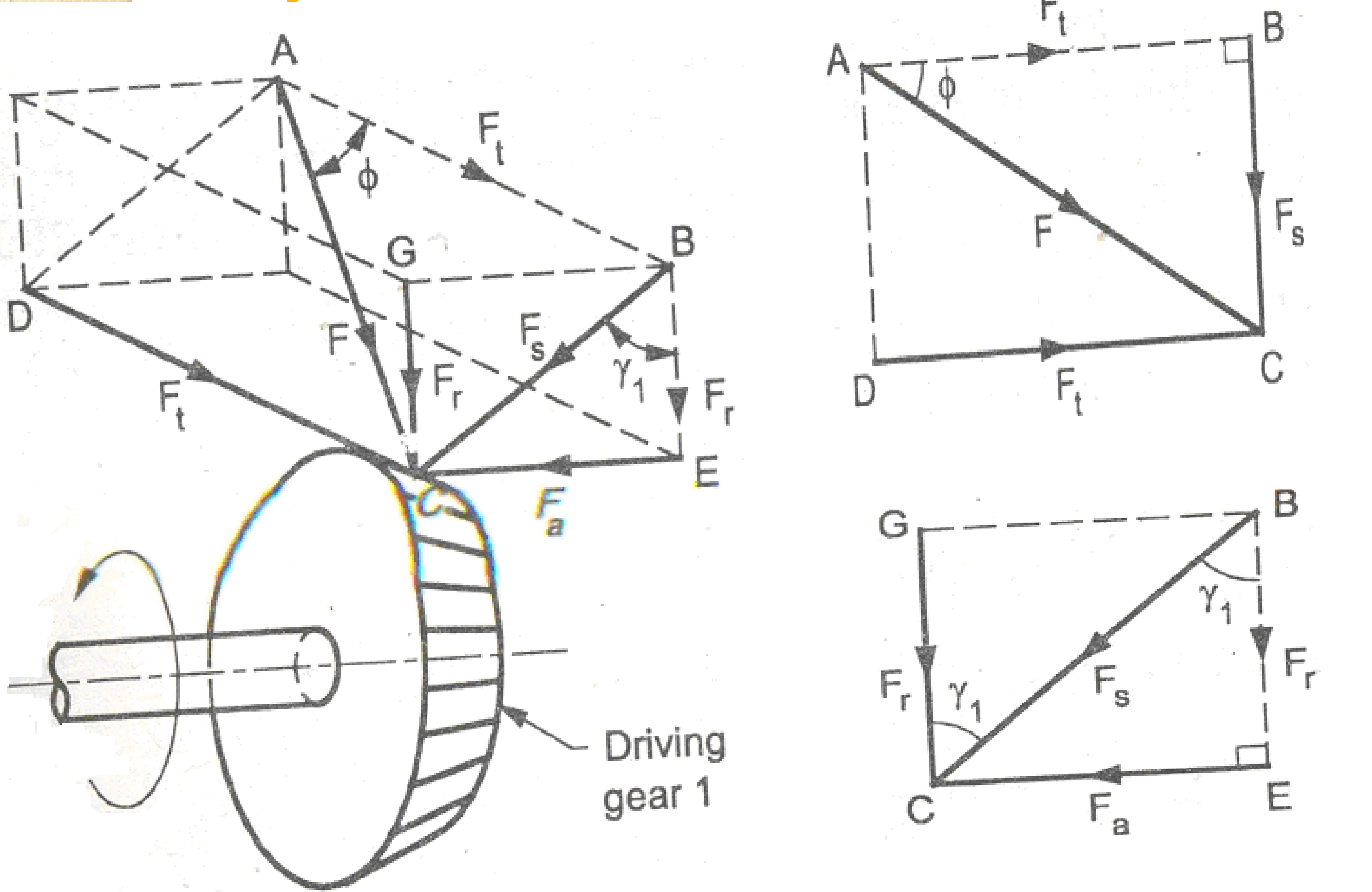
Frm fig.

$$r_m = \left[r_1 - \frac{b \sin \gamma_1}{2} \right]$$

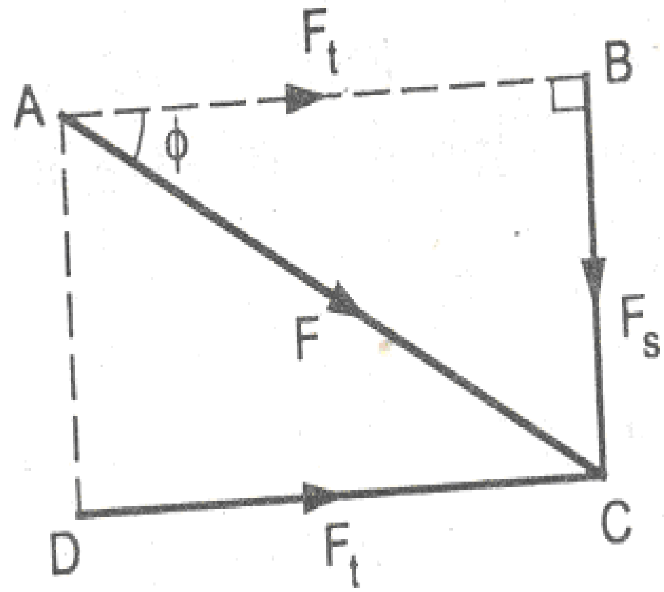
Force Analysis Of Bevel Gears



Force Analysis Of Bevel Gears



Force Analysis Of Bevel Gears



Consider plane ABCD……From ΔABC

$$F_s = F_t \tan\phi ....(1)$$

Consider plane BEGC……From ΔBEC

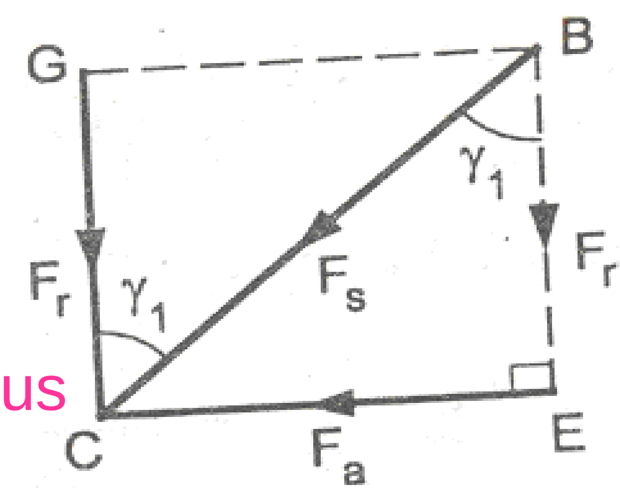
$$F_a = F_s \sin\gamma_1$$

$$F_r = F_s \cos\gamma_1$$

From eq.(1), we get F_a & F_r as

$$F_a = F_t \tan\phi \cdot \sin\gamma_1 ....(2)$$

$$F_r = F_t \tan\phi \cdot \cos\gamma_1 ....(3)$$



Eq(1), (2) & (3) are used to determine verious forces acting on driving gear 1

Force Analysis Of Bevel Gears

The forces acting on driven gear 2 can be determined by considering actions & reactions in equal & opposite directions.

Therefore,

axial force of driving gear 2 = radial force of driving gear 1

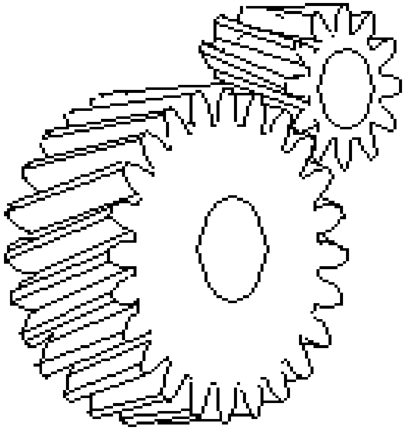
i.e., $F_{a2} = F_{r1}$

radial force of driving gear2 = axial force of driving gear

i.e., $F_{r2} = F_{a1}$

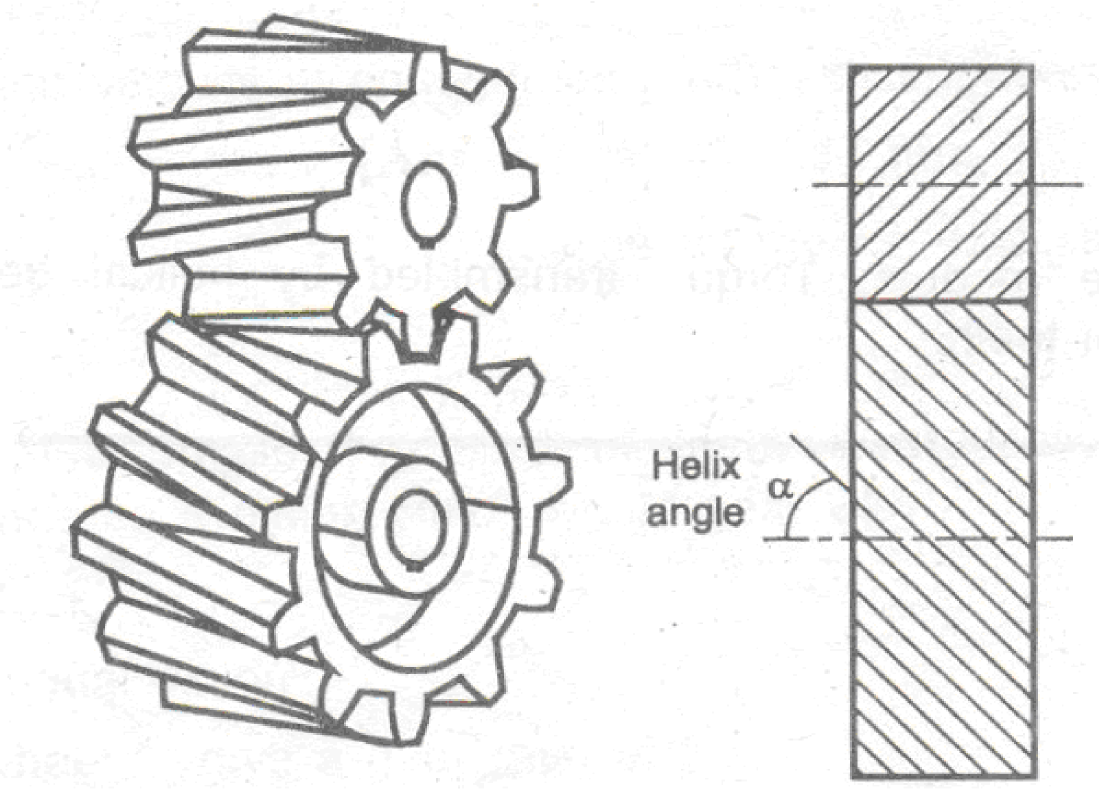
Helical Gear

- Teeth are at an angle; Helix angle (α)7^0 to 23^0)
- Gradual engagement of teeth reduces shocks & Stresses
- More smooth & quiet operation
- Used for high speed transmission & efficiency is frm
- Tooth strength is greater because the teeth are longer
- Greater surface contact on the teeth thus carry more load than a spur gear
- Used in automobiles

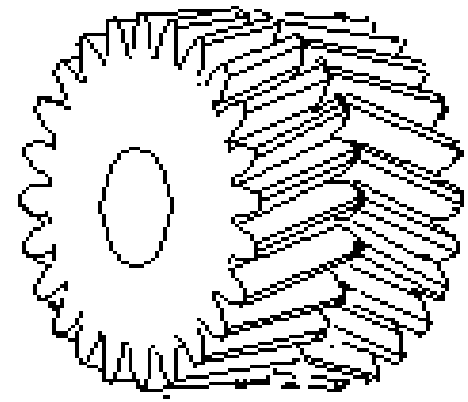


HELICAL GEARS

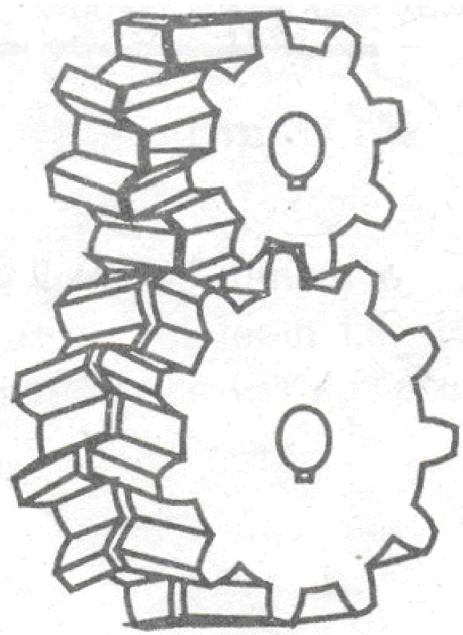
- **Disadvantage:**
  - -Longer surface of contact reduces the efficiency of a helical gear relative to a spur gear
  - -They induce axial thrust in one direction on bearing

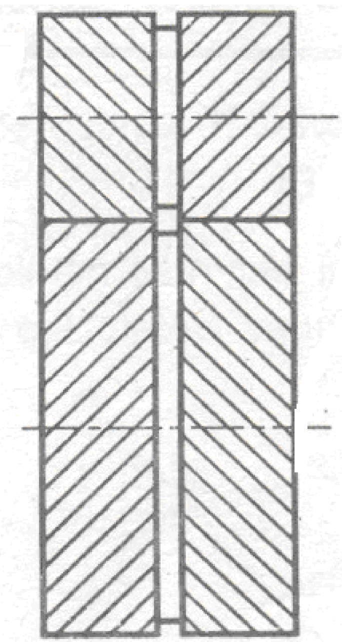


Herringbone Gears

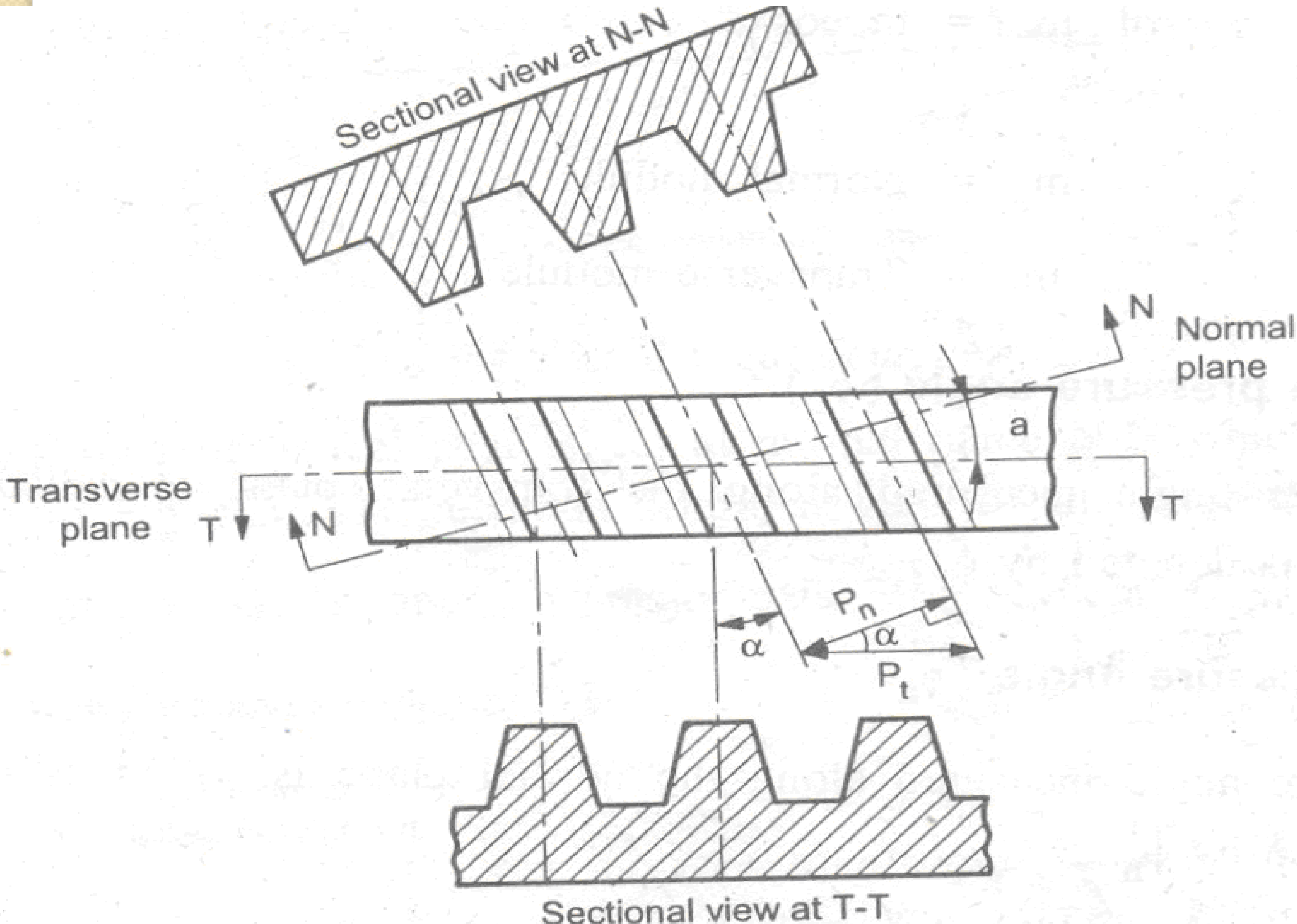


- Two helical gears of identical pitch & of opposite hand
- Axial thrust of two gears act in opposite direction, thus…
- Problem of axial thrust is eliminated

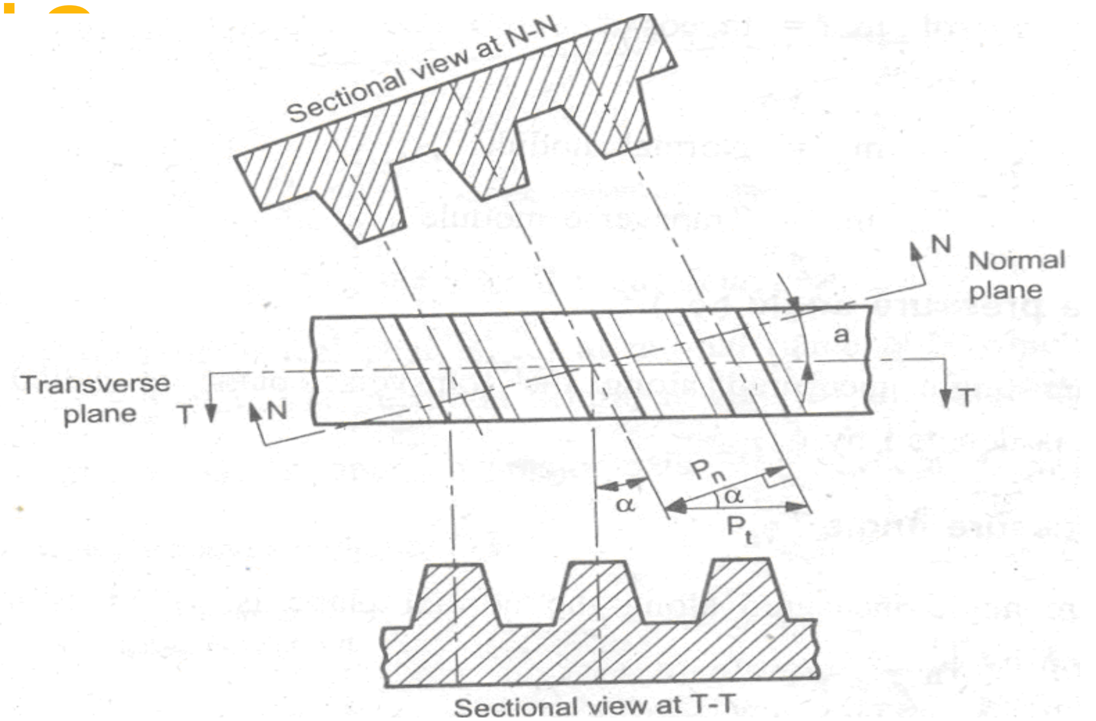




Terminology Of Helical Gears



Terminology Of Helica



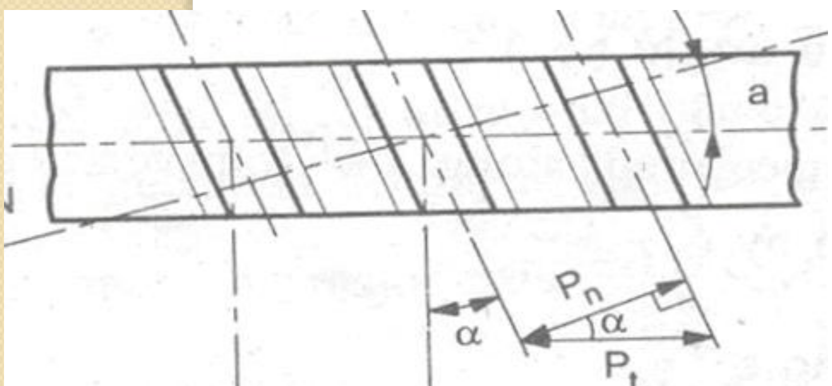
- **Helix angle (α):-**
  It is the angle between axis of shaft and center line of teeth. It is the angle at which teeth are inclined to the axis of the gear. It is denoted by α

- **Transverse circular pitch (P_t):-**
  It is the distance between the corresponding points on adjacent teeth measured along PC in transverse plane. It is denoted by P_t

Terminology Of Helical Gears

- **Normal circular pitch (P_n):-**

It is the distance between the corresponding points on adjacent teeth measured along PC in normal plane. It is denoted by P_n



$$\cos\alpha = \frac{P_n}{P_t}$$

$$P_n = P_t \cos\alpha ....(1)$$

$$But,$$

$$P_c = \pi \cdot m$$

$$\therefore eq(1).....$$

$$\pi \cdot m_n = \pi \cdot m_t \cdot \cos\alpha$$

$$\boxed{\therefore m_n = m_t \cos\alpha}$$

Where, m_n = Normal module

m_t = Transverse module

Terminology Of Helical Gears

- Axial Pitch (**P_a):-** It is the distance between the corresponding points on adjacent teeth measured along axial direction.
- **Transverse pressure angle (Φ_t)**

  The pressure angle measured along the transverse plane.

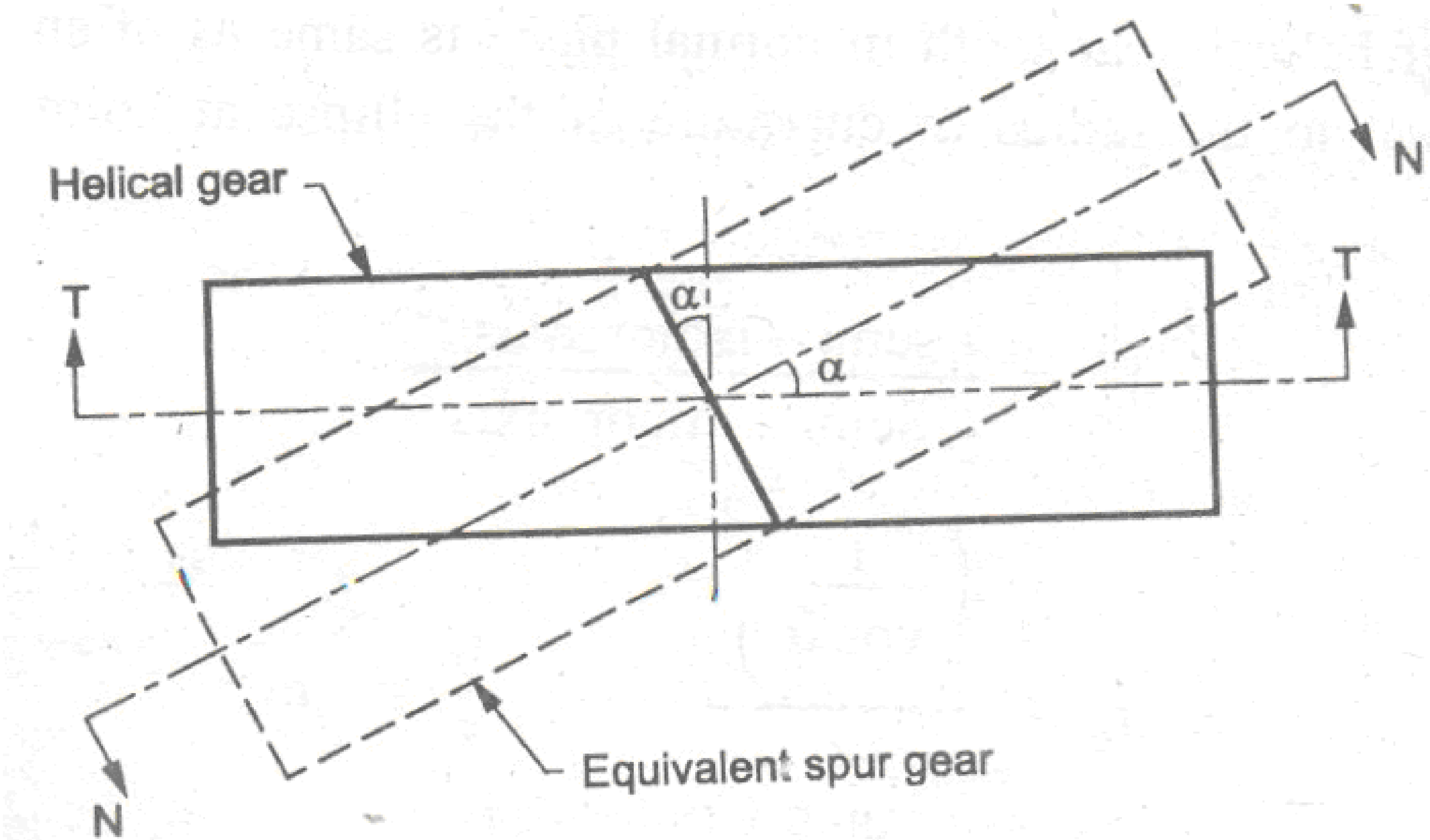
- **Normal pressure angle (Φ_n)**

  The pressure angle measured along the normal plane.

$$\boxed{\cos\alpha = \frac{\tan\phi_n}{\tan\phi_t}}$$

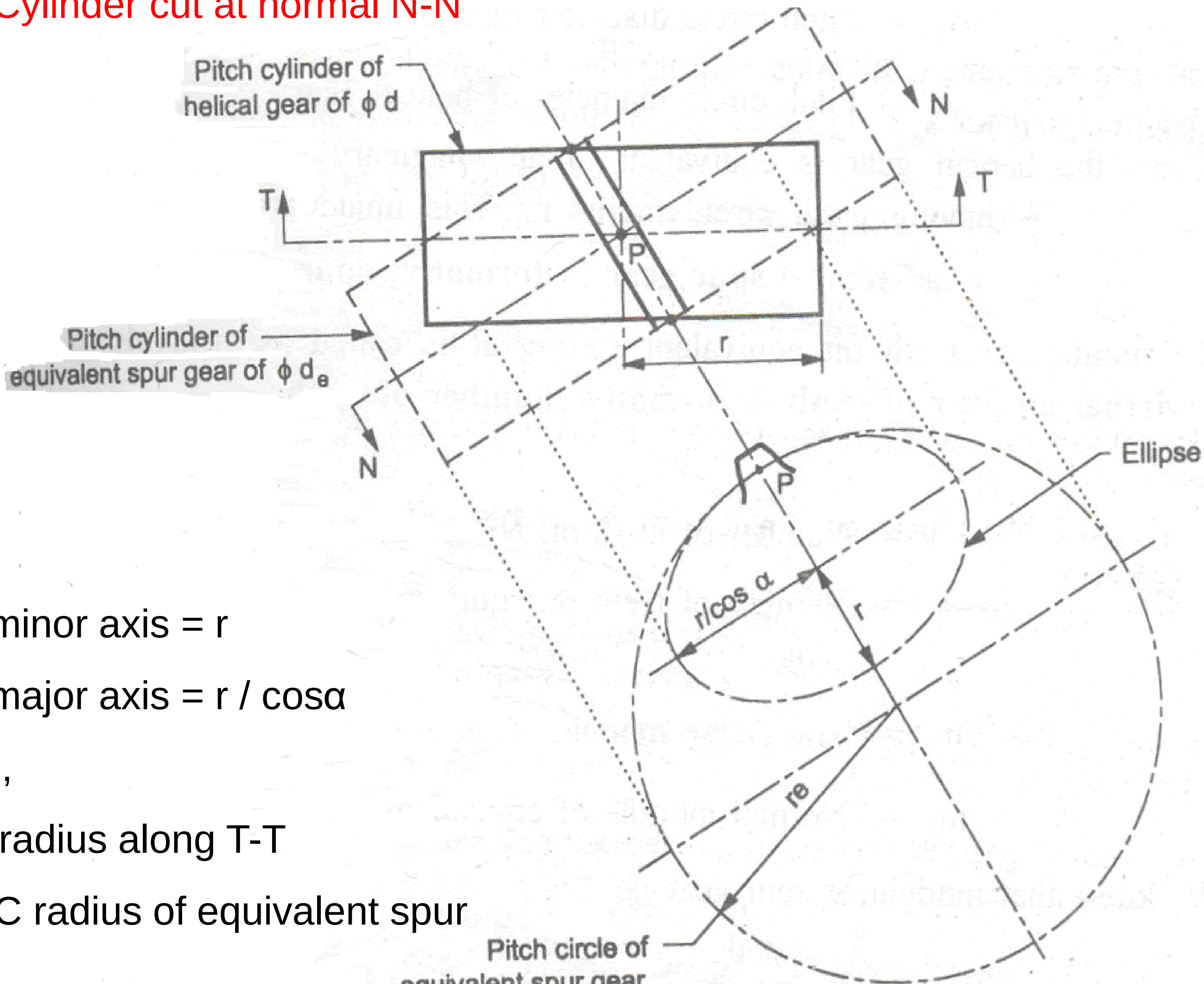
Centre distance
Gear ratio

Virtual Number Of Teeth/Number Of Teeth On Equivalent Spur Gear



If the helical gear is viewed along the normal plane, it will appear as a spur gear

Pitch Cylinder cut at normal N-N



Semi-minor axis = r

Semi-major axis = r / cosα

Where,

r- PC radius along T-T

r_e – PC radius of equivalent spur gear

Virtual Number Of Teeth

Let, r_e – PC radius of equivalent spur gear
d_e – PC diameter of equivalent spur gear
r– PC radius of helical gear

$$r_e = \frac{(\text{Semi-major axis})^2}{\text{Semi-minor axis}}$$

$$r_e = \frac{\left[\dfrac{r}{\cos\alpha}\right]^2}{r}$$

$$\boxed{r_e = \frac{r}{\cos^2\alpha}}$$

OR

$$\boxed{d_e = \frac{d}{\cos^2\alpha}}$$

Virtual Number Of Teeth

Hence, the helical gear is equivalent to an imaginary spur gear, which is in normal plane N-N, having pitch circle radius r. This imaginary spur gear is called equivalent spur gear or virtural spur gear or formative spur gear.

The number of teeth on equivalent spur gear is called as equivalent number teeth or virtual number of teeth or formative number of teeth.

Virtual Number Of Teeth

Let,

t = Number of teeth on helical gear

t_e = Number of teeth on equivalent spur gear or virtual number of teeth

m_t = Transverse module of helical gear

m_n = Normal module of equivalent spur gear

Virtual Number Of Teeth

- We know that module of spur gear is,

$$m_n = \frac{d_e}{t_e}$$

or

$$t_e = \frac{d_e}{m_n}$$

or

$$t_e = \frac{d}{m_n \cos^2 \alpha} \qquad \text{...........}\left[\because de = \frac{d}{\cos^2 \alpha}\right]$$

or

$$t_e = \frac{d}{m_t \cos^3 \alpha} \qquad \text{......(1)......}\left[\because m_n = m_t \cos\alpha\right]$$

Virtual Number Of Teeth

- We know that module of helical gear is,

$$m_t = \frac{d}{t}$$

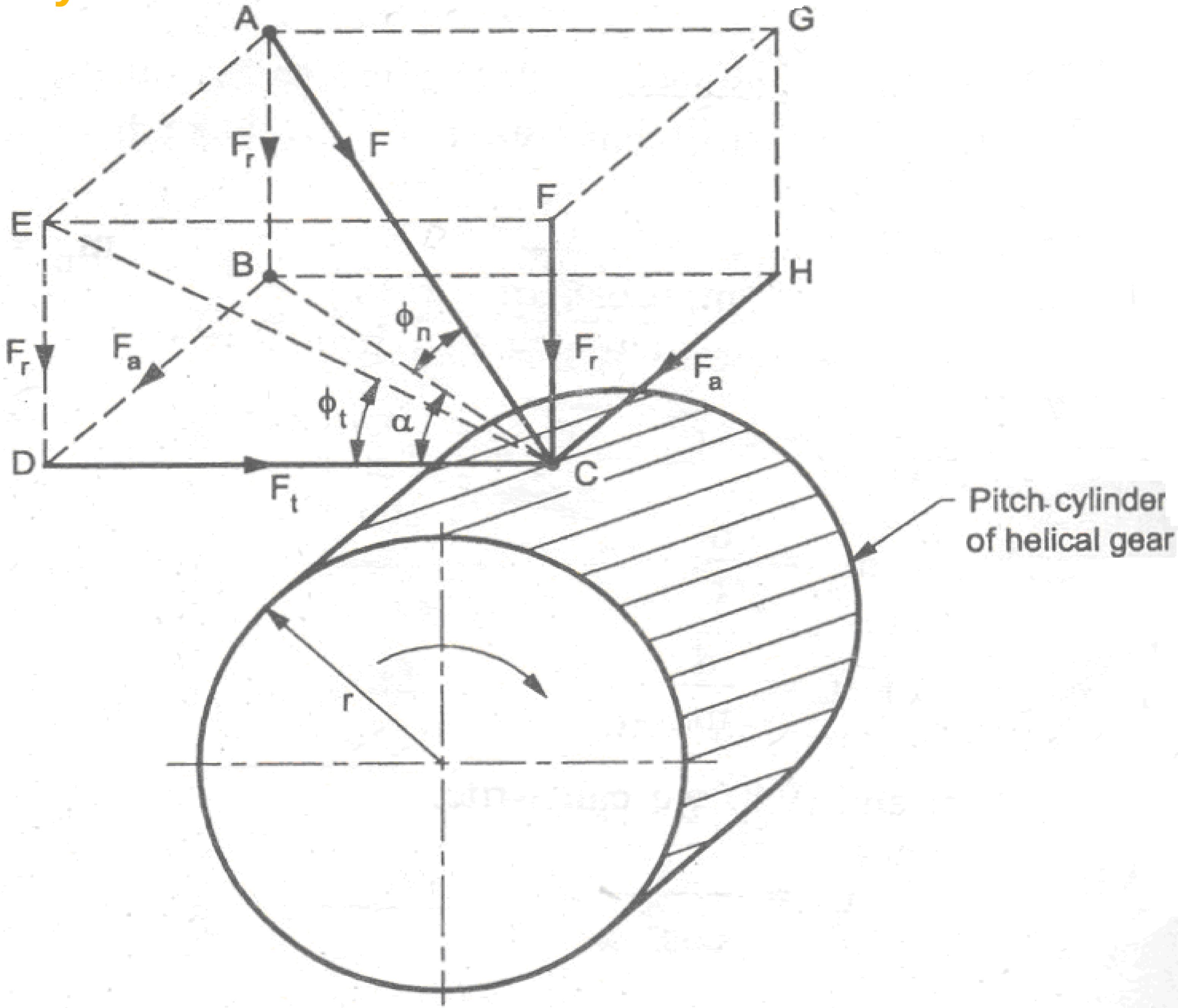
or

$$t = \frac{d}{m_t} \qquad \qquad \ldots\ldots\ldots\ldots(2)$$

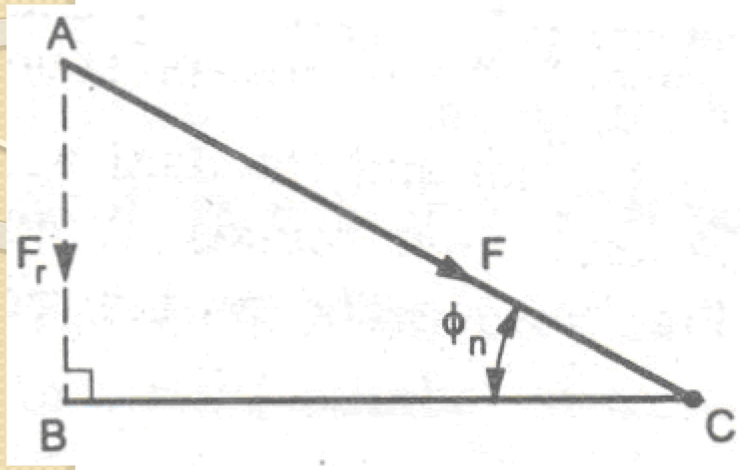
From eq (1) & (2)

$$\boxed{t_e = \frac{t}{\cos^3 \alpha}}$$

Force Analysis Of Helical Gears



Force Analysis Of Helical Gears

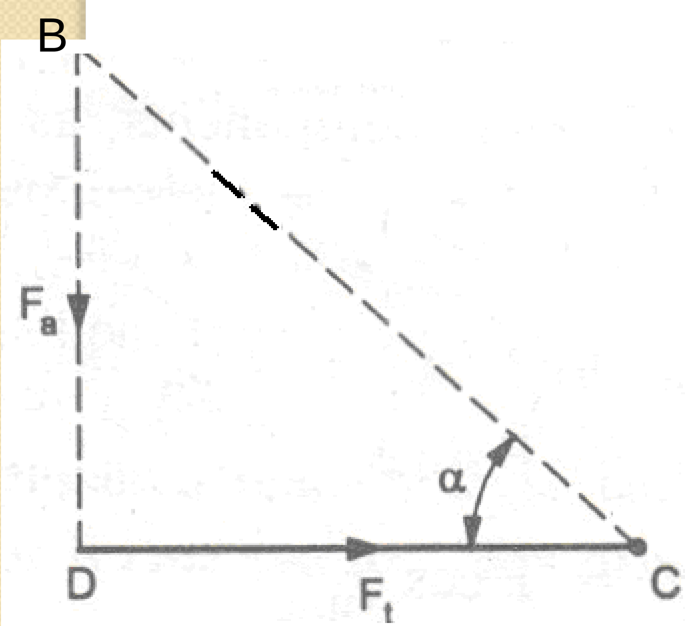


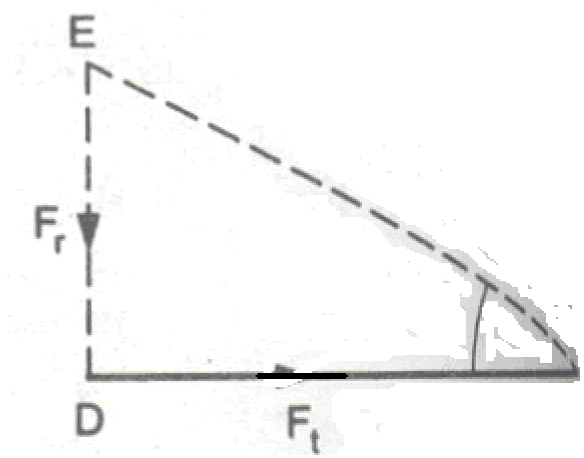
Resultant force F resolved into three components:-

Ft - Tangential Force

Fr - Radial Force

Fa - Axial Force





Force Analysis Of Helical Gears

From fig (a);

$$F_r = F\sin\phi_n .....(1)$$

$$BC = F\cos\phi_n$$

From fig (b); $F_t = BC \times \cos\alpha$

$$F_t = F\cos\phi_n \cdot \cos\alpha ....(2)$$

$$F_a = BC\sin\alpha$$

$$F_a = F\cos\phi_n \cdot \sin\alpha ....(3)$$

$$Dividing (3) by (2)...,$$

$$\frac{F_a}{F_t} = \frac{F\cos\phi_n \cdot \sin\alpha}{F\cos\phi_n \cdot \cos\alpha}$$

$$F_a = F_t \tan\alpha$$

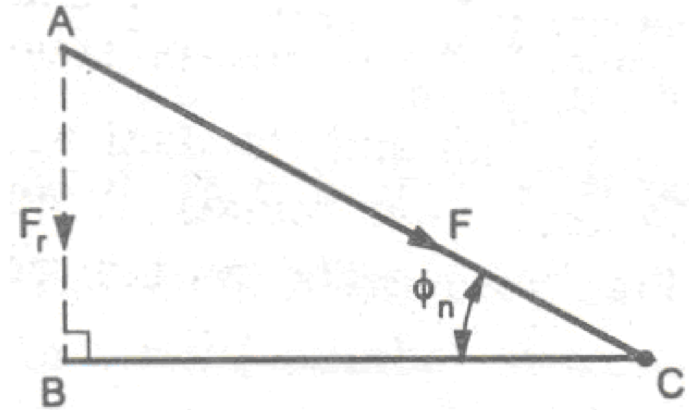


Fig. (a)

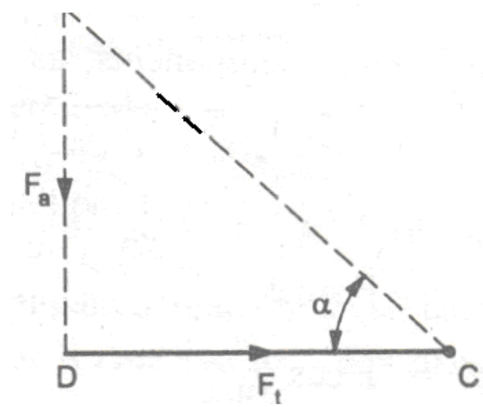


Fig. (b)

Force Analysis Of Helical Gears

From eq (1) & (2)……

$$F_r = \frac{F_t}{\cos\phi_n \cdot \cos\alpha} \cdot \sin\phi_n$$

$$\therefore F_r = F_t\left[\frac{\tan\phi_n}{\cos\alpha}\right]\ldots\ldots\ldots\ldots(4)$$

Also tangential force acting on tooth is,

$$F_t = \frac{T}{r}$$

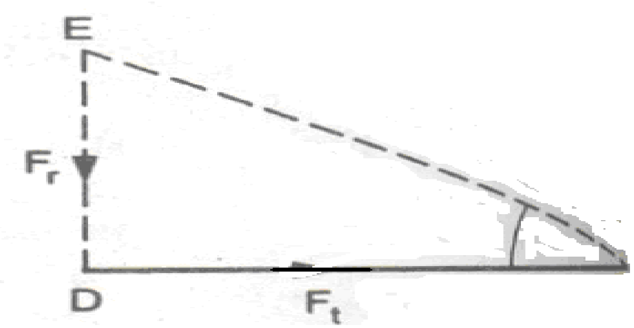
Where,

T= torque transmitted by helical gear

r =pitch circle radius of helical gear

From fig(3)……………

$$\tan\phi_t = \frac{F_r}{F_t}\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

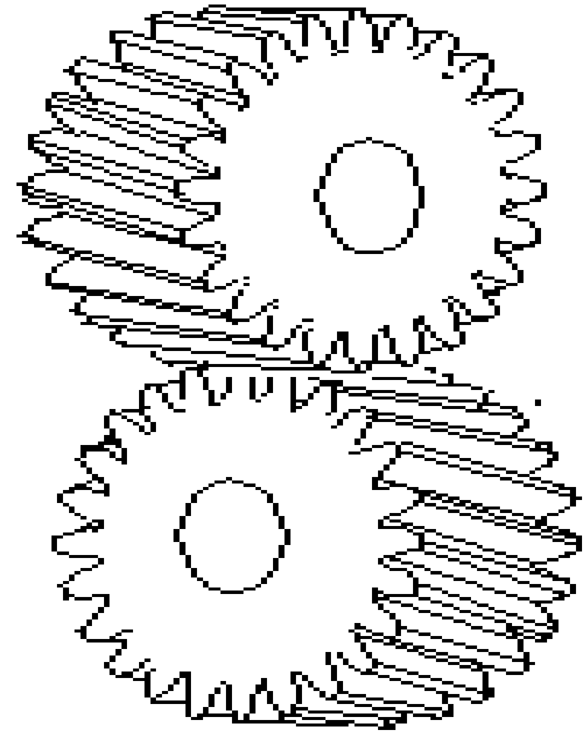


Force Analysis Of Helical Gears

From eq (4)……. $$\cos\alpha = \frac{\tan\phi_n}{\frac{F_r}{F_t}} \qquad \text{.......... ....(6)}$$

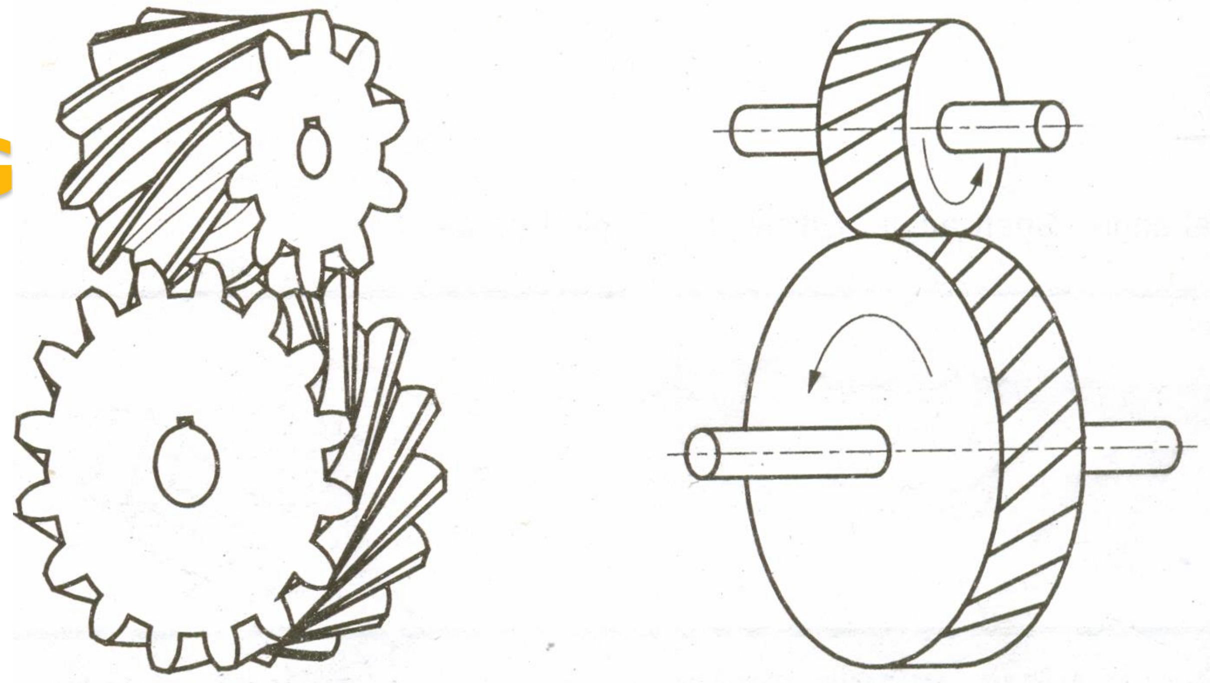
From eq (5) & (6)……. $$\boxed{\cos\alpha = \frac{\tan\phi_n}{\tan\phi_t}}$$

Spiral Gears



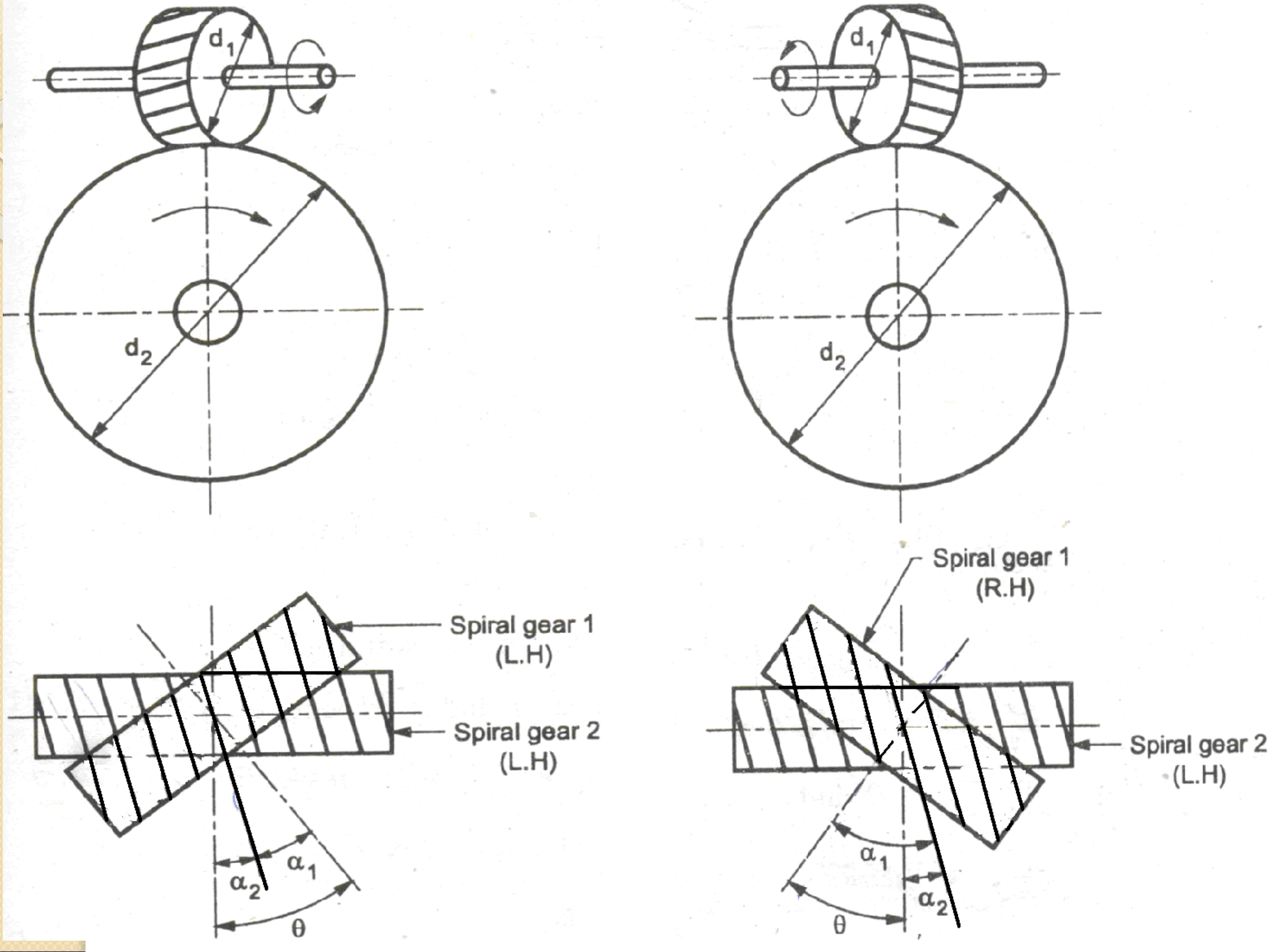
- Skew Gears / Crossed Helical Gears
- Used for Non-parallel & Non-intersecting shafts
- Point contact between mating teeth
- Low load transmission
- Used in distribution devices of automobile engines/ measurement instruments

Spiral G



- Two meshing gears can have same or opposite hands
- Helix angle in case of spiral angle is known as *Spiral Angle*
- If spiral angles are different, Transverse module m_t is also different
- For specifying size of spiral gears Normal module m_n is always used

Shaft Angle Of Spiral Gears



Shaft Angle Of Spiral Gears

- The shaft angle (θ) is the angle through which one of the shaft must be rotated so that it is parallel to the other shaft
- If the hands of two meshing spiral gears are same, then the shaft angle θ is given by,

$$\theta = \alpha_1 + \alpha_2$$

- If the hands of two meshing spiral gears are different, then the shaft angle θ is given by,

$$\theta = \alpha_1 - \alpha_2$$

Centre Distance Between Spiral Gears

l**=** C.D. between spiral gears

$\boldsymbol{\alpha_1, \alpha_2}$= Spiral angles for gear 1 & 2

$\mathbf{Z_1, Z_2}$ = No. of teeth on gear 1 & 2

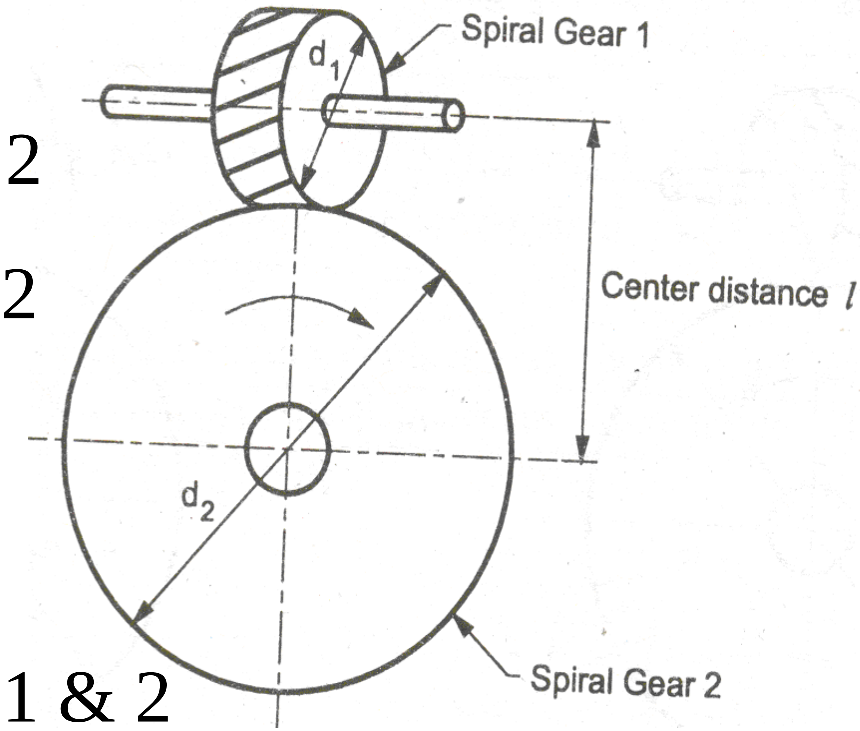
$\mathbf{N_1, N_2}$= Speed of gear 1 & 2

G= Gear ratio=$\mathbf{Z_2/Z_1=N_1/N_2}$

$\mathbf{mt_1, mt_2}$= Transverse module for 1 & 2

$\mathbf{m_n}$= Normal module for 1& 2

$\mathbf{r_1, r_2}$ = PC radius for 1 & 2



Centre Distance Between Spiral Gears

We know that pitch circle radius of gear is,

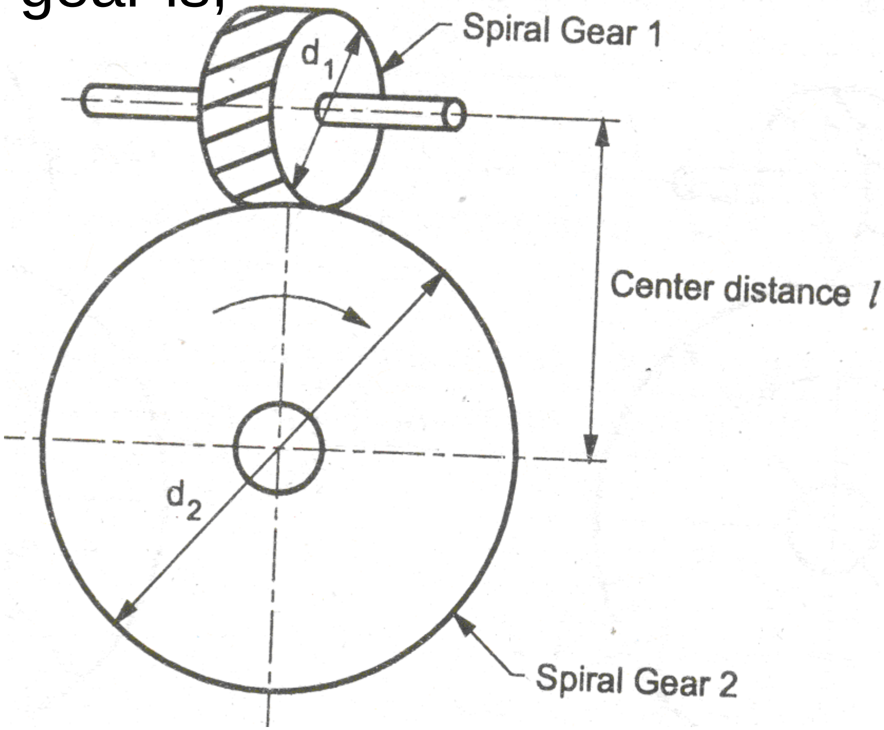
$$r = \frac{m_t Z}{2}$$

Pitch circle radius of gear 1 is,

$$r_1 = \frac{m_{t1} Z_1}{2}$$

Pitch circle radius of gear 2 is,

$$r_2 = \frac{m_{t2} Z_2}{2}$$



Centre Distance Between Spiral Gears

The centre distance between two spiral gear is,

$$l = r1 + r2$$

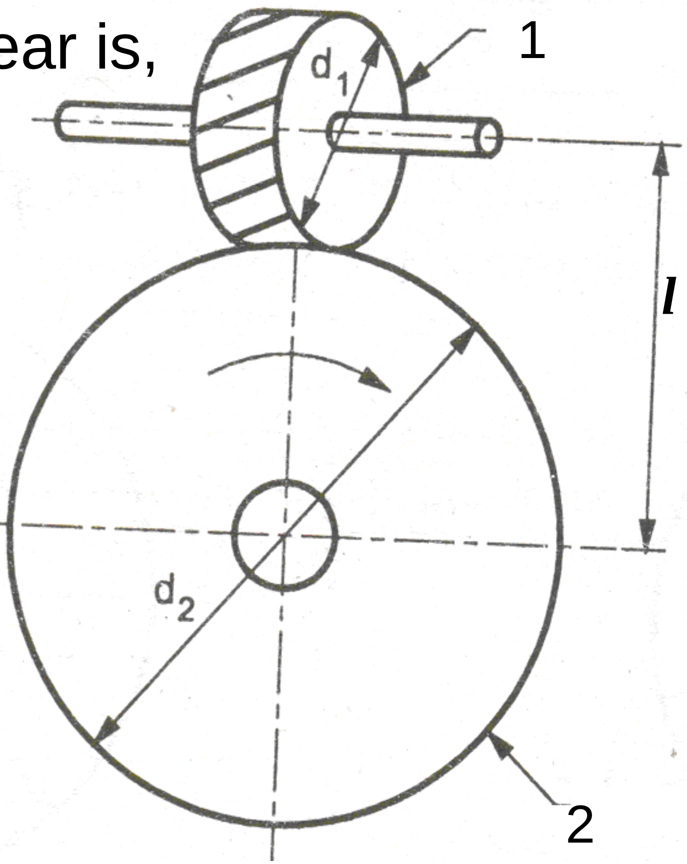
$$\therefore l = \frac{m_{t1} \cdot T_1}{2} + \frac{m_{t2} \cdot T_2}{2}$$

$$\therefore l = \frac{m_n}{\cos\alpha_1} \cdot \frac{T_1}{2} + \frac{m_n}{\cos\alpha_2} \cdot \frac{T_2}{2}$$

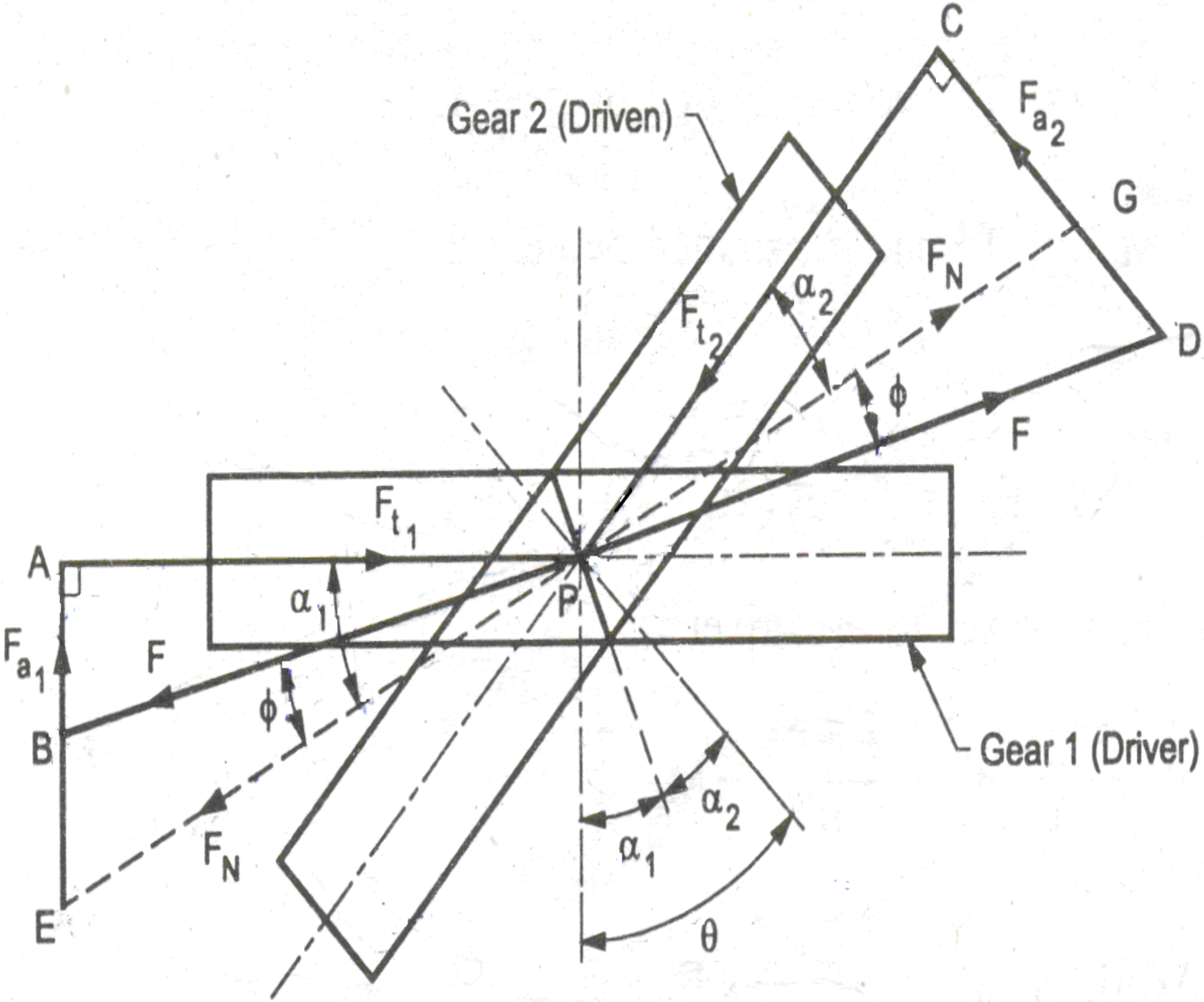
$$\therefore l = \frac{m_n}{2} \cdot \left[\frac{T_1}{\cos\alpha_1} + \frac{T_2}{\cos\alpha_2} \right]$$

$$\therefore l = \frac{m_n T_1}{2} \cdot \left[\frac{1}{\cos\alpha_1} + \frac{T_2 / T_1}{\cos\alpha_2} \right]$$

$$\therefore l = \frac{m_n T_1}{2} \cdot \left[\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2} \right]$$



Efficiency Of Spiral Gears



Efficiency Of Spiral Gears

F_{t1} = Tangential force acting on gear 1

F_{t2} = Tangential force acting on gear 2

F_a = Axial force acting on gear 1

F_{a2} = Axial force acting on gear 2

F_N = Normal reaction at the point of contact

F = Resultant force/ Resultant reaction at pt of contact

α_1 = Spiral angles for gear 1

α_2 = Spiral angles for gear 2

Efficiency Of Spiral Gears

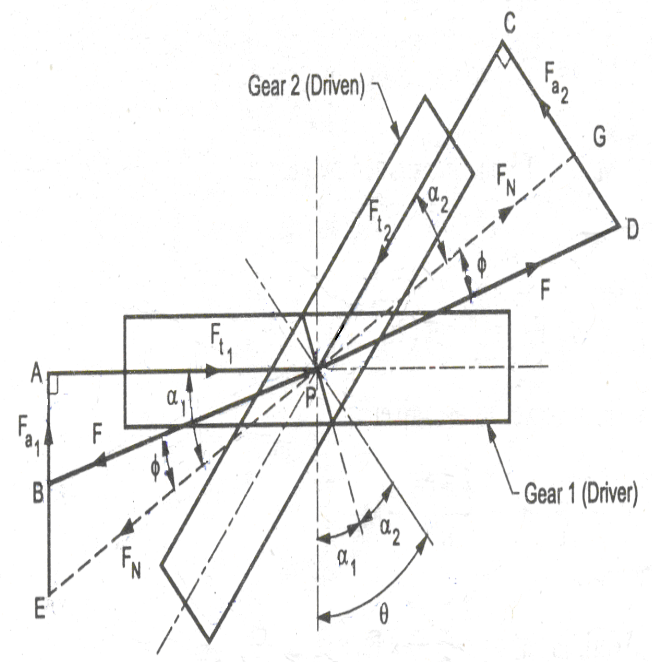
- θ = Shaft angle = $\alpha 1 + \alpha 2$
- Φ = Angle of friction
- N1 = Speed of gear 1
- N2 = Speed of gear 2
- G = Gear ratio = T2/T1 = N1/N2
- mt1 = Transverse module for 1
- mt2 = Transverse module for 2
- mn = Normal module for 1 & 2
- d1 = PC diameter for 1
- d2 = PC diameter for 2

Efficiency Of Spiral Gears

$$From....\Delta PAB,$$

$$F_{t1} = F\cos(\alpha_1 - \phi)$$

Work i/p or i/p power to the driver is;



$$\therefore work(i/p) = \frac{2\pi N_1}{60} \times M_{t1}$$

$$\therefore work(i/p) = \frac{2\pi N_1}{60} \times F_{t1} \times r_1$$

$$\therefore work(i/p) = \frac{2\pi N_1}{60} \times F_{t1} \times \frac{d_1}{2}$$

$$\therefore work(i/p) = \frac{\pi d_1 N_1}{60} \times F_{t1}$$

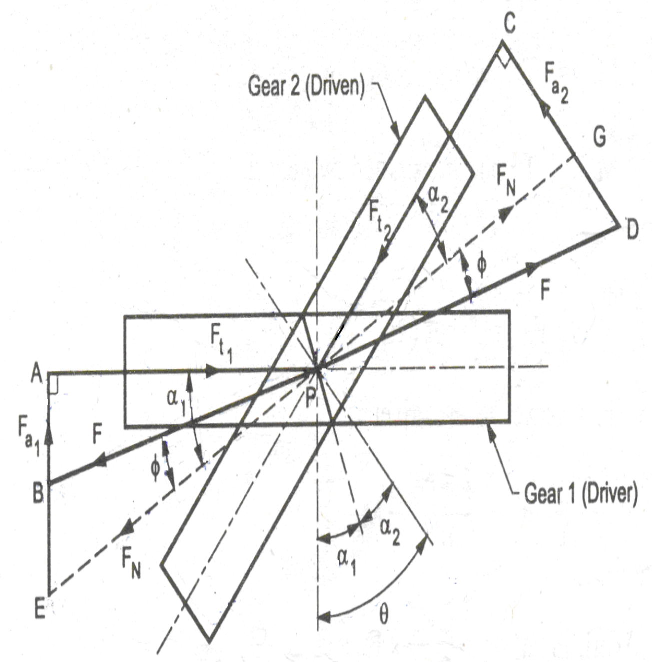
$$\therefore work(i/p) = \frac{\pi d_1 N_1}{60} \times F(\cos\alpha_1 - \phi)..................(1)$$

Efficiency Of Spiral Gears

$$From....\Delta PCD,$$

$$F_{t2} = F\cos(\alpha_2 + \phi)$$

Work i/p or i/p power to the driver is;



$$\therefore work(o/p) = \frac{2\pi N_2}{60} \times M_{t2}$$

$$\therefore work(o/p) = \frac{2\pi N_2}{60} \times F_{t2} \times r_2$$

$$\therefore work(o/p) = \frac{2\pi N_2}{60} \times F_{t2} \times \frac{d_2}{2}$$

$$\therefore work(o/p) = \frac{\pi d_2 N_2}{60} \times F_{t2}$$

$$\therefore work(o/p) = \frac{\pi d_2 N_2}{60} \times F(\cos\alpha_2 + \phi)................(2)$$

Efficiency Of Spiral Gears

The Efficiency of Spiral gear is;

$$\eta = \frac{work(o/p)}{work(i/p)}$$

$$\eta = \frac{\frac{\pi d_2 N_2}{60} \times F(\cos\alpha_2 + \phi)}{\frac{\pi d_1 N_1}{60} \times F(\cos\alpha_1 - \phi)}$$

$$\eta = \frac{d_2 N_2 (\cos\alpha_2 + \phi)}{d_1 N_1 (\cos\alpha_1 - \phi)}$$

Efficiency Of Spiral Gears

We know that;

$$m_n = m_{t1} \cos\alpha_1$$

$$\therefore m_n = \left[\frac{d_1}{T_1}\right]\cos\alpha_1 \ldots\ldots\ldots\ldots\ldots \left[\because m_t = \frac{d}{T}\right]$$

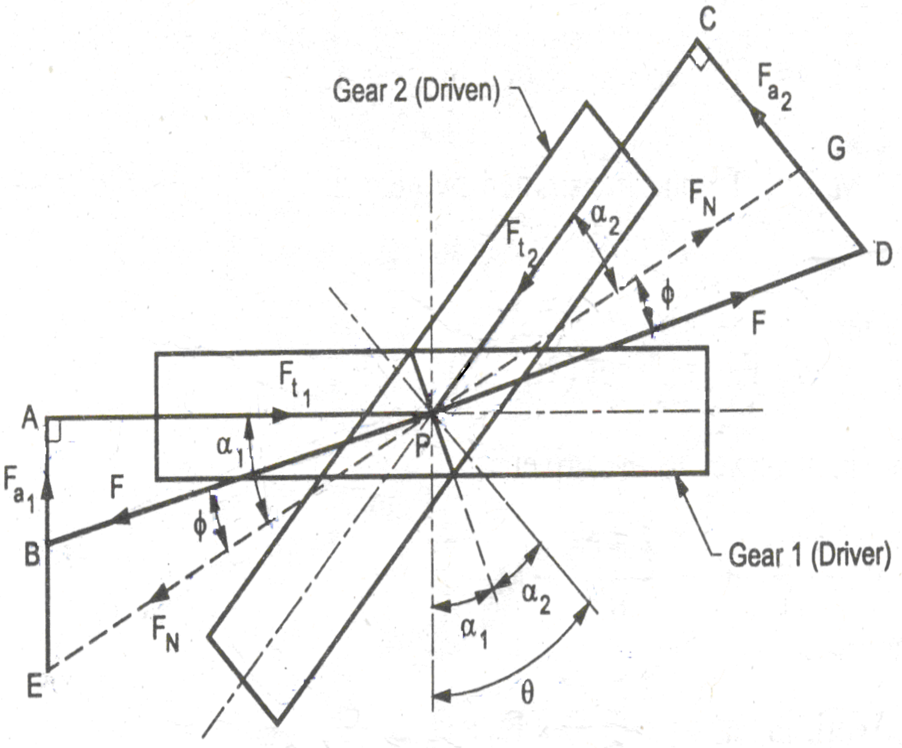
$$\therefore d_1 = \frac{m_n T_1}{\cos\alpha_1} \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\therefore d_2 = \frac{m_n T_2}{\cos\alpha_2} \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Efficiency Of Spiral Gears

From eq (3) & (4);

$$\frac{d_2 N_2}{d_1 N_1} = \frac{\dfrac{m_n T_2}{\cos\alpha_2} \times N_2}{\dfrac{m_n T_1}{\cos\alpha_1} \times N_1}$$



$$\frac{d_2 N_2}{d_1 N_1} = \frac{T_2 N_2}{T_1 N_1} \cdot \frac{\cos\alpha_1}{\cos\alpha_2}$$

$$\frac{d_2}{d_1} = \frac{T_2}{T_1} \frac{\cos\alpha_1}{\cos\alpha_2}$$

$$\frac{d_2 N_2}{d_1 N_1} = \frac{\cos\alpha_1}{\cos\alpha_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots \left[\because G = \frac{T_2}{T_1} = \frac{N_1}{N_2}\right] \ldots\ldots\ldots\ldots\ldots(5)$$

Efficiency Of Spiral Gears

Substituting value of eq (5) in η;

$$\eta = \frac{\cos\alpha_1 \bullet (\cos\alpha_2 + \phi)}{\cos\alpha_2 \bullet (\cos\alpha_1 - \phi)}$$

$$\eta = \frac{1/2\cos(\alpha_1 + \alpha_2 + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{1/2\cos(\alpha_2 + \alpha_1 - \phi) + \cos(\alpha_2 - \alpha_1 + \phi)}$$

$$\eta = \frac{\cos(\theta + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\theta - \phi) + \cos(\alpha_2 - \alpha_1 + \phi)} \text{..........} [\because \theta = \alpha_1 + \alpha_2] \text{.....} (A)$$

Efficiency Of Spiral Gears

Angles Φ & θ are constants,

Therefore, Efficiency will be maximum when

$\cos(\alpha_1 - \alpha_2 - \Phi)$ is maximum i.e.,

$$\cos(\alpha_1 - \alpha_2 - \phi) = 1$$

$$\therefore \alpha_1 - \alpha_2 - \phi = 0$$

$$\therefore \alpha_1 - \alpha_2 = \phi$$

$$\therefore \alpha_1 = \alpha_2 + \phi$$

$$OR$$

$$\alpha_2 = \alpha_1 - \phi$$

Efficiency Of Spiral Gears

Substituting value of α1 & α2 in η;

$$\eta_{\max} = \frac{\cos(\theta+\phi)+\cos(\alpha_2+\phi-\alpha_2-\phi)}{\cos(\theta-\phi)+\cos(\alpha_1-\phi-\alpha_1+\phi)}$$

$$\eta_{\max} = \frac{\cos(\theta+\phi)+\cos(0)}{\cos(\theta-\phi)+\cos(0)}$$

$$\eta_{\max} = \frac{\cos(\theta+\phi)+1}{\cos(\theta-\phi)+1}$$

$$\theta = \alpha_1 + \alpha_2$$

$$\alpha_1 = \theta - \alpha_2$$

$$\alpha_1 = \theta - (\alpha_1 - \phi)$$

$$\alpha_1 = \theta - \alpha_1 + \phi$$

$$2\alpha_1 = \theta + \phi$$

$$\alpha_1 = \frac{\theta+\phi}{2} \text{.......}Similarly, \text{......}\alpha_2 = \frac{\theta-\phi}{2}$$

Efficiency Of Spiral Gears

$$\boxed{\alpha_1 = \frac{\theta + \phi}{2}} \quad \textbf{\&} \quad \boxed{\alpha_2 = \frac{\theta - \phi}{2}}$$

Are conditions for maximum efficiency of spiral gears

$$From \Delta PAB,$$

Axial thrust on driver is;

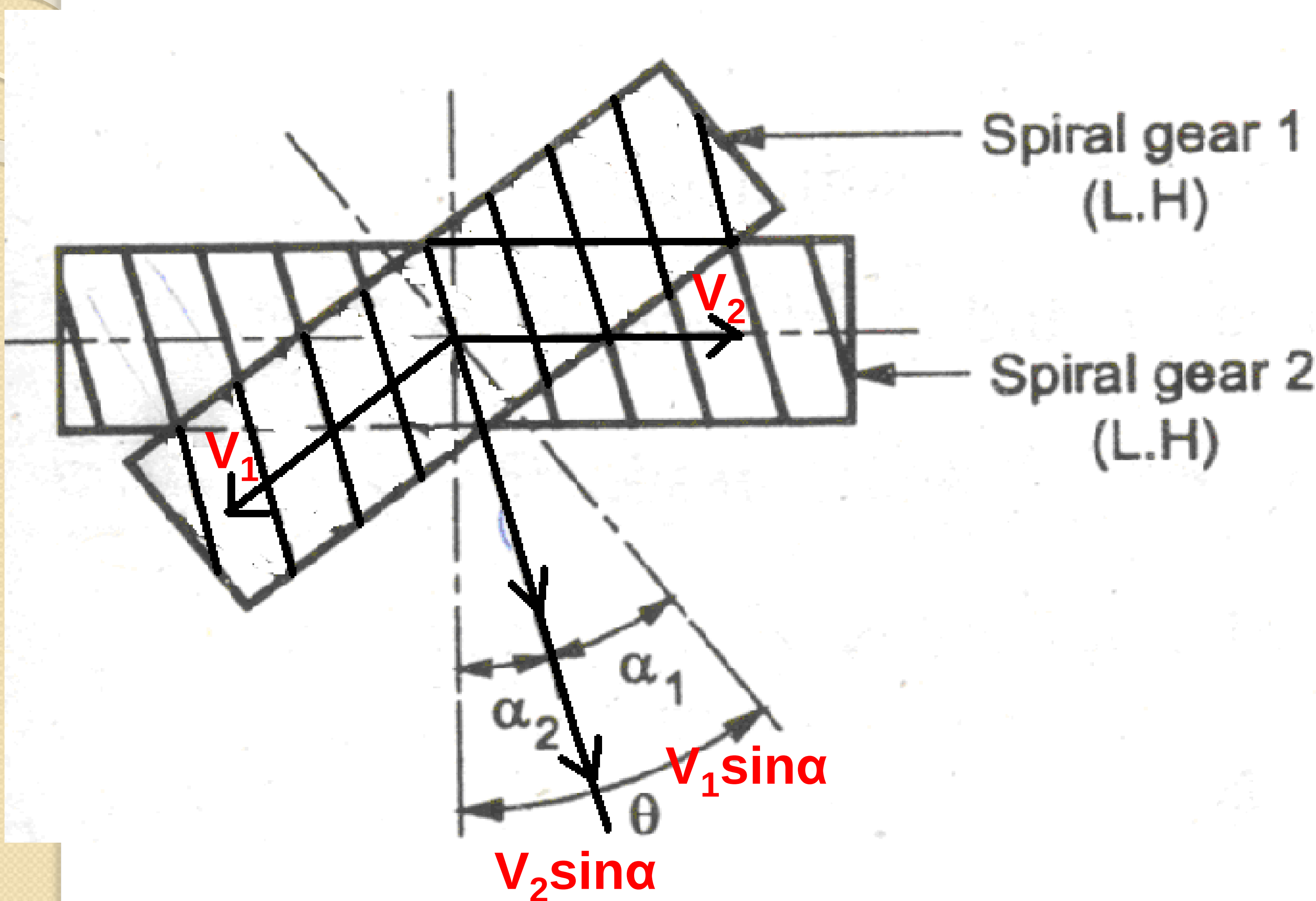
$$F_{a1} = F \sin(\alpha_1 - \phi)$$

$$From \Delta PCD,$$

Axial thrust on driven is;

$$F_{a2} = F \sin(\alpha_2 + \phi)$$

Velocity Of Sliding Between Spiral Gears



Velocity Of Sliding Between Spiral Gears

Circumferential velocity/ Tangential velocityof gear 1 at pitch pt;

$$V_1 = \omega_1 \cdot r_1$$

Circumferential velocity/ Tangential velocityof gear 2 at pitch pt;

$$V_2 = \omega_2 \cdot r_2$$

Component of V_1 along the tooth profile $= V_1 \sin \alpha_1$

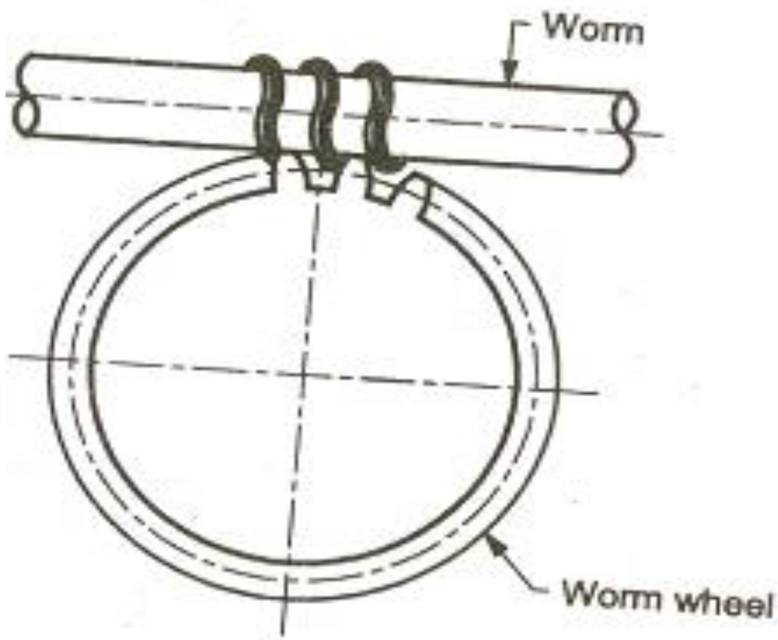
Component of V_2 along the tooth profile $= V_2 \sin \alpha_2$

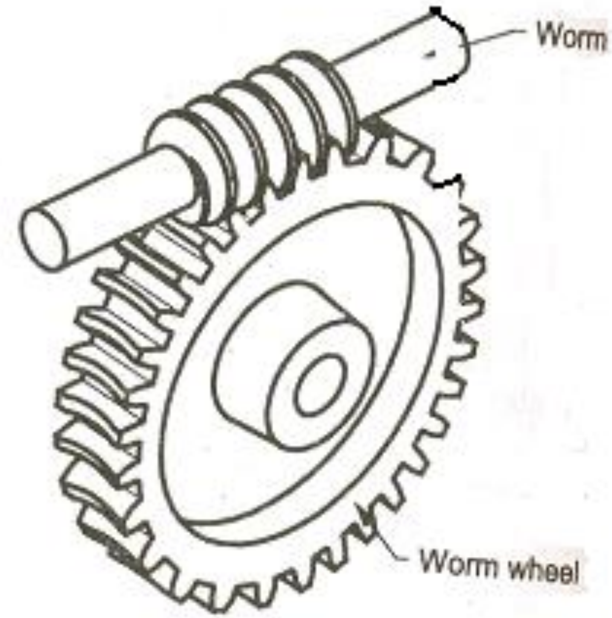
Velocity of sliding between gear 1 & 2 is;

$$V_s = V_1 \sin \alpha_1 + V_2 \sin \alpha_2$$

$$V_s = \omega_1 \cdot r_1 \cdot \sin \alpha_1 + \omega_2 \cdot r_2 \cdot \sin \alpha_2$$

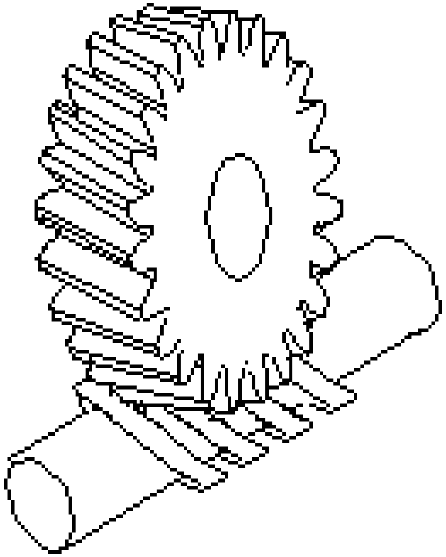
Worm & Worm Wheel





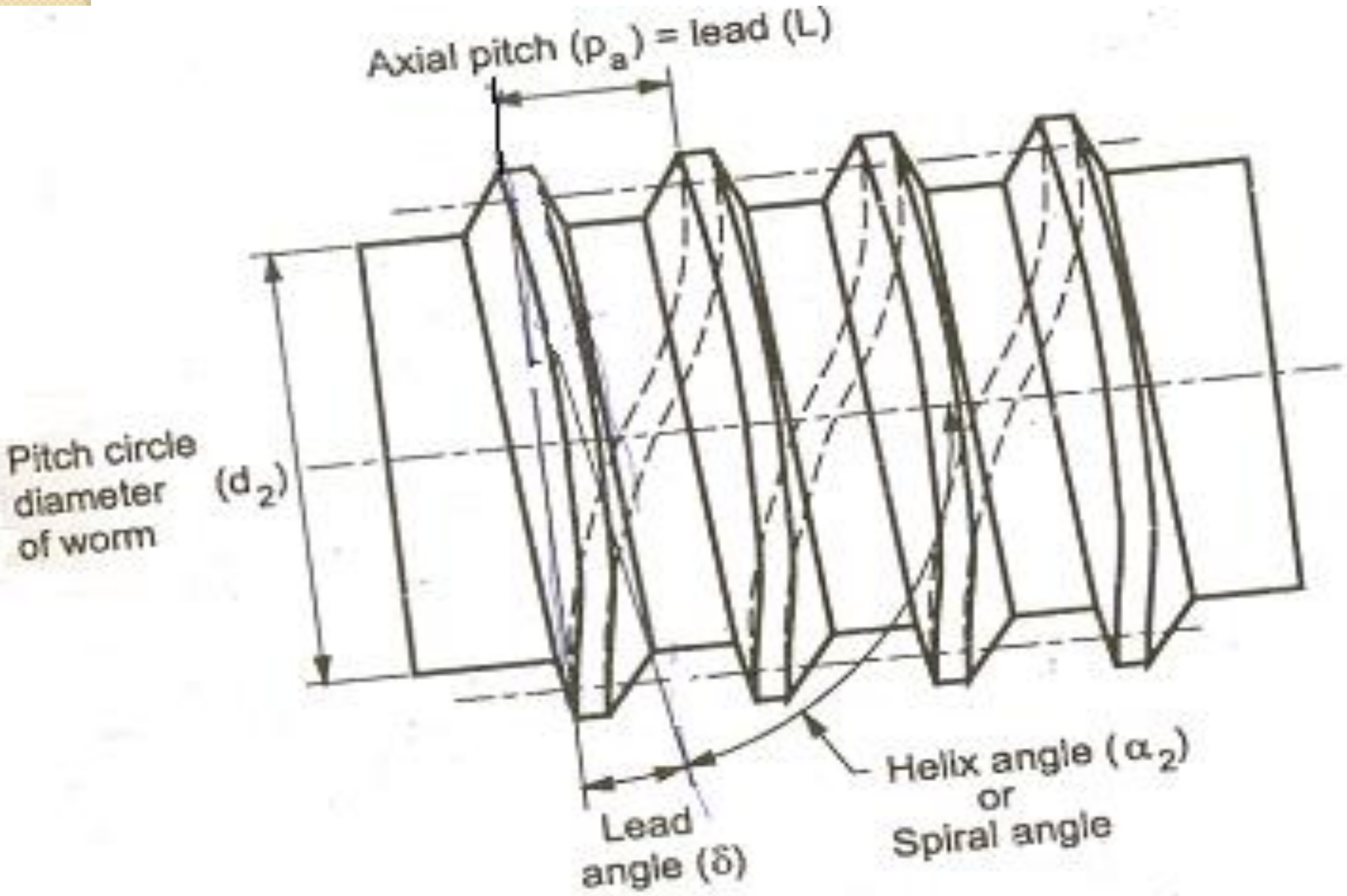
- Used for Non-parallel & Non-intersecting shafts
- It's a special case of spiral gear with shaft angle 90^0
- Worm – Threaded screw (1 to 8 teeth)
- Worm wheel –Toothed gear

Worm & Worm Wheel



- Large speed reduction upto 100:1
- Worm can easily turn the gear but…
- Gear cannot turn worm
- This locking feature acts as a brake
- Used in conveyor systems/ lifting devices like cranes

Terminology of Worm & Worm Wheel



Terminology of Worm & Worm Wheel

1. Axial Pitch (p_a) :-

Distance measured along axis , frm a point on one tooth to the corresponding point on adjacent tooth.

$$\boxed{\text{Axial Pitch of worm = Circular Pitch of worm gear}}$$

2. Lead (L) :-

Distance moved by a point on the helical profile, whn worm is rotated through one revolution

$$\boxed{L = n \cdot p_a}$$

where,

L = Lead of worm

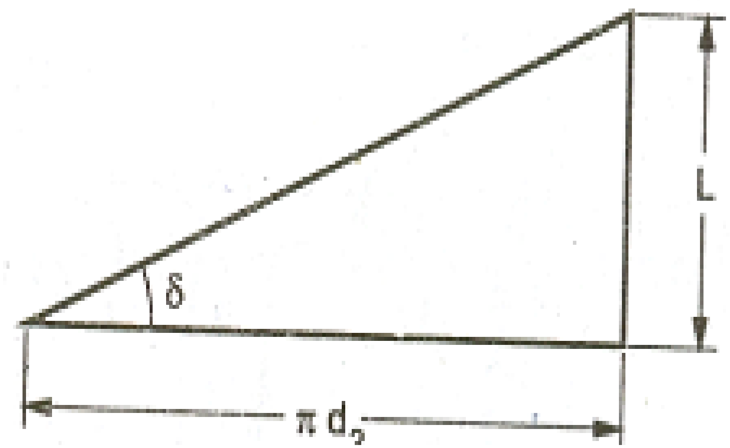
n = no. of starts

pa = Axial pitch

Terminology of Worm & Worm Wheel

3.Lead Angle (δ) :-

Angle between the tangent to the helix & line normal to the axis.



$$\boxed{\delta = \frac{\pi}{2} - \alpha_2 = 90^0 - \alpha_2}$$

where,

α_2=spiral angle of worm

If one thread of worm is developed,

Hypotenuse –Thread

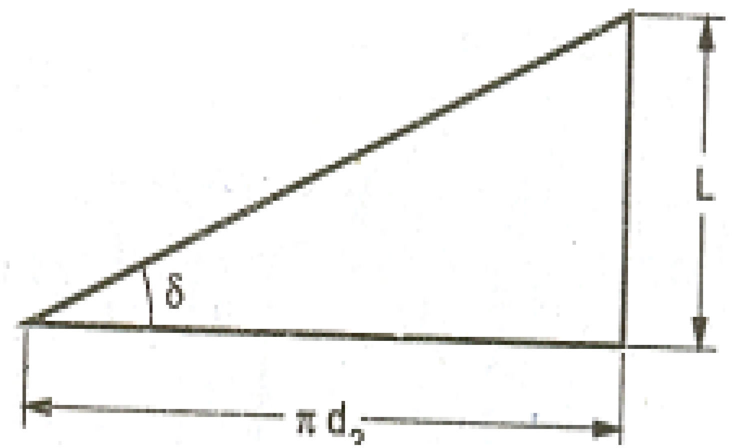
Base – circumference of wheel

Altitude – lead of worm

Terminology of Worm & Worm Wheel

Frm fig.

$$\boxed{\tan\delta = \frac{L}{\pi d_2}}$$



where,

L= Lead of worm

d_2= PCD of worm

If α1 is the spiral angle of worm gear, thn shaft angle is;

$$\alpha_1 + \alpha_2 = 90^0$$

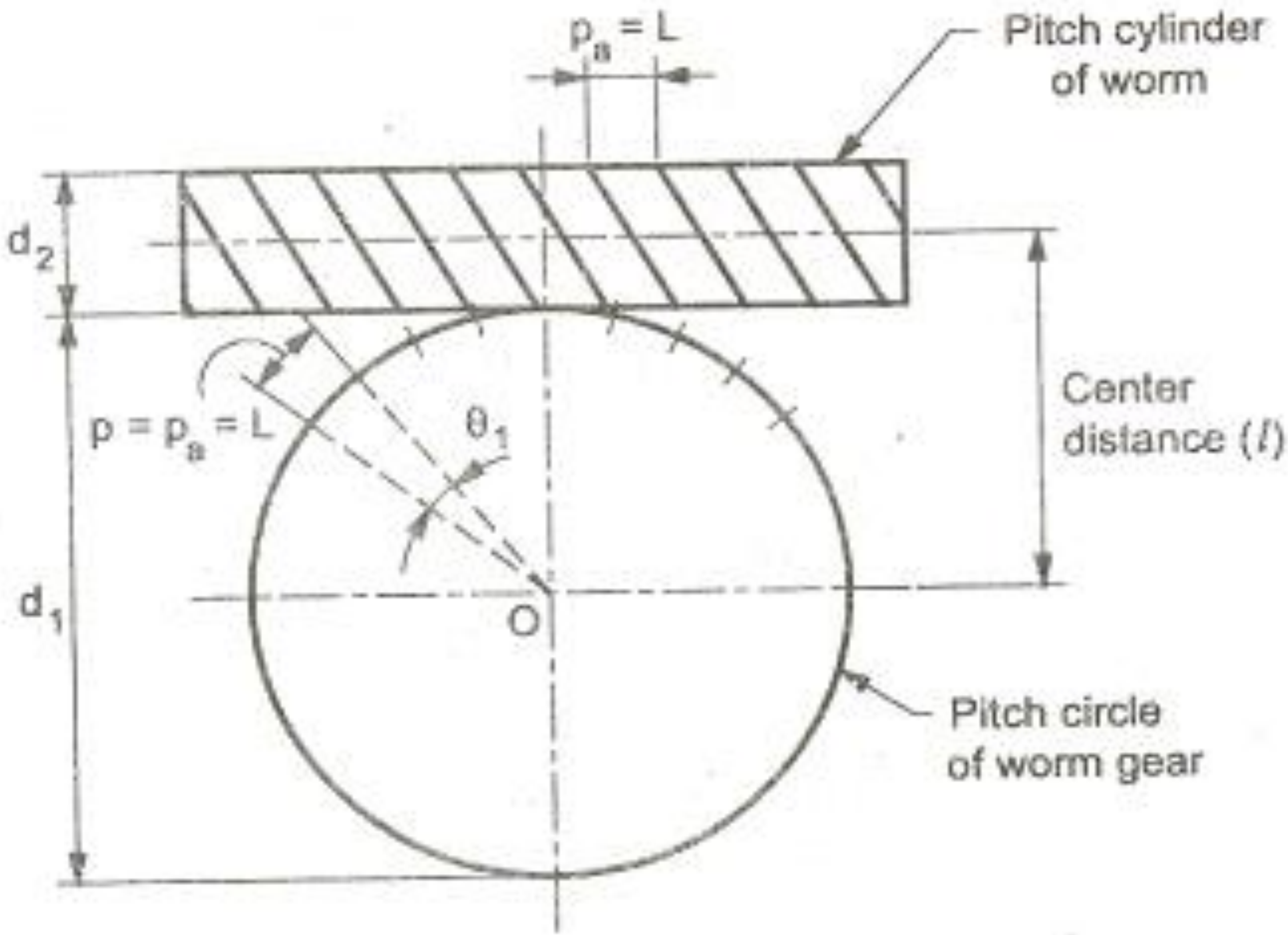
$$or...\alpha_1 = 90^0 - \alpha_2$$

$$But...,\delta = 90^0 - \alpha_2$$

$$\boxed{\therefore \alpha_1 = \delta}$$

i.e. ***Spiral angle of worm gear (α_1) = Lead angle of worm (δ)***

Velocity ratio & Centre Distance Between Worm Gears



Velocity ratio Of Worm Gears

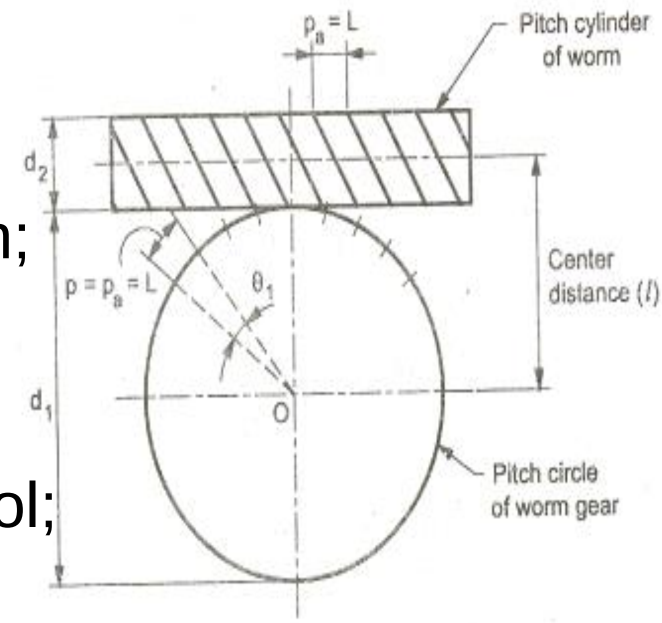
Let,

θ_2 – angle turned by worm in one revolution;

$$\theta_2 = \pi \cdot d_2 ....\dots(1)$$

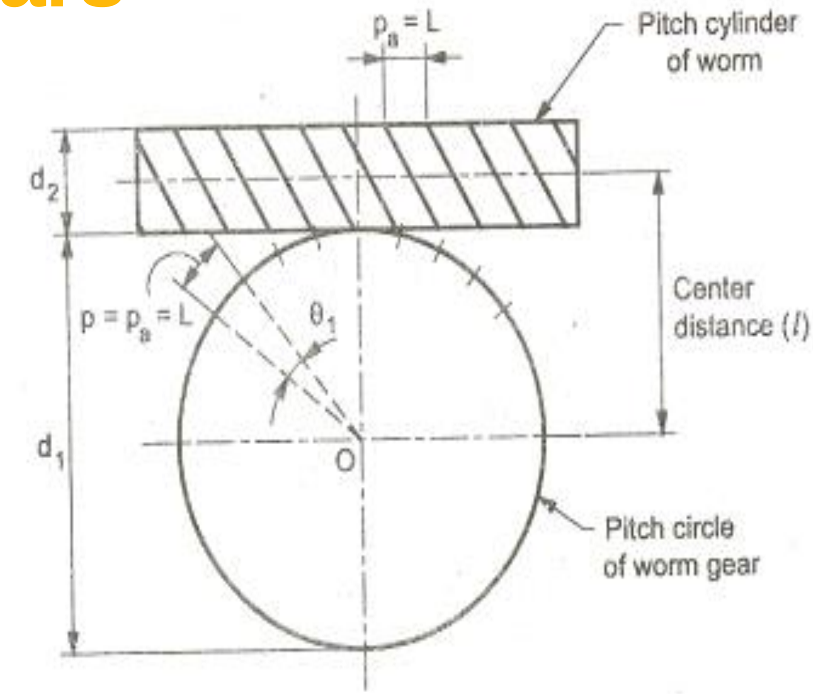
θ_1 – Angle turned by worm gear in one revol;

$$\theta_1 = \frac{L}{r_1} = \frac{L}{d_1/2} = \frac{2L}{d_1} ....(2)$$



"Velocity ratio" is the ratio of angle turned by worm gear in one revolution of worm; to the angle turned by worm in one revolution.

Velocity ratio Of Worm Gears



$$\therefore Velocity`Ratio = \frac{\theta_1}{\theta_2}$$

$$\therefore Velocity`Ratio = \frac{2L/d_2}{2\pi} = \frac{L}{\pi \cdot d_2}$$

$$or, Velocity`Ratio = \frac{np_a}{\pi \cdot d_2} \text{.........................} [\because L = np_a]$$

Centre Distance Between Worm Gears

l = C.D. between worm & worm gears

α_1 = Spiral angles for worm gear

α_2 = Spiral angles for worm

T_1 = No. of teeth on worm gear

T_2 = No. of teeth on worm

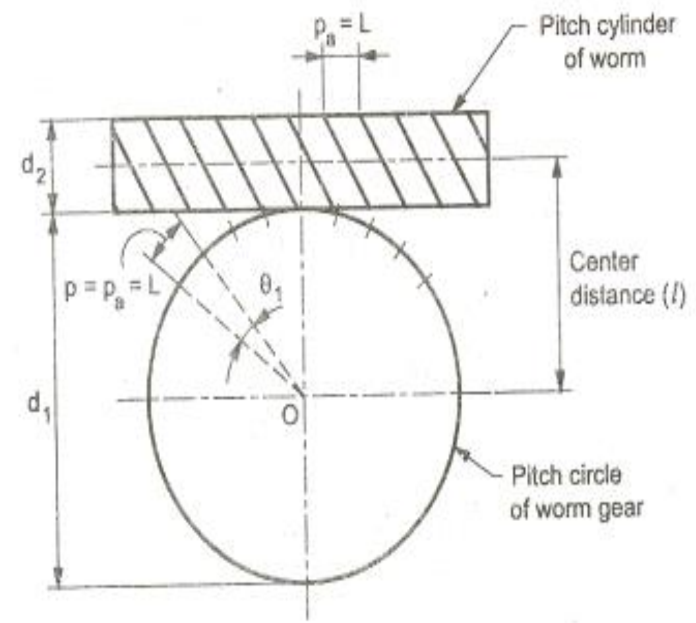
mt_1 = Transverse module for worm gear

mt_2 = Transverse module for worm

m_n = Normal module for worm & worm gear

r_1 = PC radius for worm gear

r_2 = PC radius for worm



Centre Distance Between Worm Gears

We know that pitch circle radius of

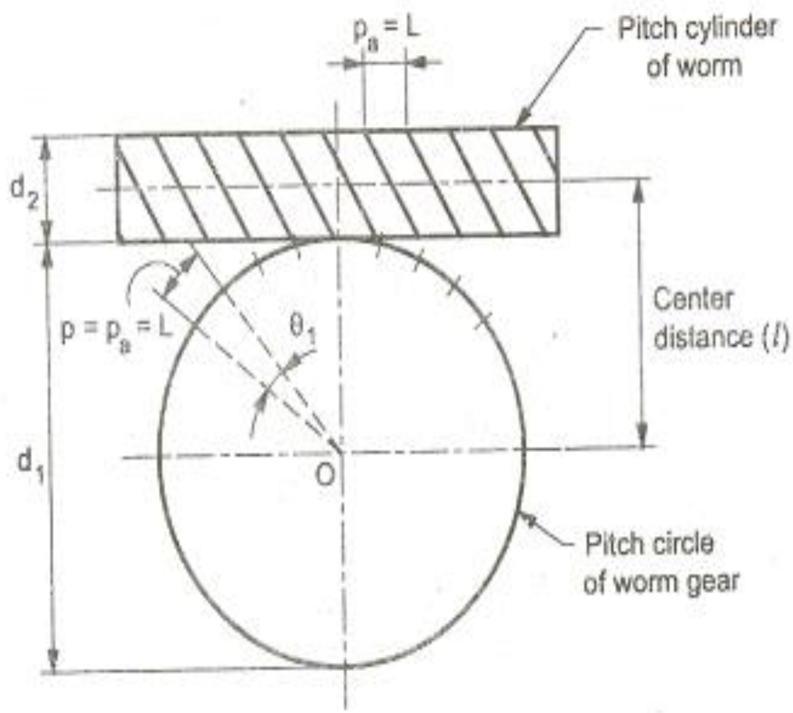
$$r = \frac{m_t \cdot T}{2}$$

Pitch circle radius of worm gear is,

$$r_1 = \frac{m_{t1} \cdot T_1}{2}$$

Pitch circle radius of worm is,

$$r_2 = \frac{m_{t2} \cdot T_2}{2}$$



Centre Distance Between Worm Gears

The centre distance between worm & worm gear is,

$$l = r1 + r2$$

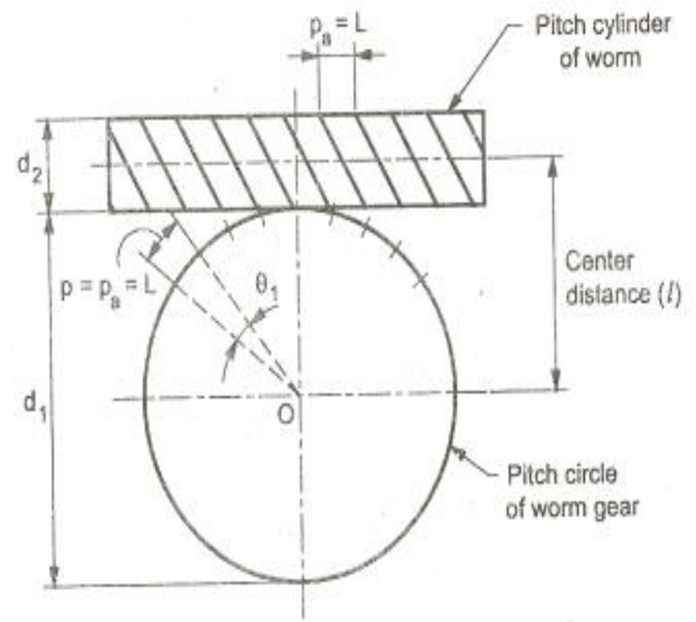
$$\therefore l = \frac{m_{t1} \cdot T_1}{2} + \frac{m_{t2} \cdot T_2}{2}$$

$$\therefore l = \frac{m_n}{\cos\alpha_1} \cdot \frac{T_1}{2} + \frac{m_n}{\cos\alpha_2} \cdot \frac{T_2}{2}$$

$$\therefore l = \frac{m_n}{2} \cdot \left[\frac{T_1}{\cos\alpha_1} + \frac{T_2}{\cos\alpha_2} \right]$$

$$\therefore l = \frac{m_{t2}\cos\alpha_2}{2} \cdot \left[\frac{T_1}{\cos\alpha_1} + \frac{T_2}{\cos\alpha_2} \right]$$

$$\therefore l = \frac{m_{t2}}{2} \cdot \left[\frac{T_1\cos\alpha_2}{\cos\alpha_1} + \frac{T_2\cos\alpha_2}{\cos\alpha_2} \right]$$



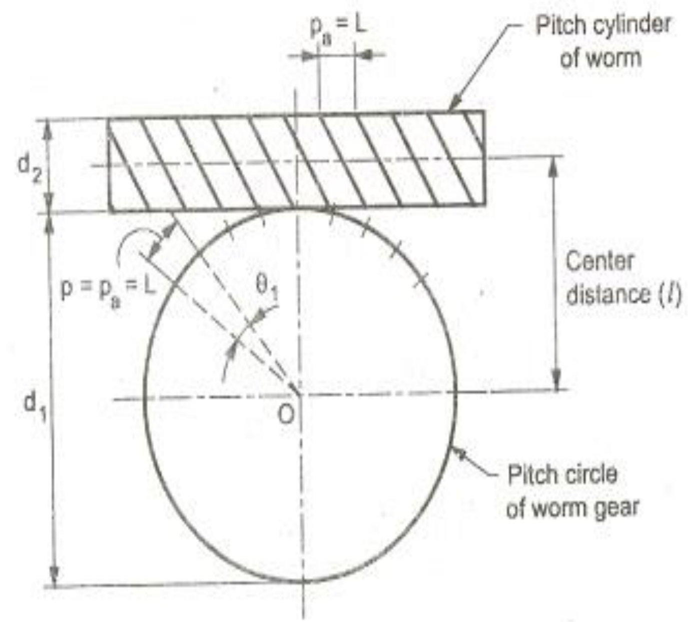
Centre Distance Between Worm Gears

$$\therefore l = \frac{m_{t2}}{2} \cdot \left[\frac{T_1 \cos(90^0 - \alpha_1)}{\cos \alpha_1} + T_2 \right]$$

$$\therefore l = \frac{m_{t2}}{2} \cdot \left[\frac{T_1 \sin \alpha_1}{\cos \alpha_1} + T_2 \right]$$

$$\therefore l = \frac{m_{t2}}{2} \cdot [T_1 \tan \alpha_1 + T_2]$$

$$\boxed{\therefore l = \frac{m_{t2}}{2} \cdot [T_1 \tan \delta + T_2]} \ldots\ldots\ldots\ldots\ldots\ldots[\because \alpha_1 = \delta]$$



This eq. gives the centre distance between worm & worm gear

Efficiency Of Worm & Worm Gears

Worm & worm gear is a special case of spiral gears with shaft angle 90^0. Therefore, efficiency is same as spiral gears

$$\eta = \frac{\cos\alpha_1 \bullet (\cos\alpha_2 + \phi)}{\cos\alpha_2 \bullet (\cos\alpha_1 - \phi)} \ldots\ldots\ldots$$ ***[Whn worm gear is driver]***

$$Put, \alpha_1 = \delta; \ldots \& \ldots \alpha_2 = 90^0 - \alpha_1;$$

$$\eta = \frac{\cos\delta \bullet \cos(90^0 - \alpha_1 + \phi)}{\cos(90^0 - \alpha_1) \bullet \cos(\delta - \phi)}$$

$$\eta = \frac{\cos\delta \bullet \cos[90^0 - (\delta - \phi)]}{\cos(90^0 - \delta) \bullet \cos(\delta - \phi)}$$

$$\eta = \frac{\cos\delta \bullet \sin(\delta - \phi)}{\sin\delta \bullet \cos(\delta - \phi)}$$

$$\boxed{\therefore \eta = \frac{\tan(\delta - \phi)}{\tan\delta}}$$

Where,

Φ – Friction angle

Efficiency Of Worm & Worm Gears

Maximum efficiency is same as spiral gears

$$\eta_{\max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$$

$$\eta_{\max} = \frac{\cos(90^0 + \phi) + 1}{\cos(90^0 - \phi) + 1}$$

$$\eta_{\max} = \frac{-\sin\phi + 1}{\sin\phi + 1}$$

$$\boxed{\eta_{\max} = \frac{1 - \sin\phi}{1 + \sin\phi}}$$

Efficiency Of Worm & Worm Gears

Whn worm is driver & worm gear is driven;

$$\eta = \frac{\cos\alpha_2 \bullet (\cos\alpha_1 + \phi)}{\cos\alpha_1 \bullet (\cos\alpha_2 - \phi)} \text{..........}$$

$$Put, \alpha_1 = \delta; \ldots \& \ldots \alpha_2 = 90^0 - \alpha_1;$$

$$\eta = \frac{\cos(90^0 - \alpha_1)\cos(\delta + \phi)}{\cos\delta \cdot \cos[(90^0 - \alpha_1) - \phi]}$$

$$\eta = \frac{\cos(90^0 - \delta) \bullet \cos(\delta + \phi)}{\cos\delta \bullet \cos[90^0 - (\delta + \phi)]}$$

$$\eta = \frac{\sin\delta \bullet \cos(\delta + \phi)}{\cos\delta \bullet \sin(\delta + \phi)}$$

$$\boxed{\therefore \eta = \frac{\tan\delta}{\tan(\delta + \phi)}}$$

Where,

Φ – Friction angle

Efficiency Of Worm & Worm Gears

Maximum efficiency is not affected when worm is driver

$$\eta_{\max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$$

$$\eta_{\max} = \frac{\cos(90^0 + \phi) + 1}{\cos(90^0 - \phi) + 1}$$

$$\eta_{\max} = \frac{-\sin\phi + 1}{\sin\phi + 1}$$

$$\boxed{\eta_{\max} = \frac{1 - \sin\phi}{1 + \sin\phi}}$$

Force Analysis Of Worm & Worm Gears

Force analysis of worm & worm gears is same as that of the spiral gears

Thus, tangential force acting on worm gear is given by;

$$F_{t1} = F\cos(\alpha_1 - \phi)$$

& Torque transmitted by worm gear is;

$$M_{t1} = F_{t1} \times r_1$$

$$\therefore M_{t1} = F\cos(\alpha_1 - \phi) \times r_1$$

$$\boxed{\therefore M_{t1} = F\cos(\delta - \phi) \times r_1}\ldots\ldots\ldots\ldots\ldots\ldots[\because \alpha_1 = \delta]$$

Force Analysis Of Worm & Worm Gears

Force analysis of worm & worm gears is same as that of the spiral gears

Thus, tangential force acting on worm is given by;

$$F_{t2} = F\cos(\alpha_2 + \phi)$$

& Torque transmitted by worm is;

$$M_{t2} = F_{t2} \times r_2$$

$$\therefore M_{t2} = F\cos(\alpha_2 + \phi) \times r_2$$

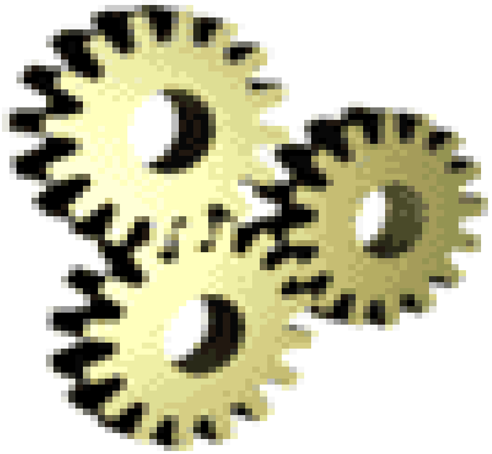
$$\therefore M_{t2} = F\cos(90^0 - \alpha_1 + \phi) \times r_2$$

$$\therefore M_{t2} = F\cos[(90^0 - (\alpha_1 - \phi)] \times r_2$$

$$\therefore M_{t2} = F\sin(\alpha_1 - \phi) \times r_2$$

$$\boxed{\therefore M_{t2} = F\sin(\delta - \phi) \times r_2}$$

Gear Trains



A gear train is two or more gears working together by meshing their teeth and turning each other in a system to generate power and speed.

- It reduces speed and increases torque.
- It creates large gear ratio
- Nature of train depends upon
    - velocity ratio required &
    - *the relative position of axes of shafts*

Types Of Gear Trains

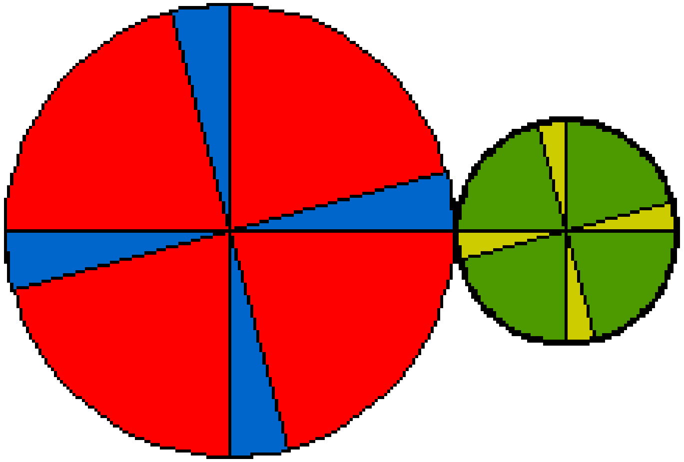
Depending upon arrangement of wheels:

1] Simple Gear Train

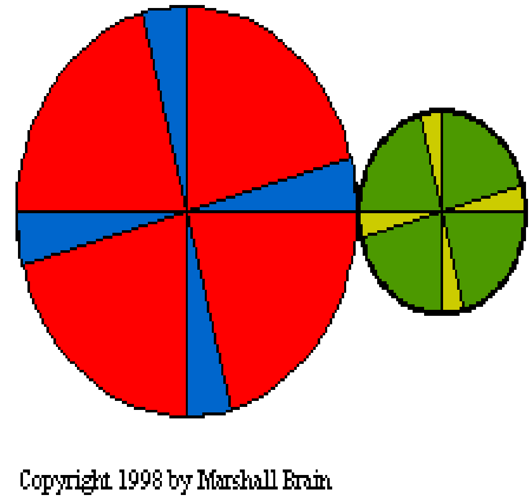
2] Compound Gear Train

3] Reverted Gear Train

4] Epicyclic Gear Train



Types Of Gear Trains



1] Simple Gear Train - Only one gear on each shaft

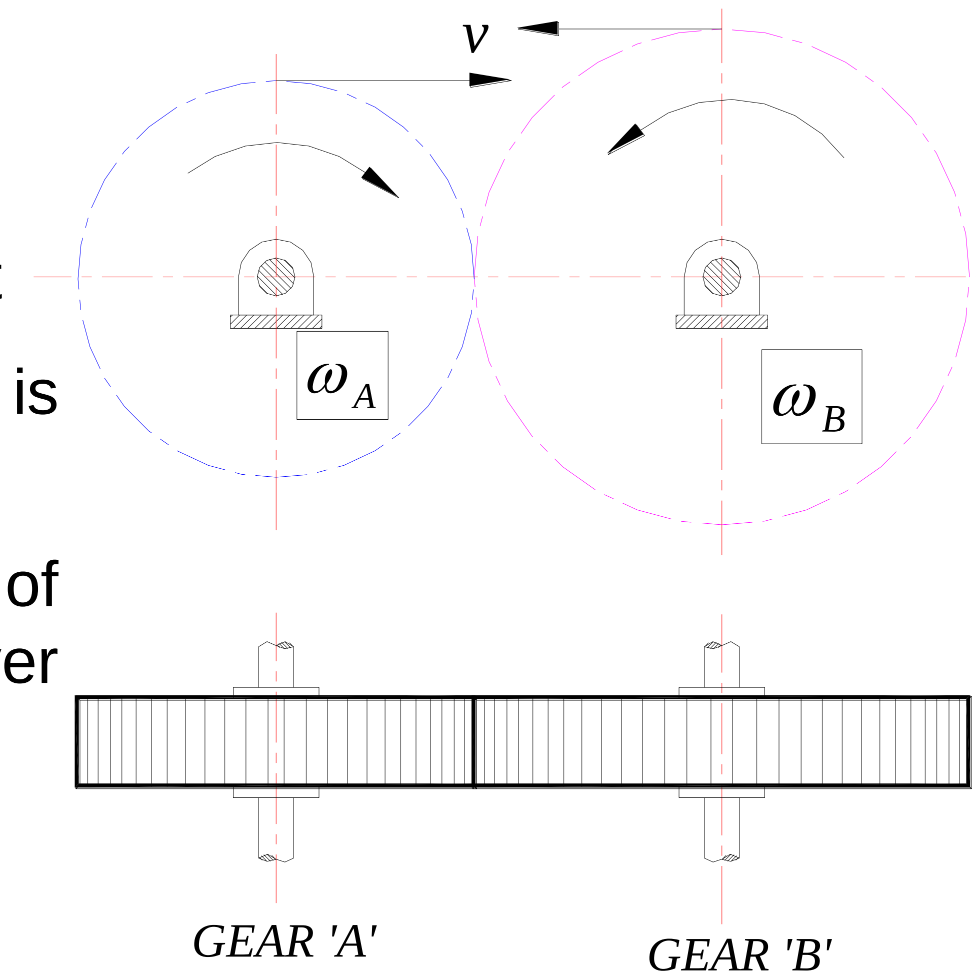
2] Compound Gear Train - more than one gear on a shaft

3] Reverted Gear Train - First & last gear are co-axial

4] Epicyclic Gear Train - axis move over the other fixed axis

Simple Gear Trains

- Only one gear on each shaft
- Distance between two shaft is small
- The direction of motion of driven gear is opposite to driver gear.



Simple Gear Trains

Let,

NA = Speed of gear A(driver) in rpm

NB = Speed of gear B(driven) in rpm

TA = Number of teeth on gear A

TB = Number of teeth on gear B

Simple Gear Trains

Speed Ratio/ Gear ratio/ Velocity ratio is given by;

$$Speed' Ratio = \frac{speed(driver)}{speed(driven)}$$

$$GR = \frac{N_A}{N_B} = \frac{T_B}{T_A}$$

Train Value is given by;

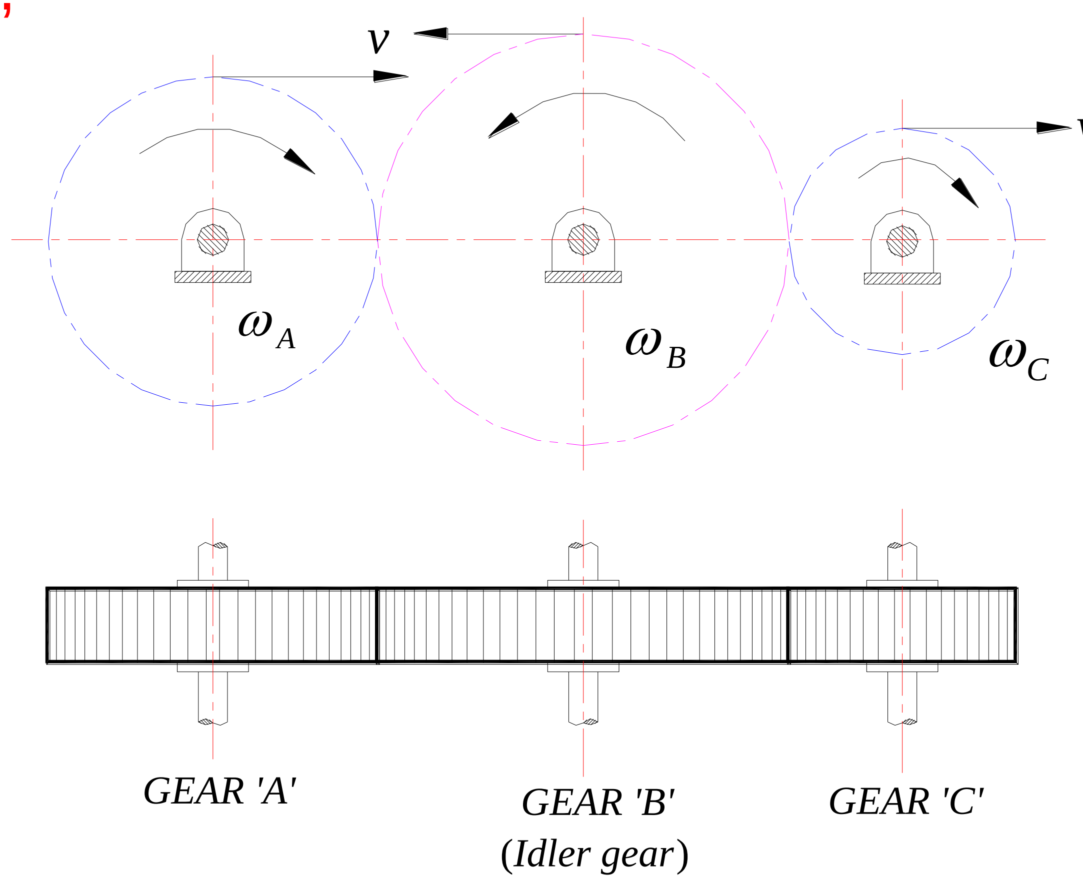
$$Train \quad Value = \frac{N_B}{N_A} = \frac{T_A}{T_B}$$

$$\therefore Train \quad Value = \frac{1}{Speed \quad Ratio}$$

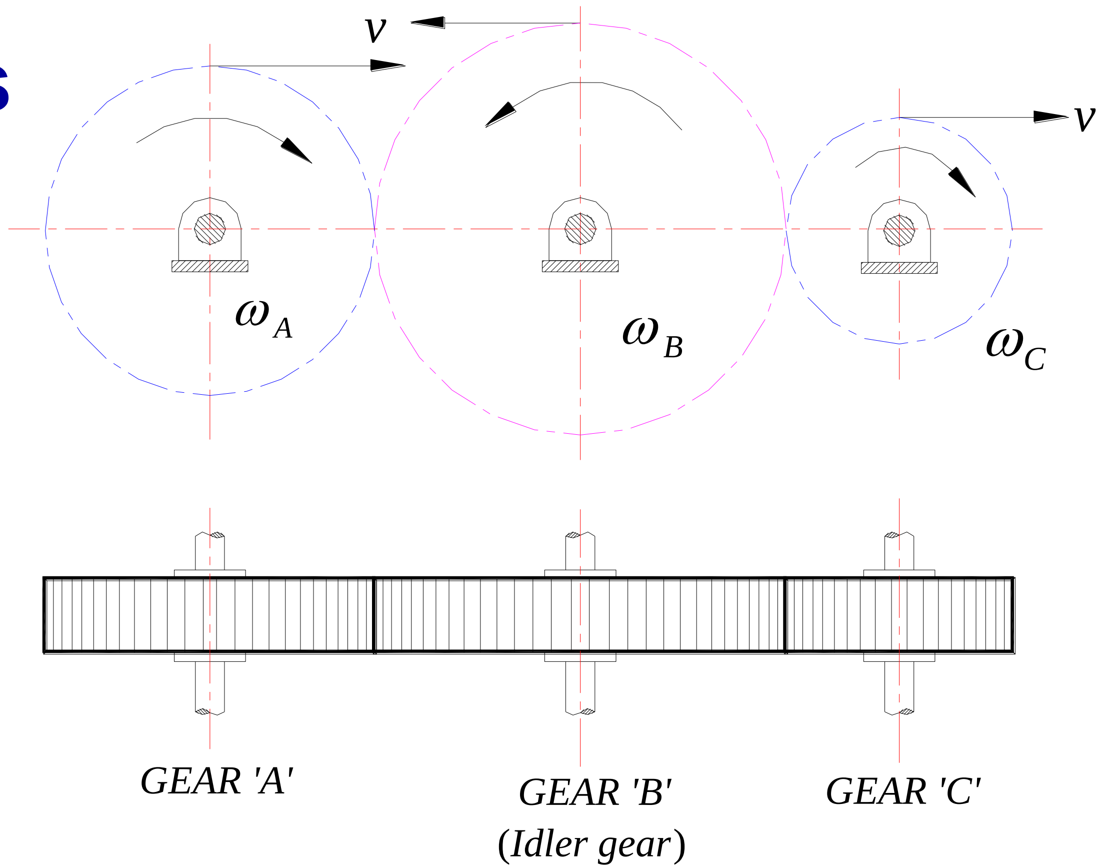
Simple Gear Trains

When distance between two gears is larger, motion is transmitted by;

1. Providing large sized gear
2. Providing no. of intermediate gears

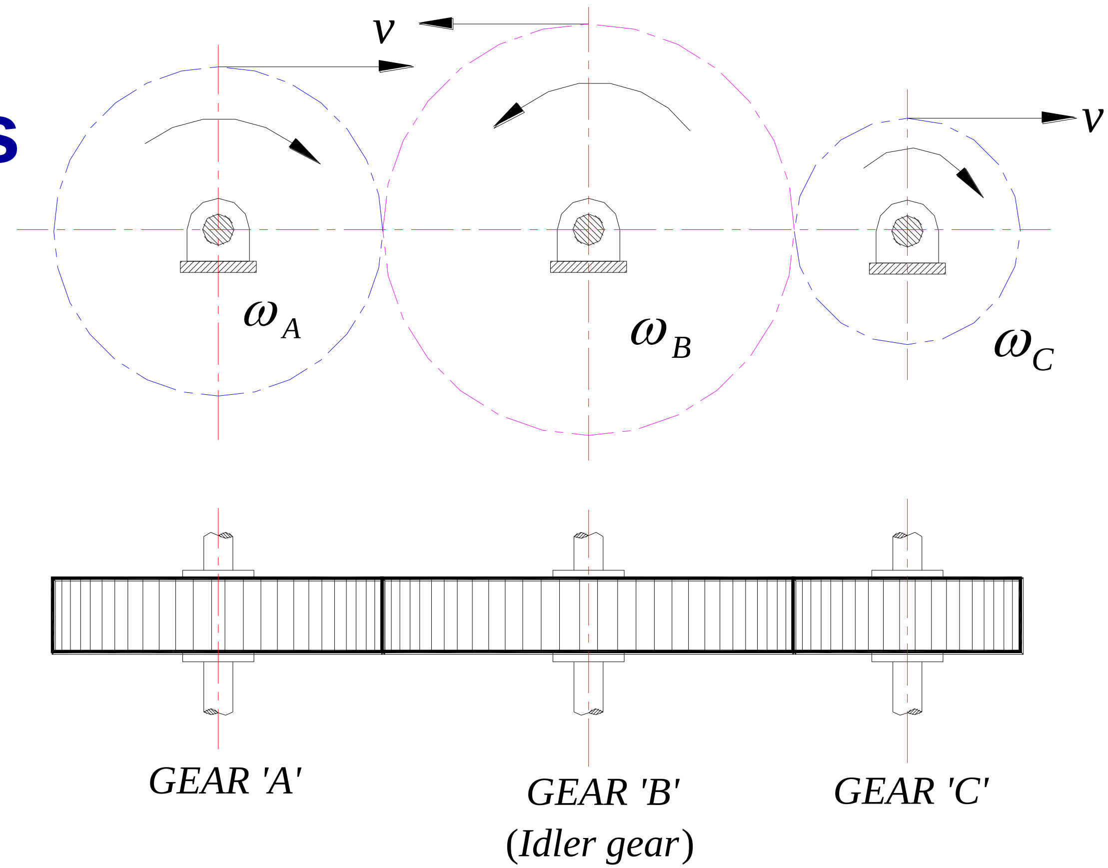


Simple Gear Trains



- Intermediate gear is known as Idler Gear
- Function of idler gear is to change the direction of rotation
- It has no effect on gear ratio

Simple Gear Trains



- Odd number of intermediate gears – motion of driver & driven is like
- Even number of intermediate gears – motion of driver & driven gear is Unlike

Simple Gear Trains

Let,

NA = Speed of driver gear in rpm

NB = Speed of intermediate gear in rpm

NC = Speed of driven gear in rpm

TA = Number of teeth on driver gear

TB = Number of teeth on intermediate gear

TC= Number of teeth on driven gear

Simple Gear Trains

Driver gear 1 is in mesh with intermediate gear 2,
Speed Ratio is given by;

$$\frac{N_A}{N_B} = \frac{T_B}{T_A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Similarly intermediate gear 2 is in mesh with driven gear 3,
Speed Ratio is given by;

$$\frac{N_B}{N_C} = \frac{T_C}{T_B} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Speed Ratio of gear train is given by; multiplying (1) & (2)

$$\frac{N_A}{N_B} \times \frac{N_B}{N_C} = \frac{T_B}{T_A} \times \frac{T_C}{T_B}$$

$$\boxed{\therefore \frac{N_A}{N_C} = \frac{T_C}{T_A}}$$

Simple Gear Trains

$$\text{i.e., Speed Ratio} = \frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver}}$$

$$\text{Train Value} = \frac{\text{Speed of driven}}{\text{Speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

Speed Ratio & Train Value is independent of size & no. of intermediate gears (Idler Gears).

Purpose is;

1) To connect gears where a large centre distance is required

2) To obtain desired direction of motion of driven gear

Simple Gear Trains

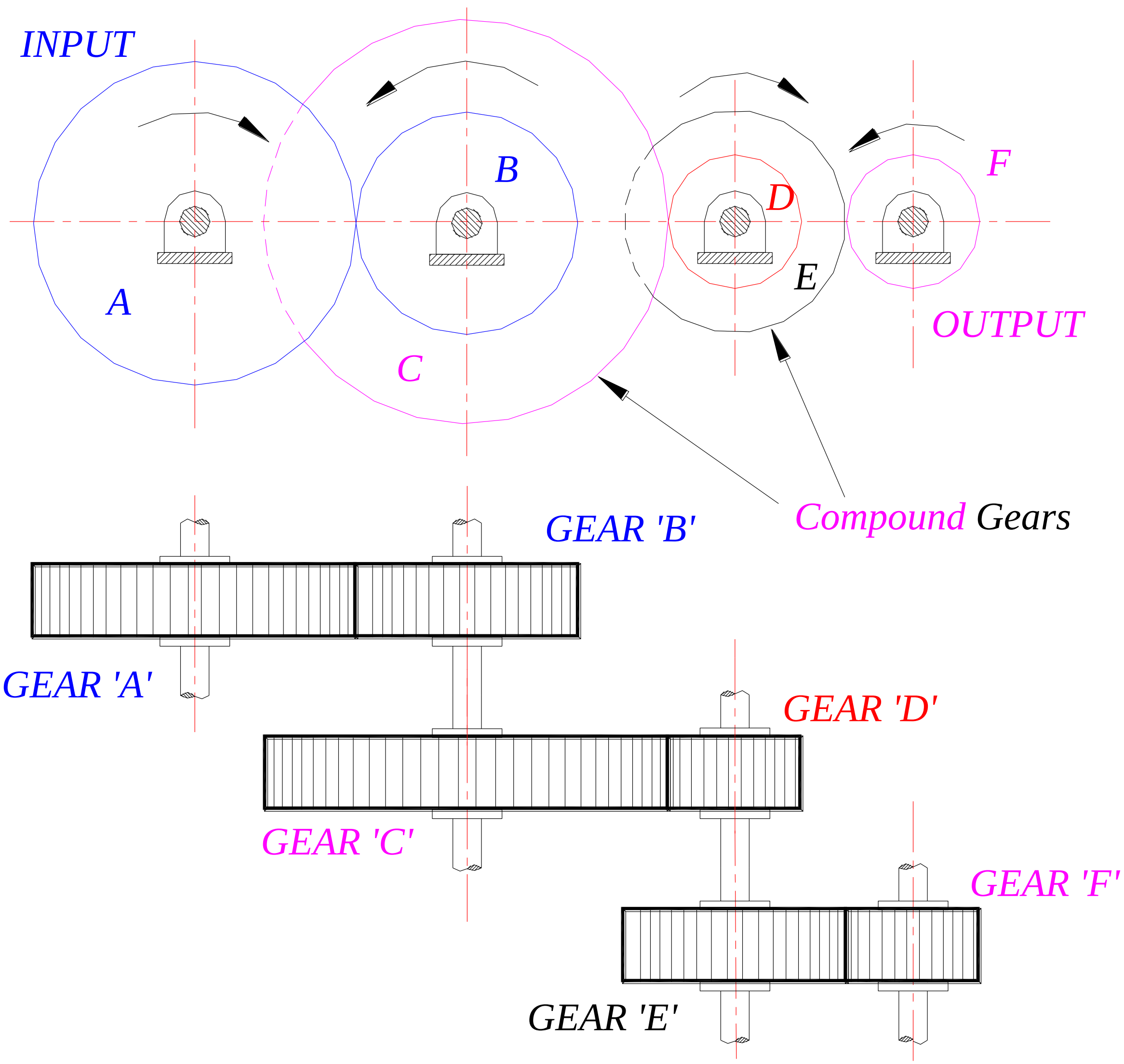
***Application**:*

a) to connect gears where a large center distance is required

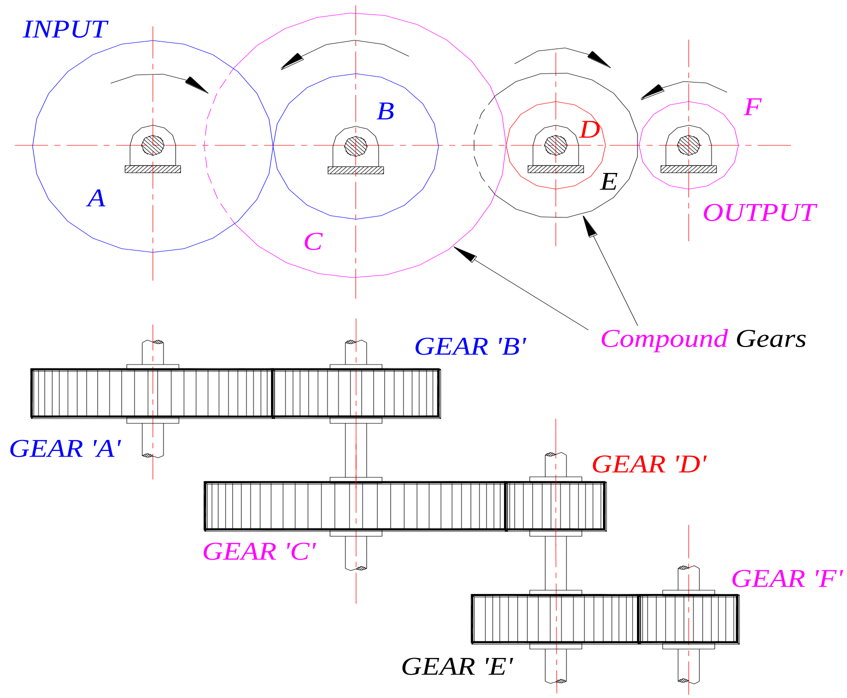
b) to obtain desired direction of motion of the driven gear (CW or CCW)

c) to obtain high speed ratio

Compound Gear train

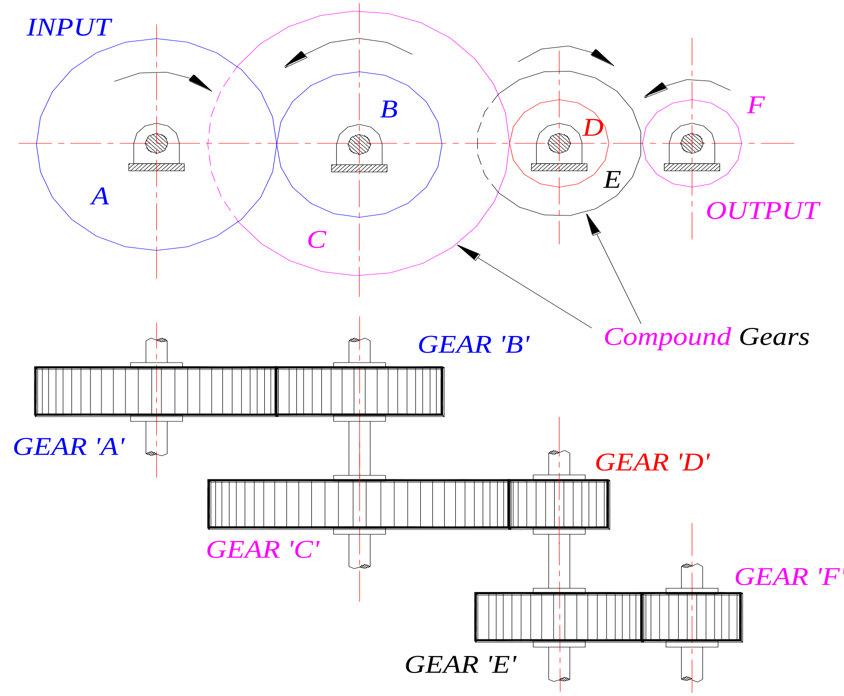


Compound Gear train



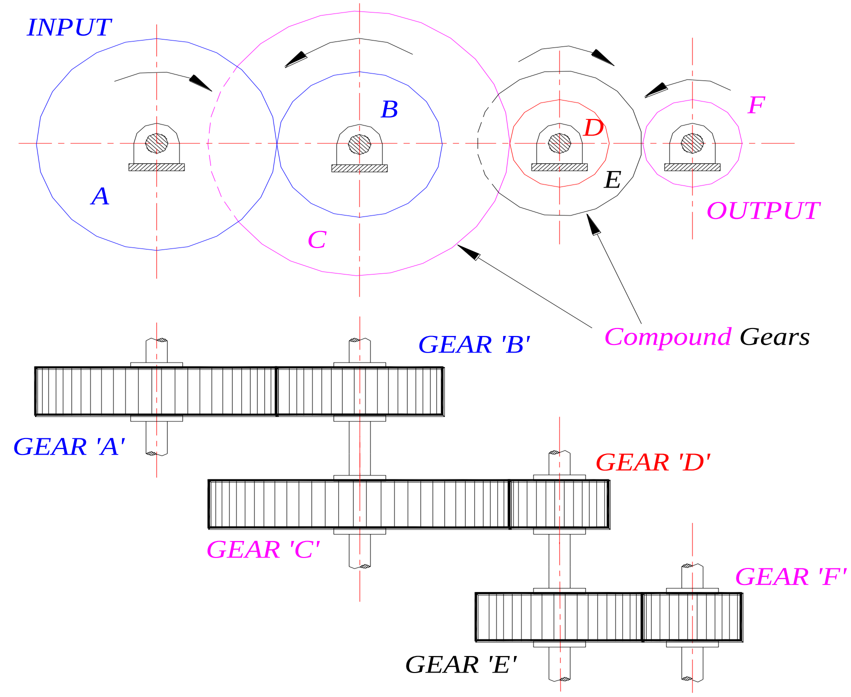
- More than one gear on a shaft
- Distance between driver & driven has to be bridged
- Great speed ratio is required

Compound Gear train



- Each intermediate shaft has two gears rigidly fixed to it , so that they may have same speed
- One of these gears meshes with driver & other with driven

Compound Gear train



Gear A is driving gear mounted on a shaft

Gear B & C are compound gears mounted on one shaft

Gear D & E are compound gears mounted on one shaft

Gear F is driven gear mounted on a shaft

Compound Gear Trains

Let,

N_A = Speed of driver gear A in rpm

T_A = Number of teeth on driver gear A

$N_B, N_C, \ldots, N_F$ = Speed of respectiver gears in rpm

$T_B, T_C, \ldots, T_F$ = Number of teeth on respective gears

Compound Gear train

Driver gear A is in mesh with gear B, Thus Speed Ratio is ;

$$\frac{N_A}{N_B} = \frac{T_B}{T_A} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Similarly,Gear C is in mesh with gear D,Speed Ratio is;

$$\frac{N_C}{N_D} = \frac{T_D}{T_C} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Similarly,Gear E is in mesh with gear F,Speed Ratio is;

$$\frac{N_E}{N_F} = \frac{T_F}{T_E} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Compound Gear train

Speed Ratio of gear train is given by; multiplying (1), (2) & (3)

$$\frac{N_A}{N_B}\times\frac{N_C}{N_D}\times\frac{N_E}{N_F}=\frac{T_B}{T_A}\times\frac{T_D}{T_C}\times\frac{T_F}{T_E}$$

But, $N_B=N_C$ [as mounted on same shaft] &
$N_D=N_E$ [as mounted on same shaft]

$$\boxed{\therefore\frac{N_A}{N_C}=\frac{T_B\times T_D\times T_F}{T_A\times T_C\times T_E}}$$

Design of Spur Gear Trains

Let,

N_A & N_B = Speed of driver & driven

T_A & T_B = Number of teeth on driver & driven

d_A & d_B = PCD of driver & driven

p_c = circular pitch

Distance between centres of two shafts;

$$x = \frac{d_A + d_B}{2}$$

& Speed Ratio/ Velocity Ratio;

$$\frac{N_A}{N_B} = \frac{d_B}{d_A} = \frac{T_B}{T_A}$$

Compound Gear Trains

$$\text{i.e., Speed Ratio} = \frac{\text{Speed of 1st driver}}{\text{Speed of last driven}}$$

$$= \frac{\text{Product of No. of teeth on drivens}}{\text{Product of No. of teeth on drivers}}$$

And

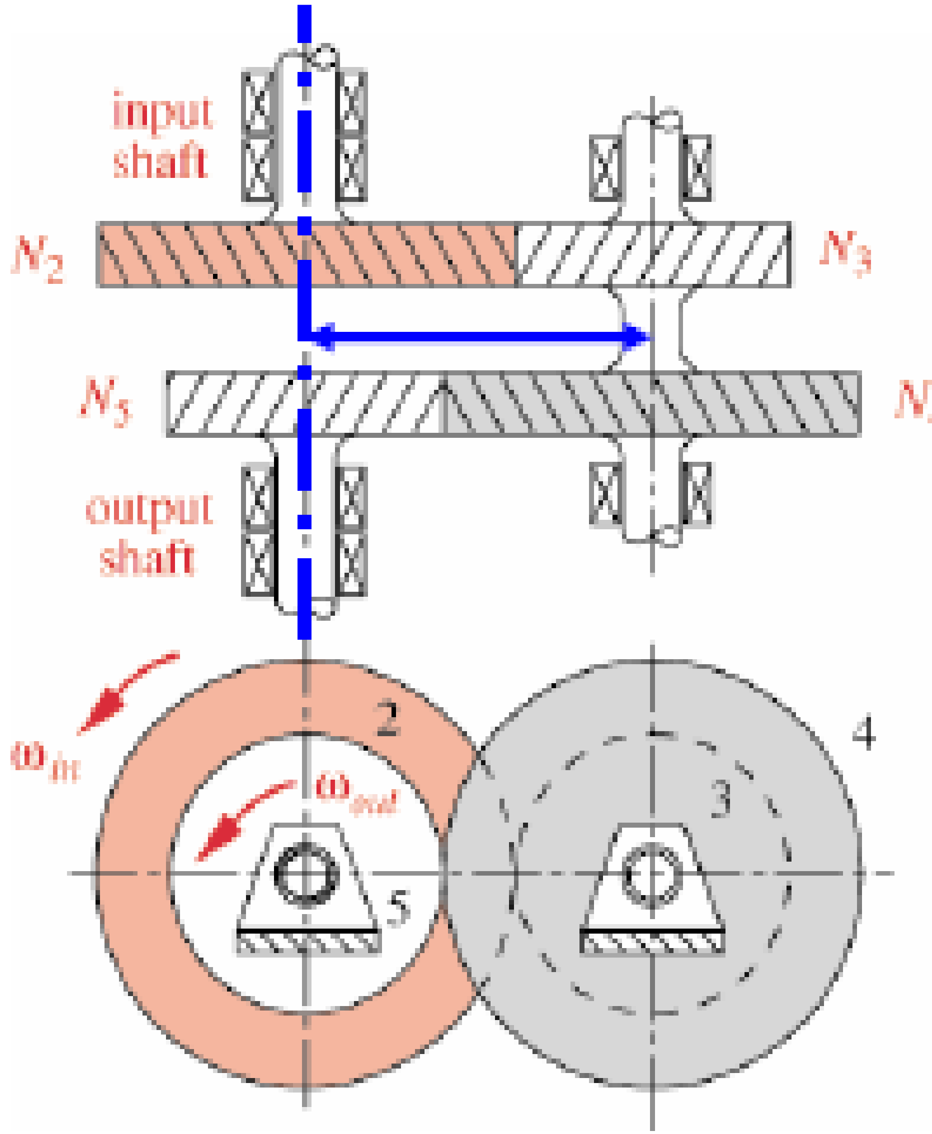
$$\text{Train Value} = \frac{\text{Speed of last driven}}{\text{Speed of 1st driver}}$$

$$= \frac{\text{Product of No. of teeth on drivers}}{\text{Product of No. of teeth on drivens}}$$

- Much larger speed reduction frm 1^{st} to last shaft
- with small gears

Reverted Gear train

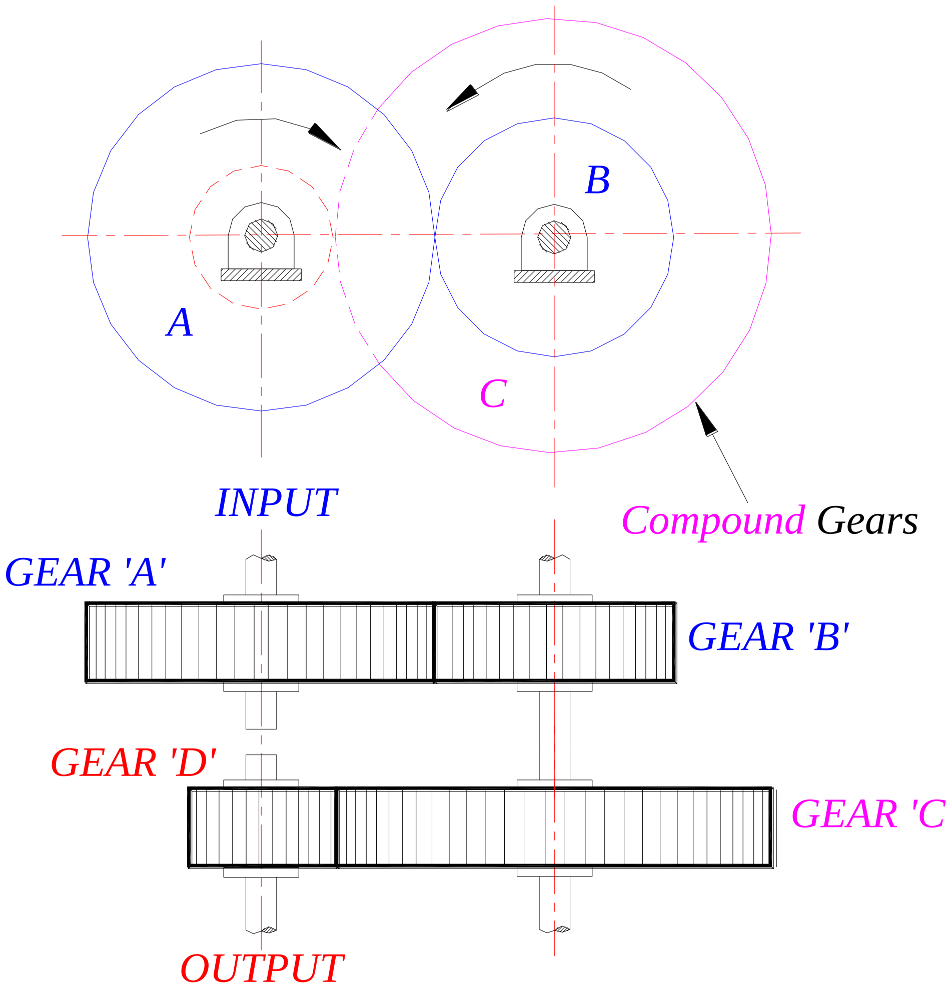
Concentric input & output shafts



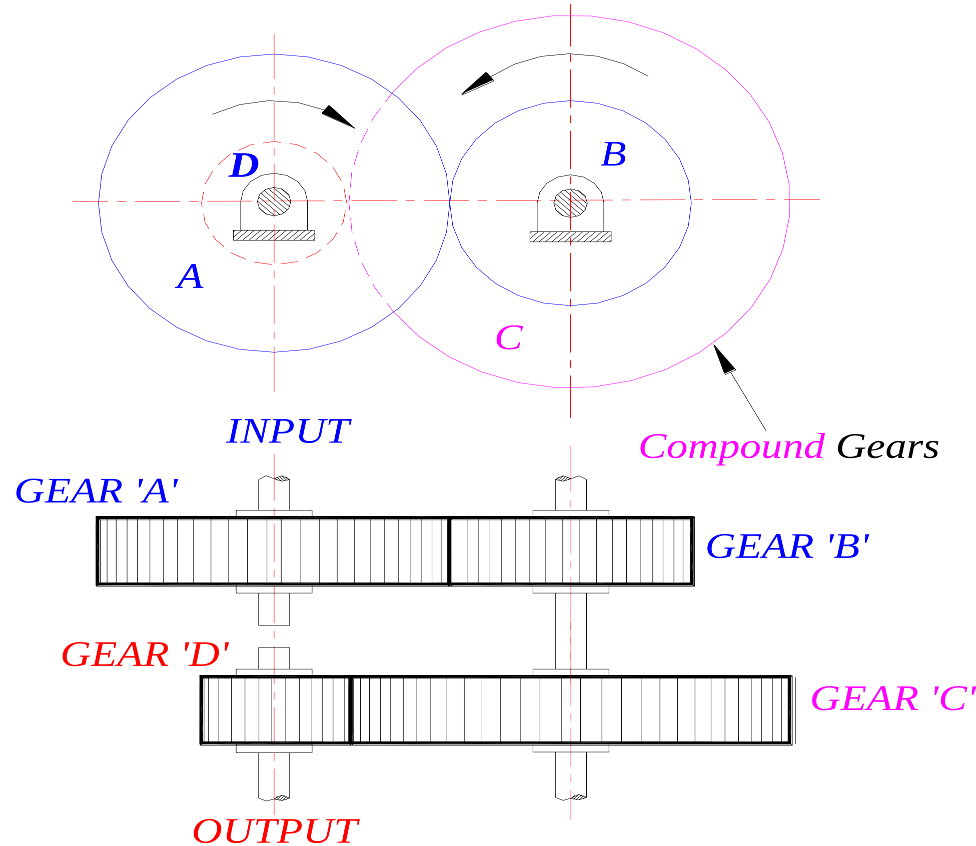
Reverted Gear train

When the axes of first driver & last driven are co-axial, then the gear train is known as Reverted Gear Train.

These are used in speed reducers, clocks and machine tools.



Reverted Gear train



Motion of *First Gear* & *Last Gear* is *Like!*

- *Gear A drives Gear B – opposite direction*
- *Gear B & Gear C are compound gear – same direction*
- *Gear C & Gear D – Opposite direction*
- *Gear D & Gear A are compound gear – same direction*

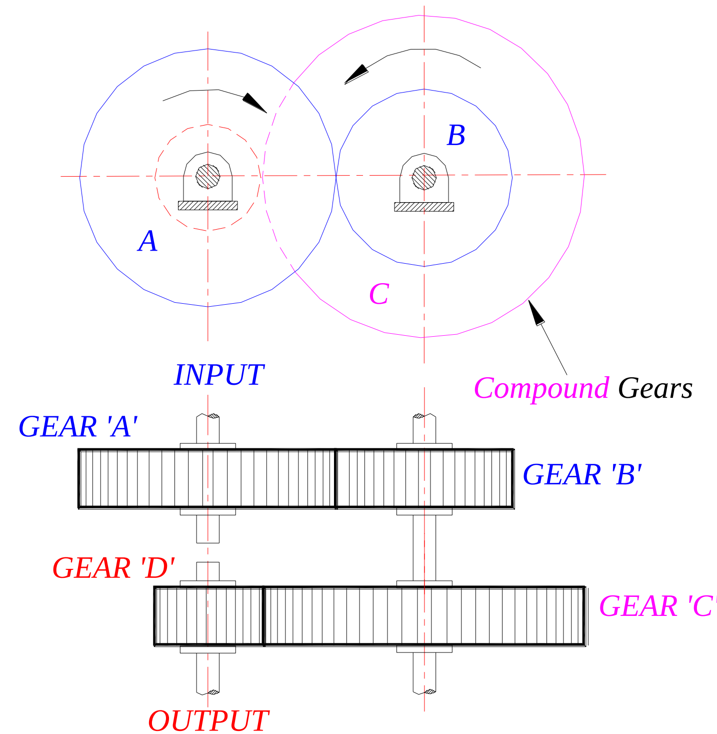
Reverted Gear train

Let,

N_A = Speed of driver gear A in rpm

T_A = Number of teeth on driver gear A

r_a= PC radius of gear A



Similarly;

N_B, N_C, N_D= Speed of respectiver gears in rpm

T_B, T_C, T_D= Number of teeth on respective gears

r_B, r_C, r_D= PC radius of respective gears

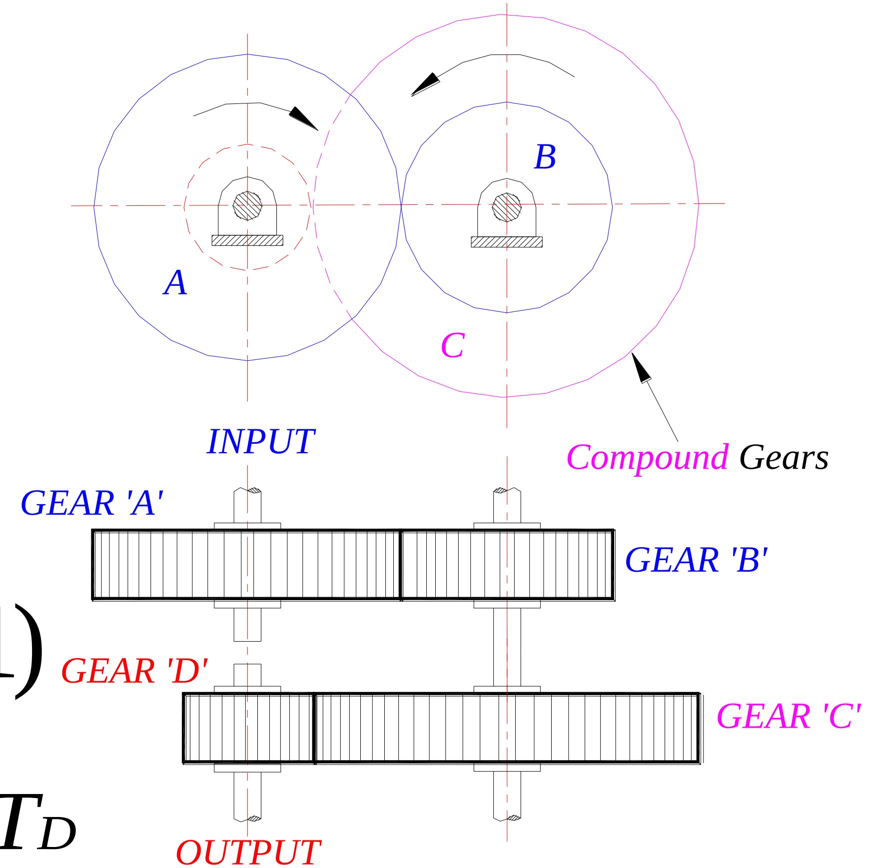
Reverted Gear train

Distance between centres of shafts of Gears A & B And Gears C & D are same.

i.e.;

$$r_A + r_B = r_C + r_D \ldots\ldots\ldots(1)$$

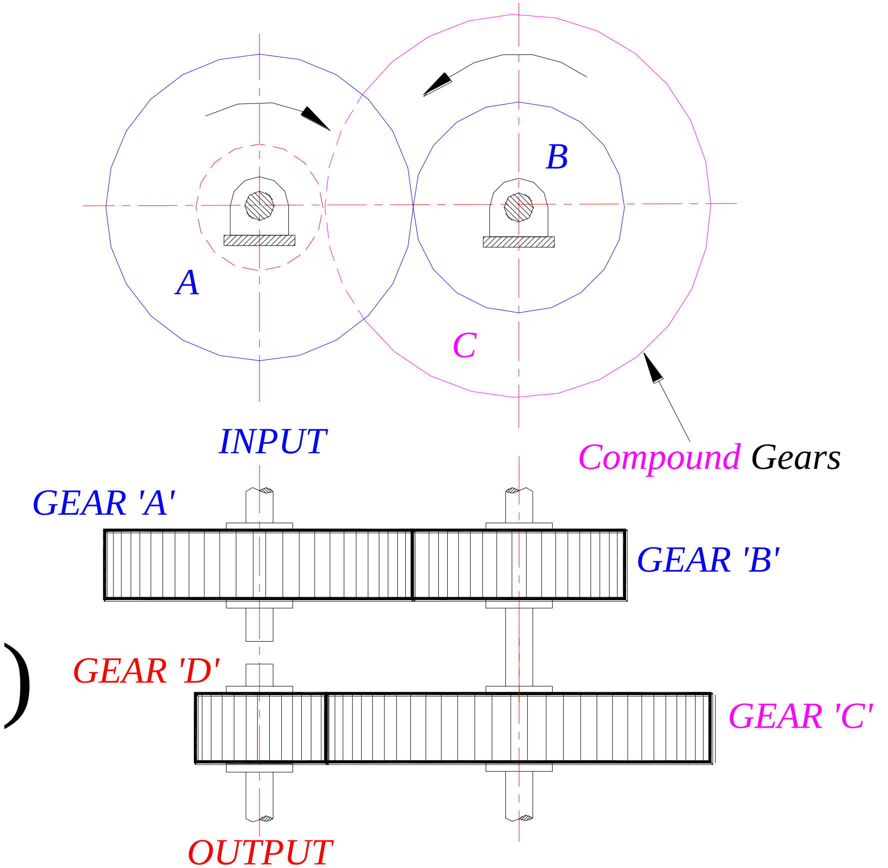
$$\therefore \frac{m \cdot T_A}{2} + \frac{m \cdot T_B}{2} = \frac{m \cdot T_C}{2} + \frac{m \cdot T_D}{2}$$



Reverted Gear train

Also module/ circular pitch for all gears is assumed to be same;

$$\therefore T_A + T_B = T_C + T_D \ldots\ldots\ldots\ldots(2)$$



Reverted Gear train

$$\text{i.e., Speed Ratio} = \frac{\text{Speed of 1st driver}}{\text{Speed of last driven}}$$

$$= \frac{\text{Product of No. of teeth on drivens}}{\text{Product of No. of teeth on drivers}}$$

And

$$\therefore \frac{N_A}{N_B} = \frac{T_B \times T_D}{T_A \times T_C} \ldots\ldots\ldots\ldots(3)$$

Frm eq. (1), (2) & (3) --- we can determine No. of teeth for given centre distance, speed ratio & module

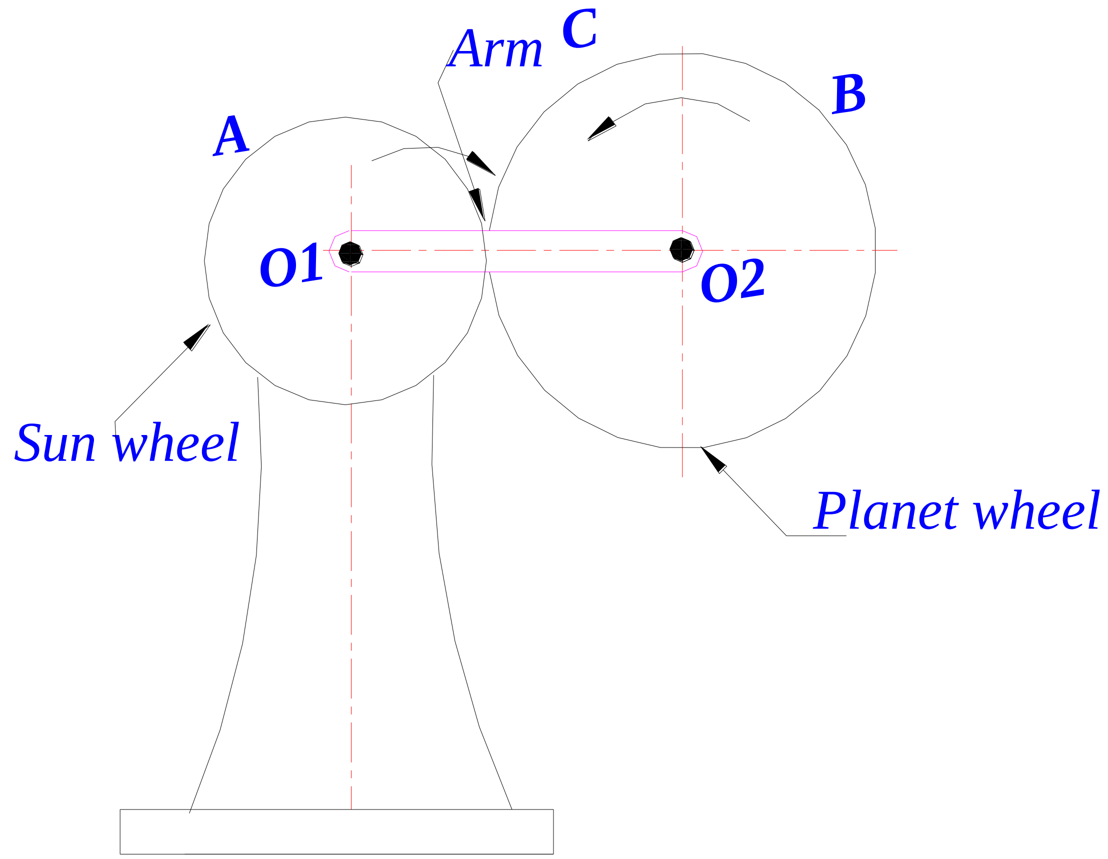
Reverted Gear train

- Used in:-
  - Automotive transmissions
  - Lathe back gears
  - Industrial speed reducers
  - Clocks

Epicyclic Gear train

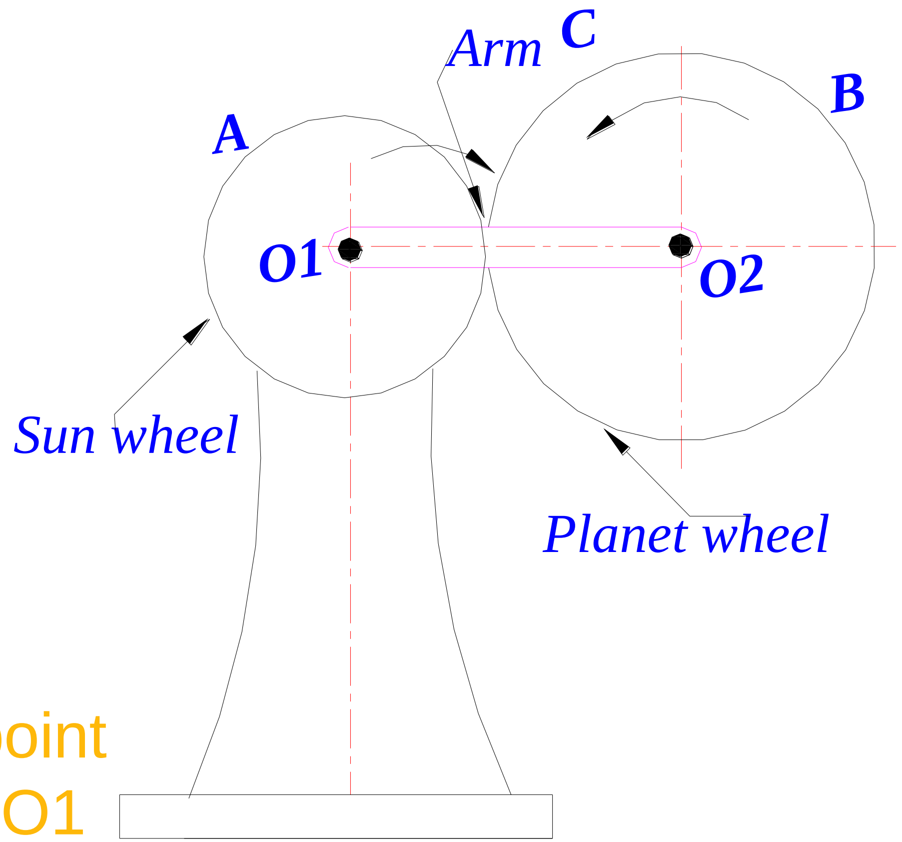
It is the system of epicyclic gears in which at least one wheel axis itself revolves around another fixed axis.

Epicyclic Gear train



Epicyclic Gear train

Epi – Upon
Cyclic – Around



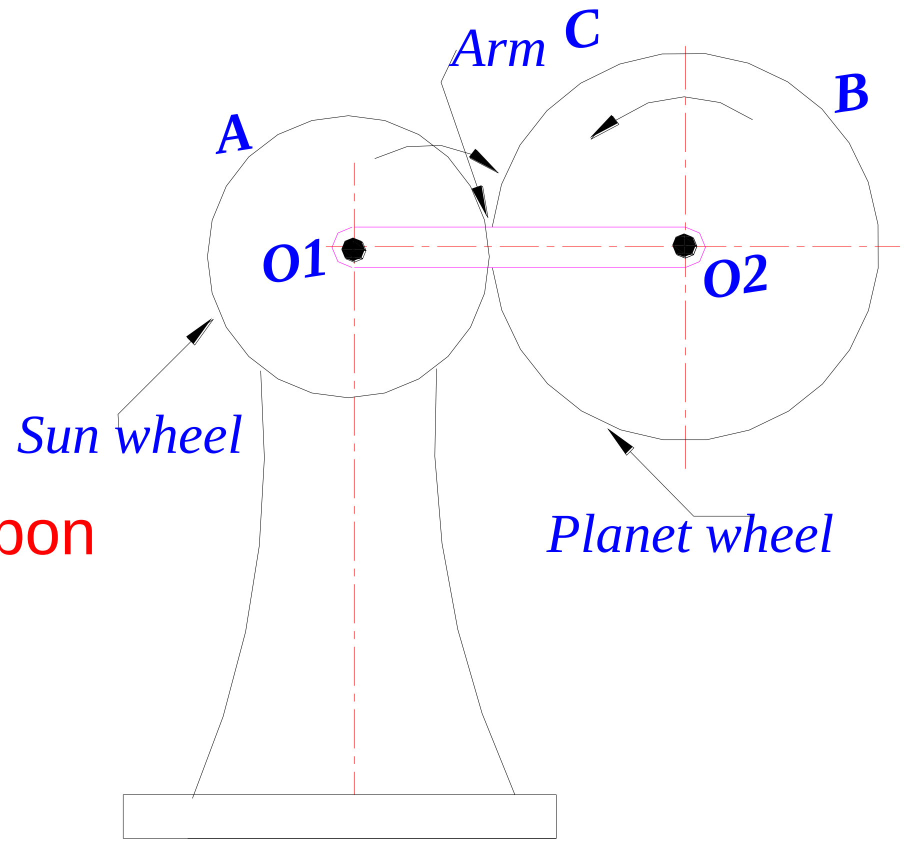
Gear A & Arm C – Rotates about common fixed axis at point O1

Gear B – Rotates about axis at point O2 also revolves upon gear A at O1

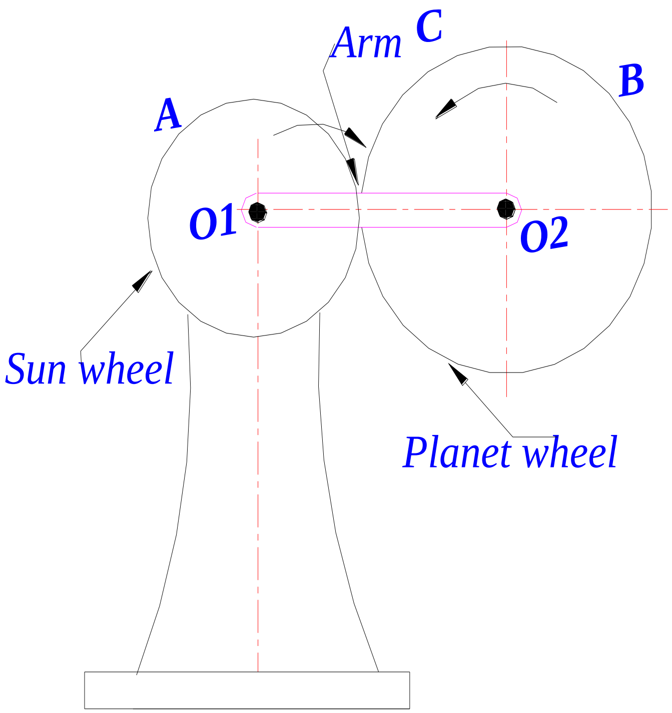
Epicyclic Gear train

Arm C fixed – Simple gear train

Gear A Fixed – Gear B rotates upon & around Gear A (Epicyclic Gear train)



Epicyclic Gear train



Used in :-

1. Transmitting high velocity ratios
2. Moderate size gears
3. Lesser space

Applications :-

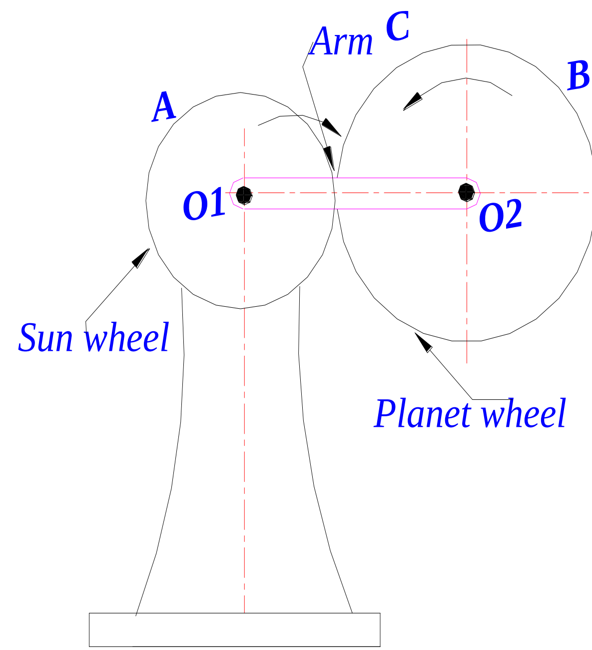
1. Back gear of lathe
2. Differential gears of automobiles
3. Hoists
4. Pulley Blocks
5. Wrist watches

Velocity Ratio of Epicyclic Gear Train

1. Tabular Method

2. Algebraic Method

Velocity Ratio of Epicyclic Gear Train



Tabular Method :-

Steps:

1) Consider arm C fixed

When Gear A makes one revolution clockwise; = +1

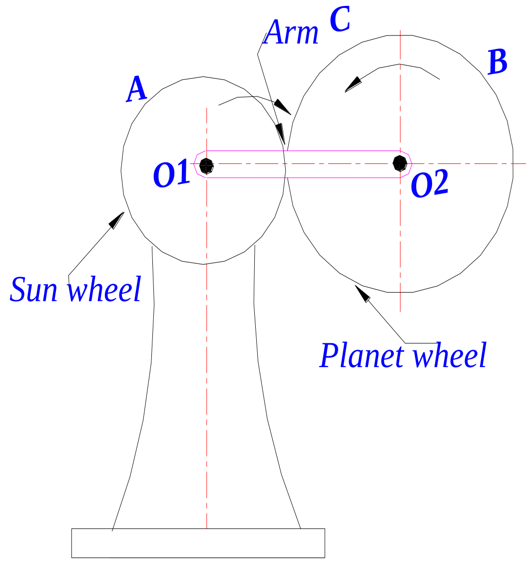
Gear B will make revolutions

= $-T_A/T_B$ (anticlockwise)

$$\left[\begin{array}{l} \because \dfrac{N_B}{N_A} = -\dfrac{T_A}{T_B} \\ \therefore N_B = -\dfrac{T_A}{T_B} \text{........}(\because N_A = 1) \end{array}\right]$$

1 st Row

Velocity Ratio of Epicyclic Gear Train



Tabular Method :-

Steps:

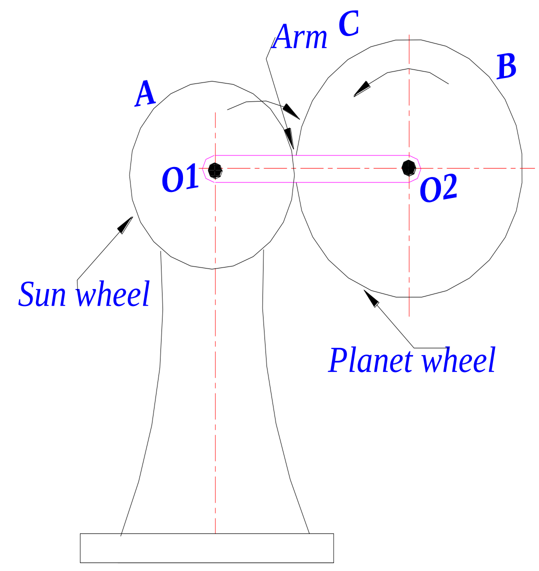
2) When Gear A makes revolution clockwise; = $+x$

Gear B will make revolutions

$= - x\, T_A/T_B$ (anticlockwise)

2nd Row

Velocity Ratio of Epicyclic Gear Train



Tabular Method :-

Steps:

3) Each element of epicyclic gear train is given revolution = +*y*

3rd Row

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$
2	Arm C Fixed – Gear A rotates through $+x$ Revolutions	0	$+x$	$-\,xT_A/T_B$

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$
2	Arm C Fixed – Gear A rotates through $+x$ Revolutions	0	$+x$	$-xT_A/T_B$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$
2	Arm C Fixed – Gear A rotates through $+x$ Revolutions	0	$+x$	$-xT_A/T_B$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4	Total Motion (row 2 + 3)	$+y$	$x+y$	$y-xT_A/T_B$

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$
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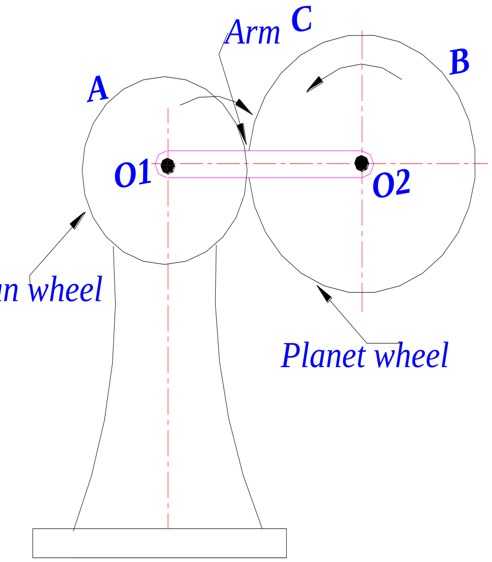
4	Total Motion (row 2 + 3)	$+y$	$x+y$	$y-xT_A/T_B$

Velocity Ratio of Epicyclic Gear Train

Algebraic Method :-

Steps:

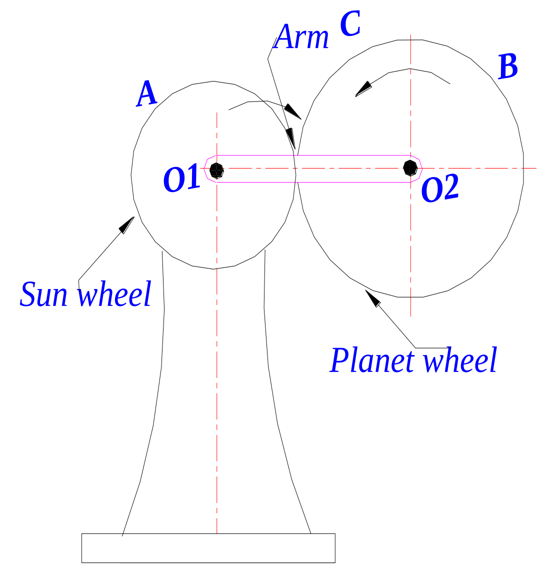


1) In this method each element of epicyclic gear train relative to arm is set down in the form of eq^n.

2) No. of eq. depends upon no. of elements in gear train

3) Two Conditions given :-
   - one element fixed
   - other has specified motion

Velocity Ratio of Epicyclic Gear Train



Algebraic Method :-

Steps:

1) Let, Arm C be fixed

Speed of gear A relative to Arm C

$$= N_A - N_C$$

Speed of gear B relative to Arm C

$$= N_B - N_C$$

Velocity Ratio of Epicyclic Gear Train

Algebraic Method :-

Gear A meshes with Gear B. thus, they revolve in Opposite Direction;

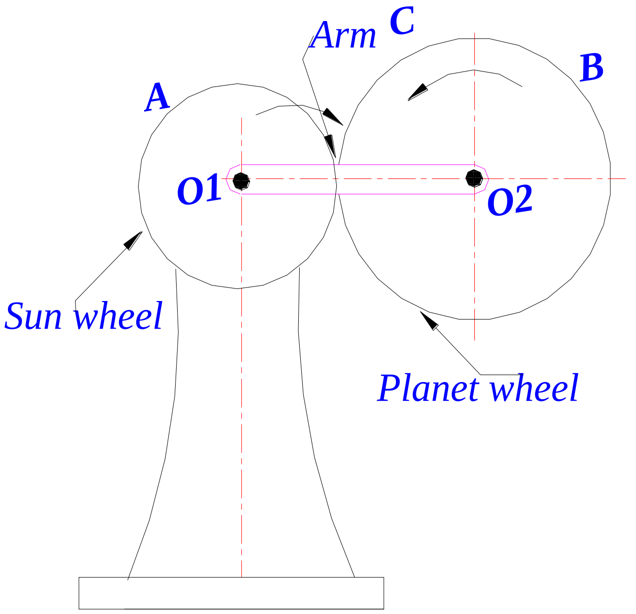
$$\therefore \frac{N_B - N_C}{N_A - N_C} = -\frac{T_A}{T_B}$$

Since Arm C is fixed; $N_C = 0$

$$\therefore \frac{N_B}{N_A} = -\frac{T_A}{T_B}$$

If Gear A is fixed; $N_A = 0$

$$\therefore \frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B}$$



Velocity Ratio of Epicyclic Gear Train

Algebraic Method :-

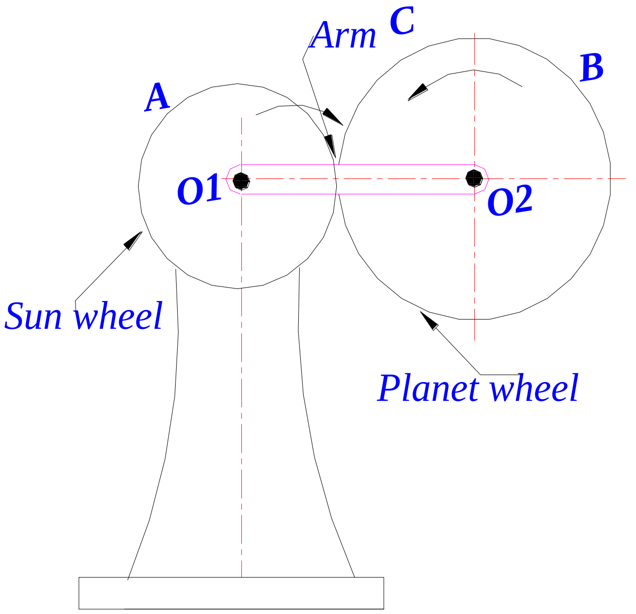
If Gear A is fixed; $N_A = 0$

$$\therefore \frac{N_B - N_C}{0 - N_C} = -\frac{T_A}{T_B}$$

$$\therefore \frac{N_B - N_C}{-N_C} = -\frac{T_A}{T_B}$$

$$-\frac{N_B}{N_C} + 1 = -\frac{T_A}{T_B}$$

$$\boxed{\therefore \frac{N_B}{N_C} = 1 + \frac{T_A}{T_B}}$$



Numerical

1) In an epicyclic gear train, an arm carries two gears A & B having 36 and 45 teeth resp.

   If the arm rotates at 150rpm, in anticlockwise direction about the centre of gear A which is fixed, determine the speed of gear B.

   If the gear A instead of being fixed, makes 300rpm in the clockwise direction, what will be the speed of gear B?

Numerical

Given :- TA=36

TB=45

Nc=150rpm(anticlockwise)

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$
2	Arm C Fixed – Gear A rotates through $+x$ Revolutions	0	$+x$	$-xT_A/T_B$

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$
2	Arm C Fixed – Gear A rotates through $+x$ Revolutions	0	$+x$	$-xT_A/T_B$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
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2	Arm C Fixed – Gear A rotates through $+x$ Revolutions	0	$+x$	$-xT_A/T_B$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4	Total Motion (row 2 + 3)	$+y$	$x+y$	$y-xT_A/T_B$

Velocity Ratio of Epicyclic Gear Train

Step No.	Conditions of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm C Fixed – Gear A rotates through +1 revolution i.e., clockwise	0	+1	$-T_A/T_B$
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3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4	Total Motion (row 2 + 3)	$+y$	$x+y$	$y-xT_A/T_B$

Numerical

Speed of gear B when A is fixed;

Speed of arm = 150rpm; Thus, frm 4th row

$$y = +150 \text{rpm}$$

Also, gear A is fixed;

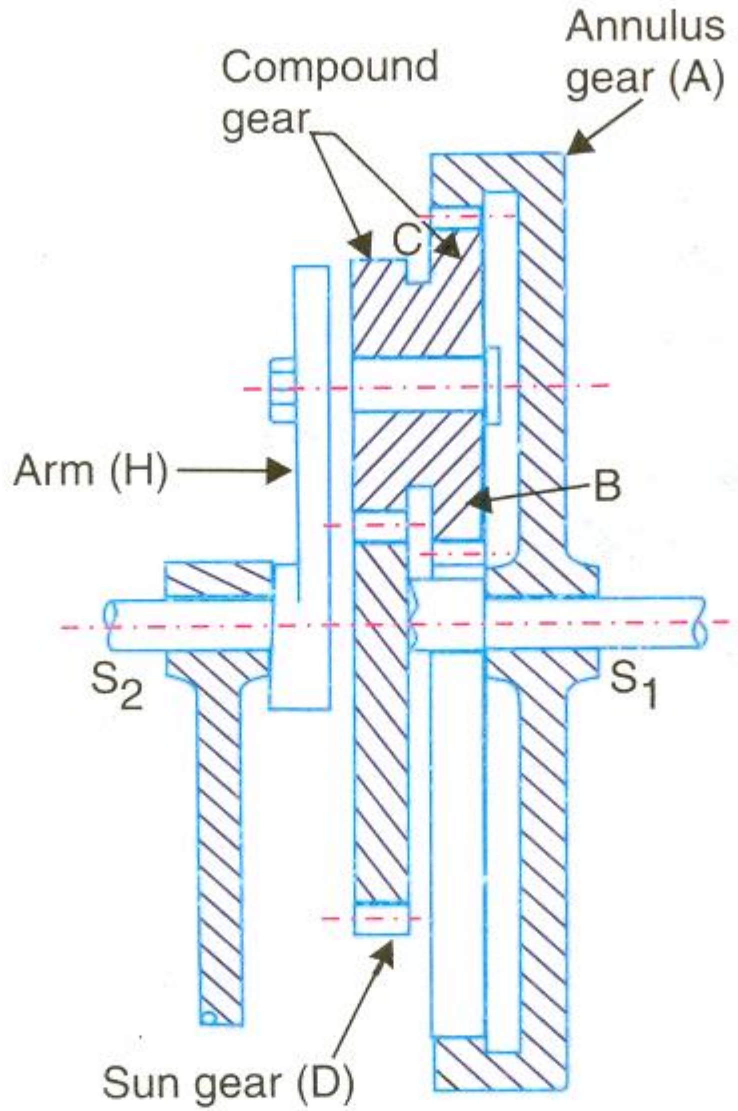
$$x + y = 0$$

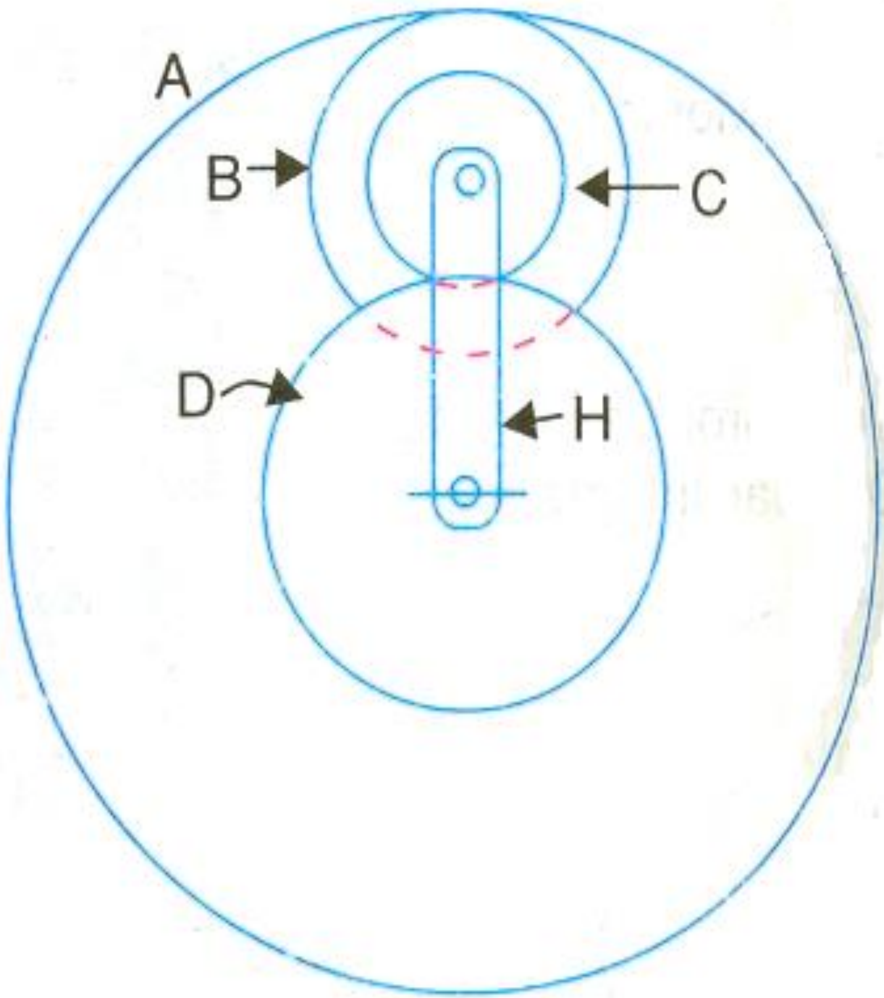
$$x = -y = -150 \text{rpm}$$

Speed of gear B,

$$N_B = y - x \times \frac{T_A}{T_B}$$

Compound Epicyclic Gear train





Compound Epicyclic Gear train

It consists of;

- Two co-axial shafts S1 & S2

- Annulus gear A which is fixed

- Compound gear/Planet gear B-C

- Sun gear D

- Arm H

- ***The Sun gear is co-axial with annulus gear & arm***

Compound Epicyclic Gear train

It consists of;

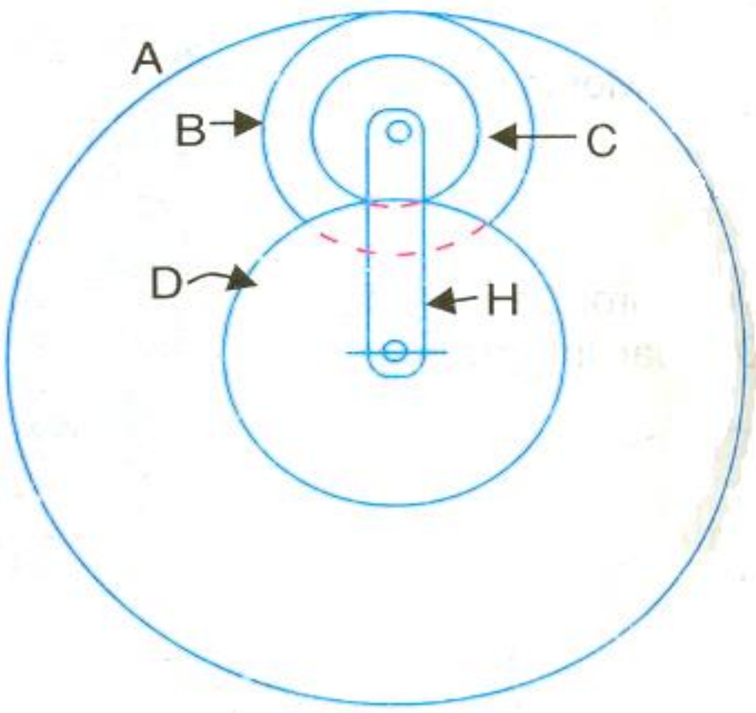
- Two co-axial shafts S1 & S2

- Annulus gear A which is fixed

- Compound gear/Planet gear B-C

- Sun gear D

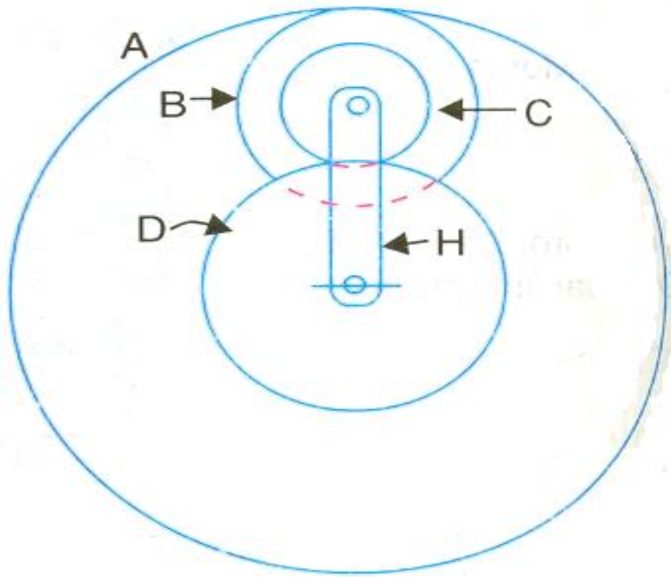
- Arm H



- *The Sun gear is co-axial with annulus gear & arm*

Compound Epicyclic Gear train

- Annulus gear A – meshes with Gear B



- Sun gear D – meshes with Gear C

- ***When annulus gear is fixed – Sun gear provides the drive***

&

- ***When Sun gear is fixed – Annulus gear provides the drive***
- ***In both cases Arm acts as a follower***

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm H	Gear D	Compound Gear B-C	Gear A

Velocity Ratio of Epicyclic Gear Train

Gear D meshes with Gear C;

$$\therefore \frac{N_C}{N_D} = -\frac{T_D}{T_C} \qquad \therefore N_C = -\frac{T_D}{T_C}$$

Gear A meshes with Gear B;

$$\therefore \frac{N_A}{N_B} = \frac{T_B}{T_A}$$

$$\therefore N_A = N_B \times \frac{T_B}{T_A}$$

N_B=N_C; Compound Gear

$$\therefore N_A = -\frac{T_D}{T_C} \times \frac{T_B}{T_A}$$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm H (N_H)	Gear D (N_D)	Comp-ound Gear B-C (N_B& N_C)	Gear A (N_A)
1	Arm fixed – Gear D rotates through +1 revolution	0	+1	$-T_D/T_C$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm H	Gear D	Comp-ound Gear B-C	Gear A
1	Arm fixed – Gear D rotates through +1 revolution	0	+1	$-T_D/T_C$	$-\dfrac{T_D}{T_C}\times\dfrac{T_B}{T_A}$
2	Arm fixed – Gear D rotates through $+x$ revolutions	**0**	$+x$	$-x\,T_D/T_C$	$-x\cdot\dfrac{T_D}{T_C}\times\dfrac{T_B}{T_A}$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm H	Gear D	Comp-ound Gear B-C	Gear A
1	Arm fixed – Gear D rotates through +1 revolution	0	+1	$-T_D/T_C$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
2	Arm fixed – Gear D rotates through $+x$ revolutions	**0**	$+x$	$-x\,T_D/T_C$	$-x\cdot\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm H	Gear D	Compound Gear B-C	Gear A
1	Arm fixed – Gear D rotates through +1 revolution	0	+1	$-T_D/T_C$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
2	Arm fixed – Gear D rotates through $+x$ revolutions	**0**	$+x$	$-x\, T_D/T_C$	$-x\cdot\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total Motion	$+y$	$x+y$	$y-x\, T_D/T_C$	$y-x\cdot\frac{T_D}{T_C}\times\frac{T_B}{T_A}$

Numerical

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm EF	Gear C	Gear B	Gear A

Velocity Ratio of Epicyclic Gear Train

Gear D meshes with Gear C;

$$\therefore \frac{N_C}{N_D} = -\frac{T_D}{T_C} \qquad \therefore N_C = -\frac{T_D}{T_C}$$

Gear A meshes with Gear B;

$$\therefore \frac{N_A}{N_B} = \frac{T_B}{T_A}$$

$$\therefore N_A = N_B \times \frac{T_B}{T_A}$$

N_B=N_C; Compound Gear

$$\therefore N_A = -\frac{T_D}{T_C} \times \frac{T_B}{T_A}$$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm EF	Gear C	Gear B	Gear A
1	Arm fixed – Gear C rotates through +1 revolution	0	+1	$-T_C/T_B$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$
2	Arm fixed – Gear D rotates through $+x$ revolutions	0	$+x$	$-xT_C/T_B$	$-x \times \frac{T_C}{T_A}$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm EF	Gear C	Gear B	Gear A
1	Arm fixed – Gear C rotates through +1 revolution	0	+1	$-T_C/T_B$	$-\frac{T_C}{T_B} \times \frac{T_B}{T_A} = -\frac{T_C}{T_A}$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm H	Gear D	Comp-ound Gear B-C	Gear A
1	Arm fixed – Gear D rotates through +1 revolution	0	+1	$-T_D/T_C$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
2	Arm fixed – Gear D rotates through $+x$ revolutions	$\mathbf{0}$	$+\ x$	$-x\ T_D/T_C$	$-x\cdot\frac{T_D}{T_C}\times\frac{T_B}{T_A}$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm H	Gear D	Comp-ound Gear B-C	Gear A
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2	Arm fixed – Gear D rotates through $+x$ revolutions	$\mathbf{0}$	$+\,x$	$-x\ T_D/T_C$	$-x \cdot \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$

Velocity Ratio of Epicyclic Gear Train

Stp No.	Conditions of Motion	Revolution of Elements			
		Arm H	Gear D	Compound Gear B-C	Gear A
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2	Arm fixed – Gear D rotates through $+x$ revolutions	**0**	$+x$	$-x\, T_D/T_C$	$-x \cdot \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total Motion	$+y$	$x+y$	$y - x\, T_D/T_C$	$y - x \cdot \frac{T_D}{T_C} \times \frac{T_B}{T_A}$

Velocity Ratio of Epicyclic Gear Train

Gear D meshes with Gear C;

$$\therefore \frac{N_C}{N_D} = -\frac{T_D}{T_C} \qquad \therefore N_C = -\frac{T_D}{T_C}$$

Gear A meshes with Gear B;

$$\therefore \frac{N_A}{N_B} = \frac{T_B}{T_A}$$

$$\therefore N_A = N_B \times \frac{T_B}{T_A}$$

N_B=N_C; Compound Gear

$$\therefore N_A = -\frac{T_D}{T_C} \times \frac{T_B}{T_A}$$

Epicyclic Gear train

A small gear at the center called the sun, several medium sized gears called the planets and a large external gear called the ring gear.

Epicyclic Gear train

Planetary gear trains have several advantages. They have higher gear ratios. They are popular for automatic transmissions in automobiles. They are also used in bicycles for controlling power of pedaling automatically or manually. They are also used for power train between internal combustion engine and an electric motor.

Epicycilc Gear train

Basic Theory

Suppose gear C is fixed and the arm A makes one revolution. Determine how many revolutions the planet gear B makes.

Step 1: revolve all elements once about the center.

Step 2: identify that C should be fixed and rotate it backwards one revolution keeping the arm fixed as it should only do one revolution in total. Work out the revolutions of B.

Step 3: add them up and we find the total revolutions of C is zero and for the arm is 1.

Torque & Efficiency

The power transmitted by a torque T N-m applied to a shaft rotating at N rev/min is given by:

$$P = \frac{2\pi N T}{60}$$

In an ideal gear box, the input and output powers are the same so;

$$P = \frac{2\pi N_1 T_1}{60} = \frac{2\pi N_2 T_2}{60}$$

$$N_1 T_1 = N_2 T_2 \quad \Rightarrow \quad \frac{T_2}{T_1} = \frac{N_1}{N_2} = GR$$

Torque & Efficiency

It follows that if the speed is reduced, the torque is increased and vice versa. In a real gear box, power is lost through friction and the power output is smaller than the power input. The efficiency is defined as:

$$\eta = \frac{Power\ out}{Power\ In} = \frac{2\pi \times N_2 T_2 \times 60}{2\pi \times N_1 T_1 \times 60} = \frac{N_2 T_2}{N_1 T_1}$$

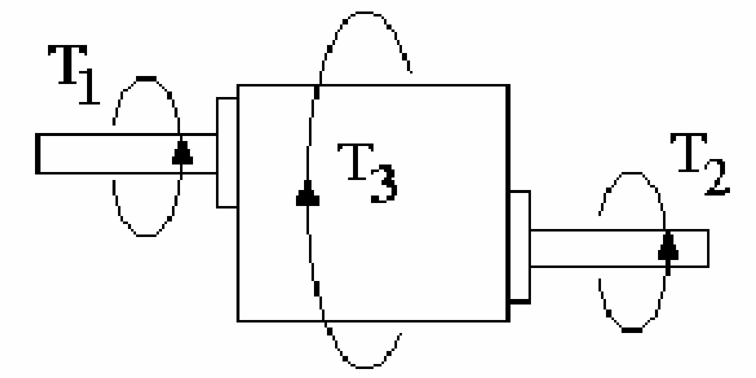
Because the torque in and out is different, a gear box has to be clamped in order to stop the case or body rotating. A holding torque T3 must be applied to the body through the clamps.

Torque & Efficiency

The total torque must add up to zero.

$$T_1 + T_2 + T_3 = 0$$

If we use a convention that anti-clockwise is positive and clockwise is negative we can determine the holding torque. The direction of rotation of the output shaft depends on the design of the gear box.



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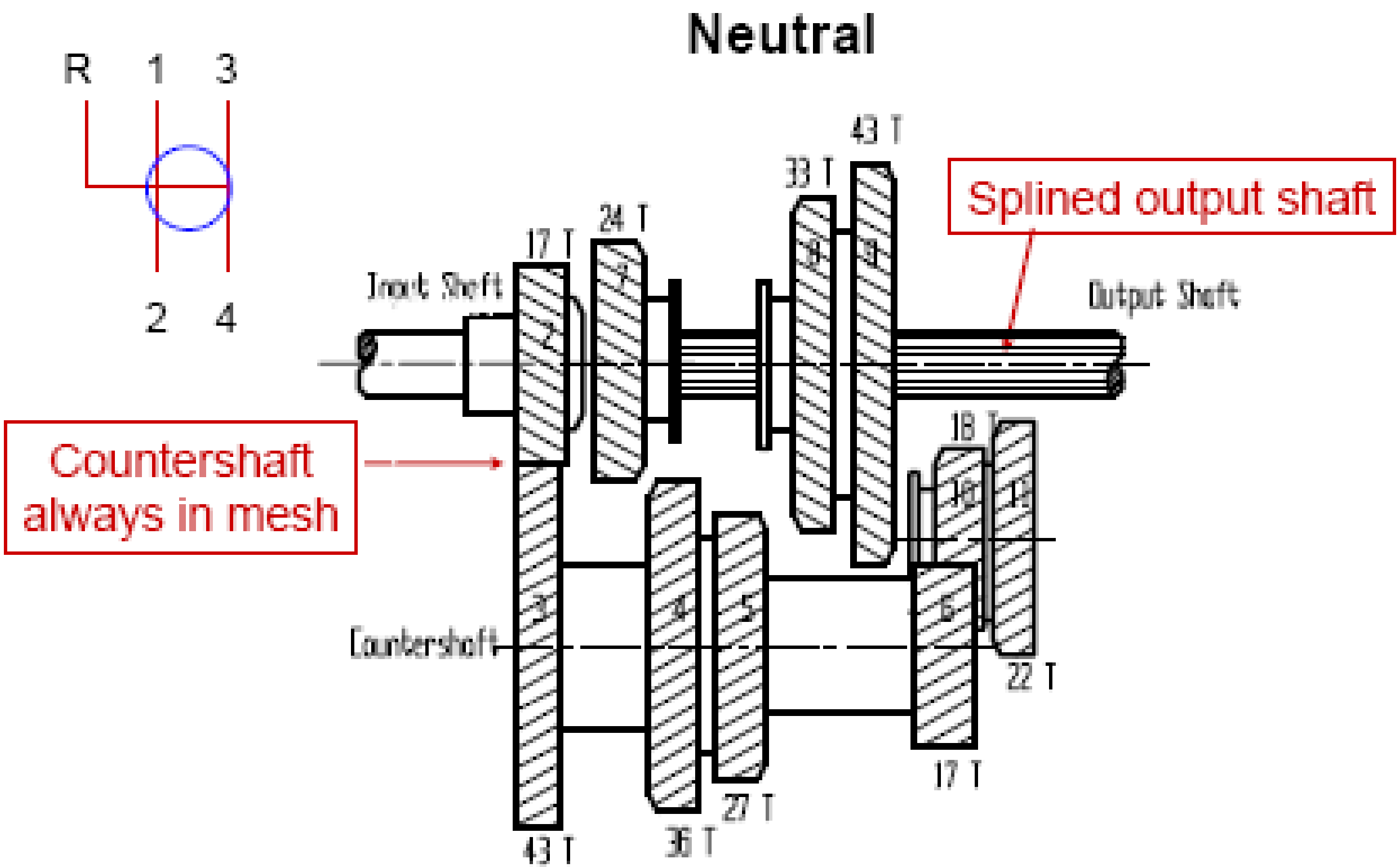
Flash Automatic Transmission Animation

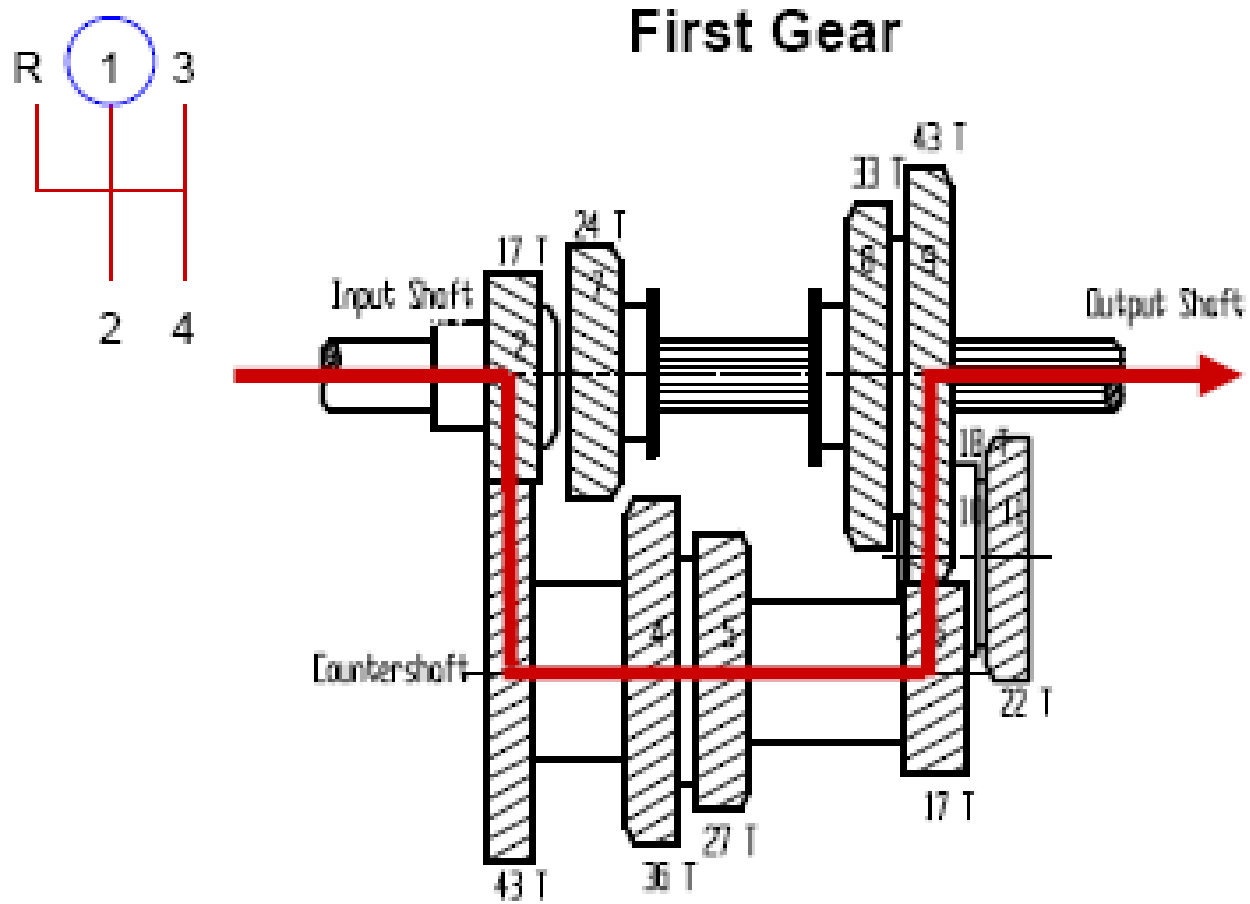
http://www.howstuffworks.com/differential2.htm

http://www.howstuffworks.com/transmission.htm

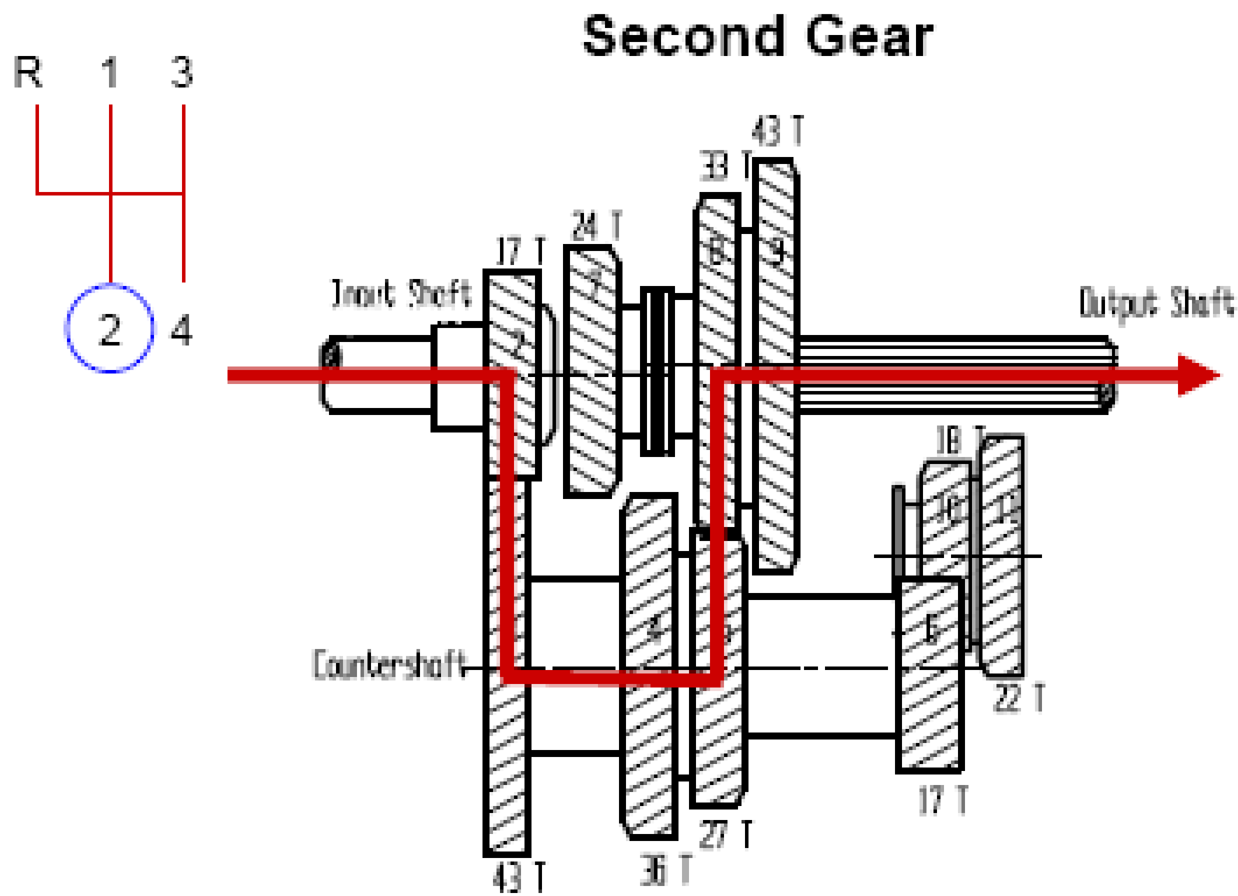
Applications



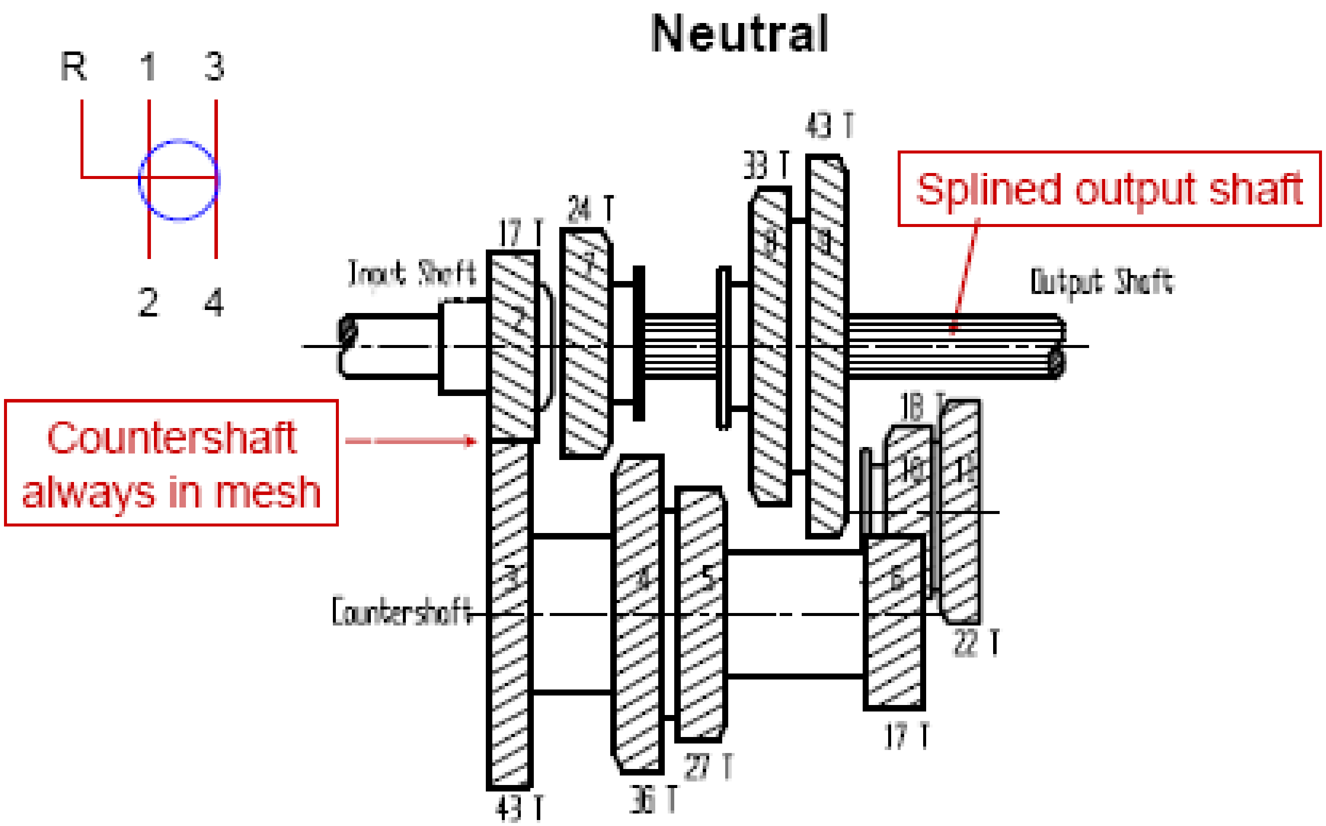
Applications



Applications



Applications



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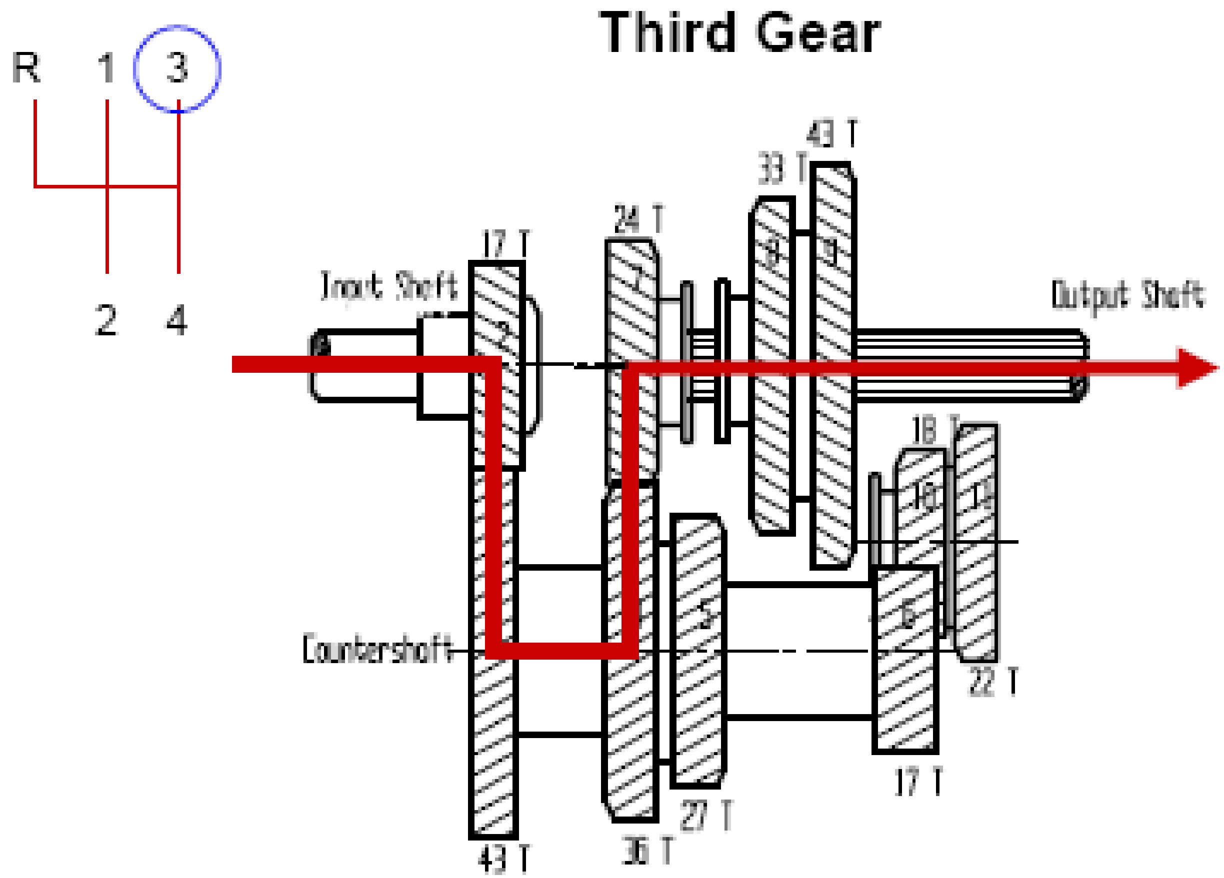

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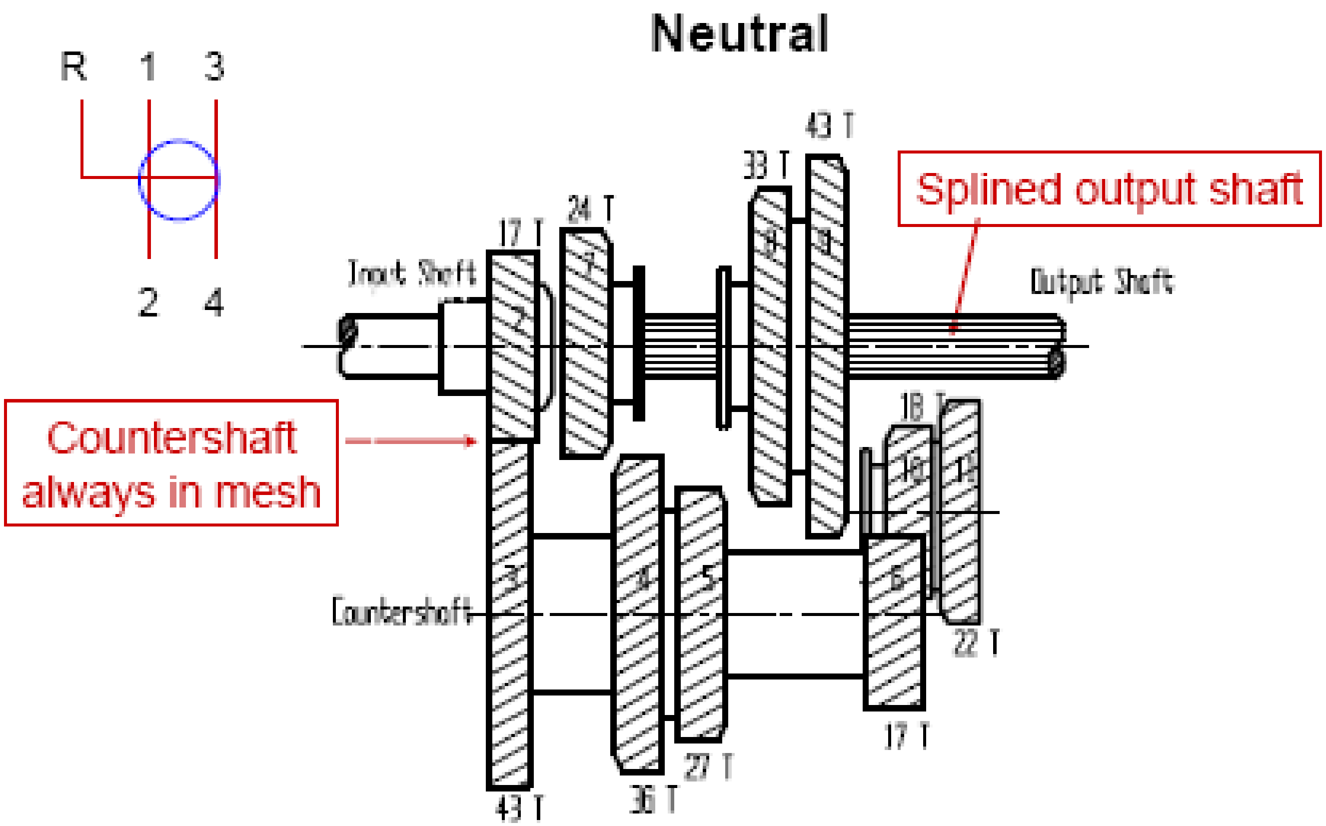

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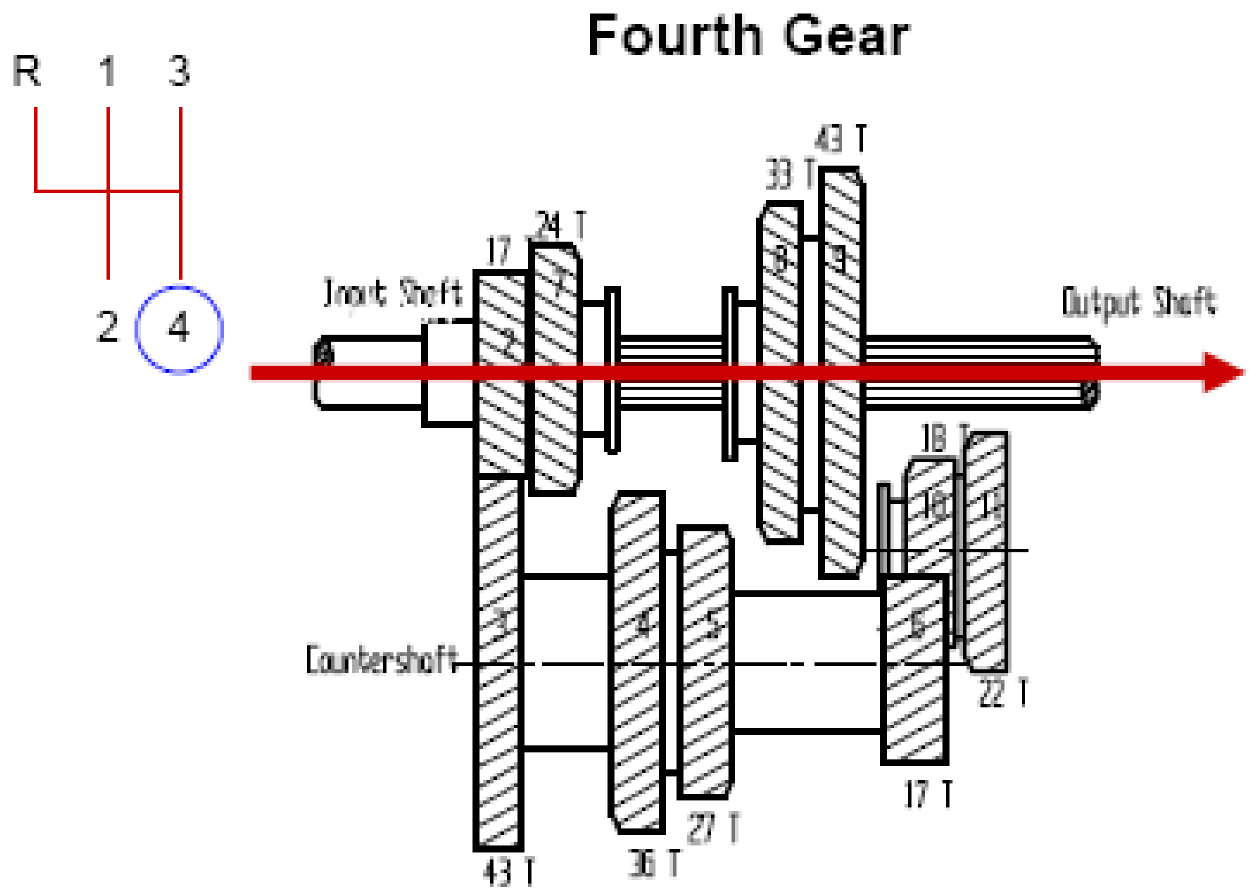

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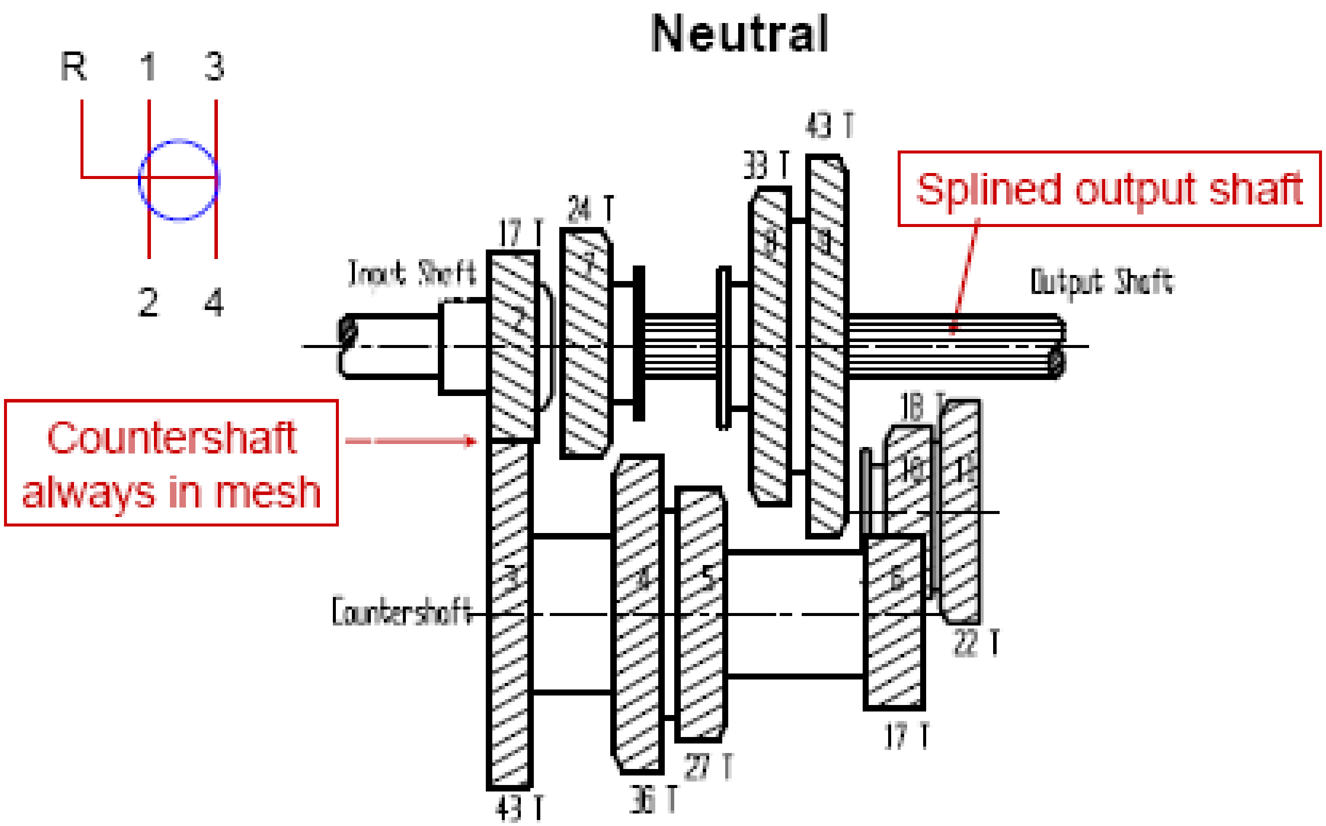

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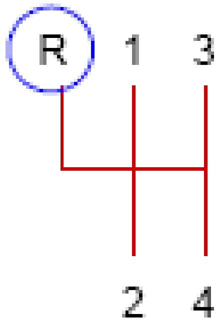
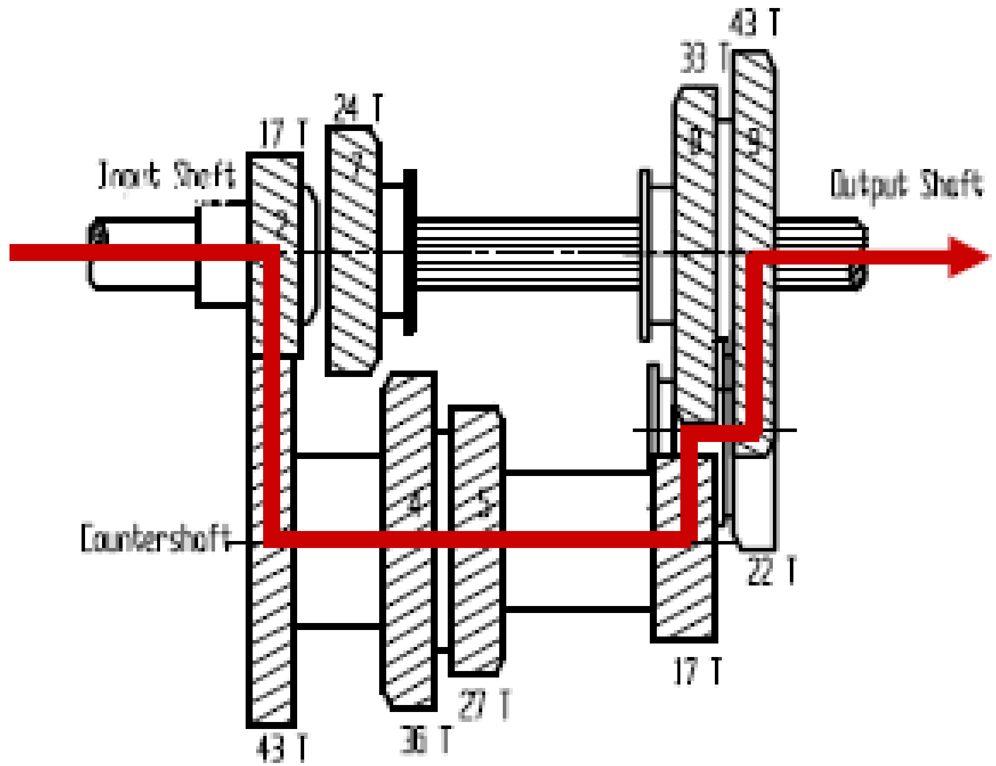
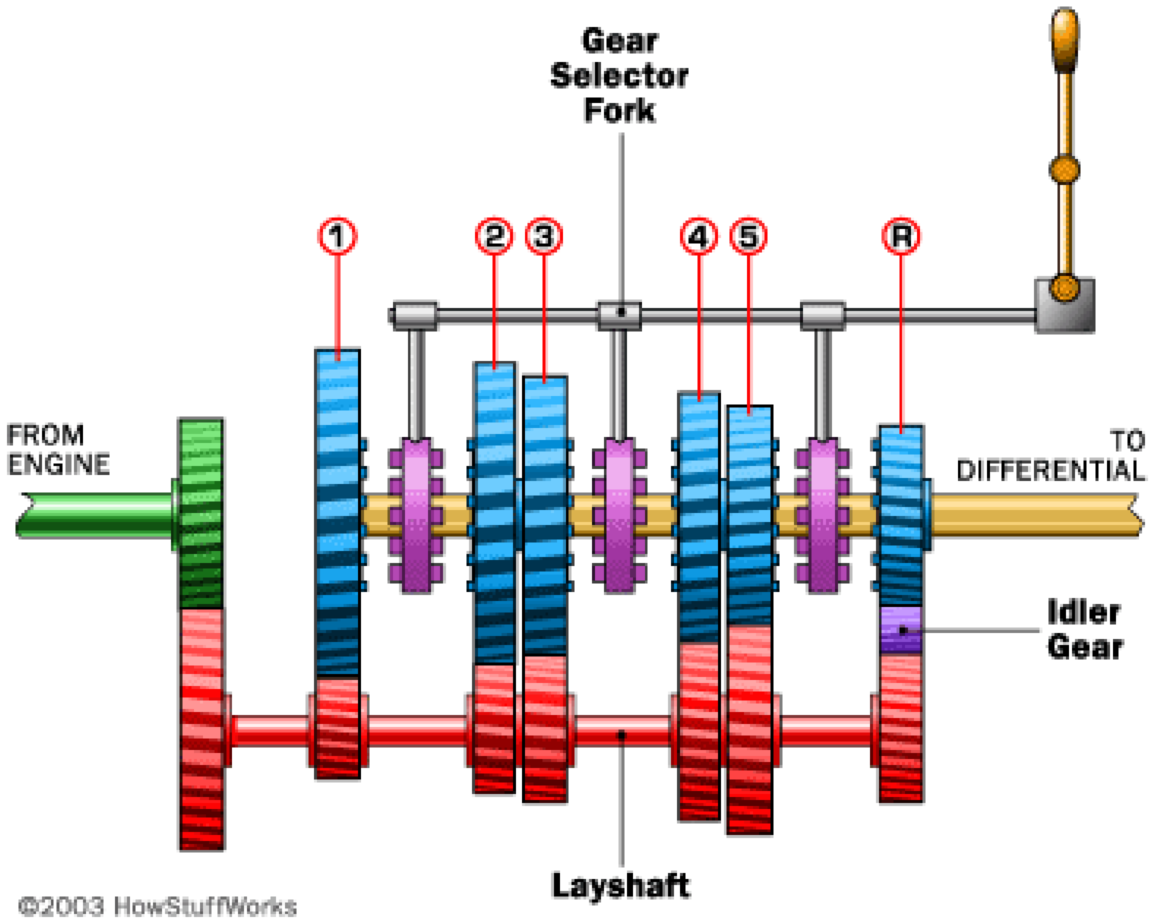


Reverse Gear

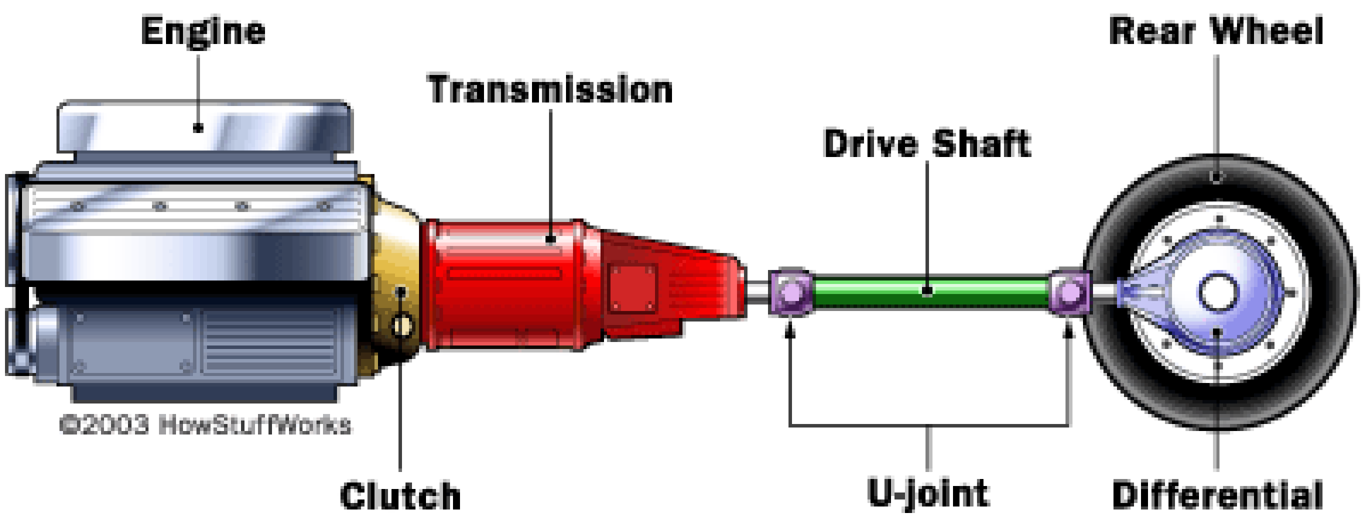


Applications



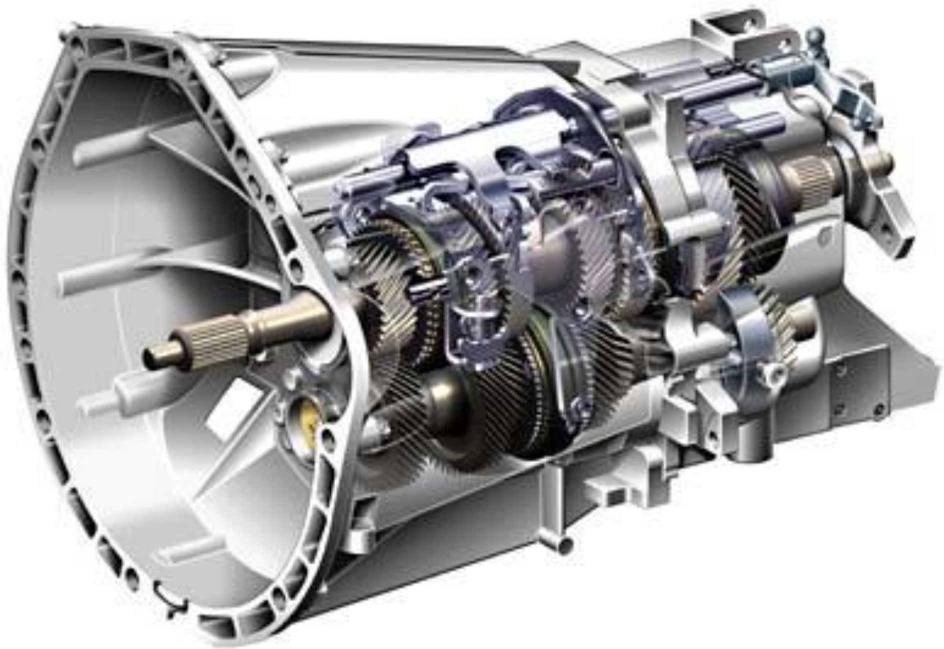
Applications

Automotive Gears: *Gears play an important role in trucks, car, buses, motor bikes and even geared cycles. These gears control speed and include gears like ring and pinion, spiral gear, hypoid gear, hydraulic gears, reduction gearbox.*

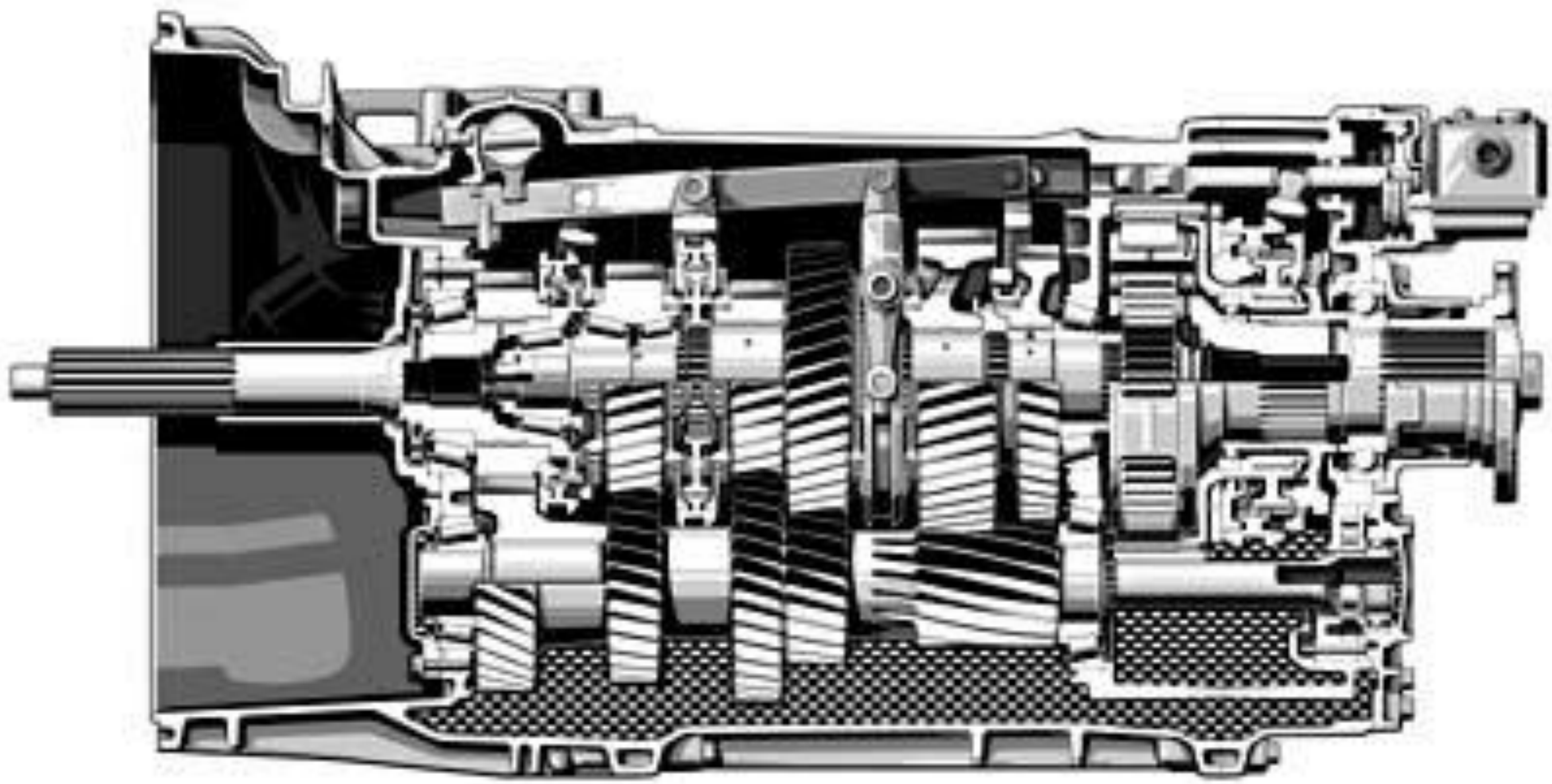


Applications

Depending on the size of the vehicles, the size of the gears also varies. There are low gears covering a shorter distance and are useful when speed is low. There are high gears also with larger number of teeth.



Applications



Applications

Conveyor Systems: Conveyor is a mechanical apparatus for carrying bulk material from place to place at a controlled rate; for example an endless moving belt or a chain of receptacles. There are various types of conveyors that are used for different material handling needs.

Applications

***Agro Industry**: All agro machinery consists of different types of gears depending upon their function and property. Different gears are used differently in the industry.*

***Wind Turbine:** When the rotor rotates, the load on the main shaft is very heavy. It runs with approximate 22 revolutions per minute but generator has to go a lot faster. It cannot use the turning force to increase the number of revolutions and that is why wind turbine uses gear to increase the speed.*

Applications

Power Station:

Helical gears - Are used to minimise noise and power losses.
Bevel gears - Used to change the axis of rotational motion.
Spur gears - Passes power from idler gears to the wheels.
Planetary gears - Used between internal combustion engine and an electric motor to transmit power.

Applications

Marine Gears: *Marine gears meet a wide variety of marine applications in a variety of configurations and installations to meet the most critical applications.*

Specific marine applications include main propulsion, centrifuges, deck machinery such as winches, windlasses, cranes, turning gears, pumps, elevators, and rudder carriers.

Applications

Mining Gears: *Mining is a process of extracting ores or minerals from the earth's surface. The gears are used for increasing the torque applied on the tool used for mining. They are used for commercial gold production, and coal mining.*

Differential Gear Box

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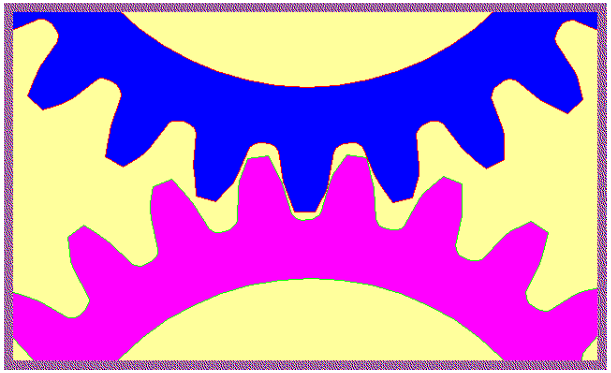
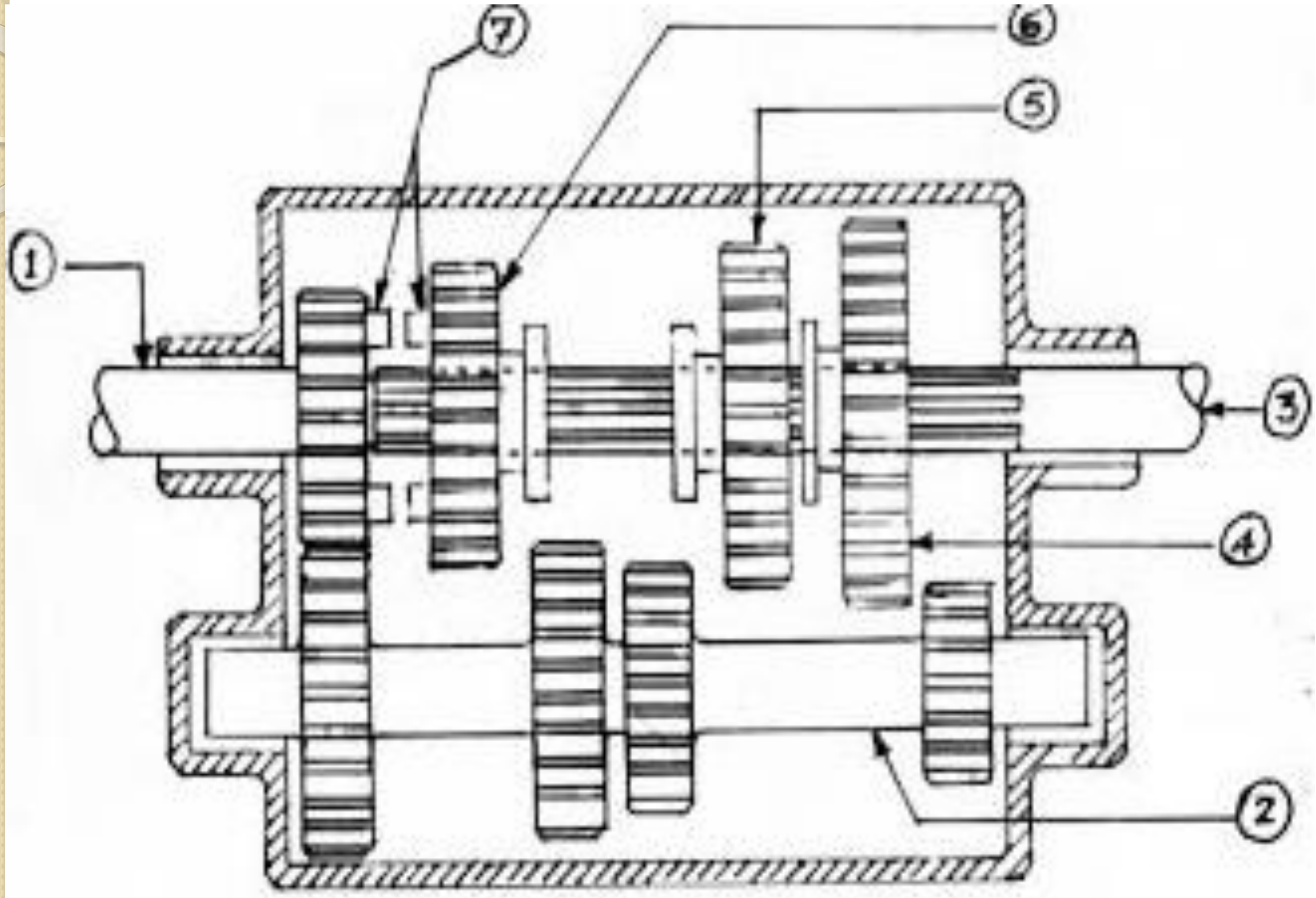
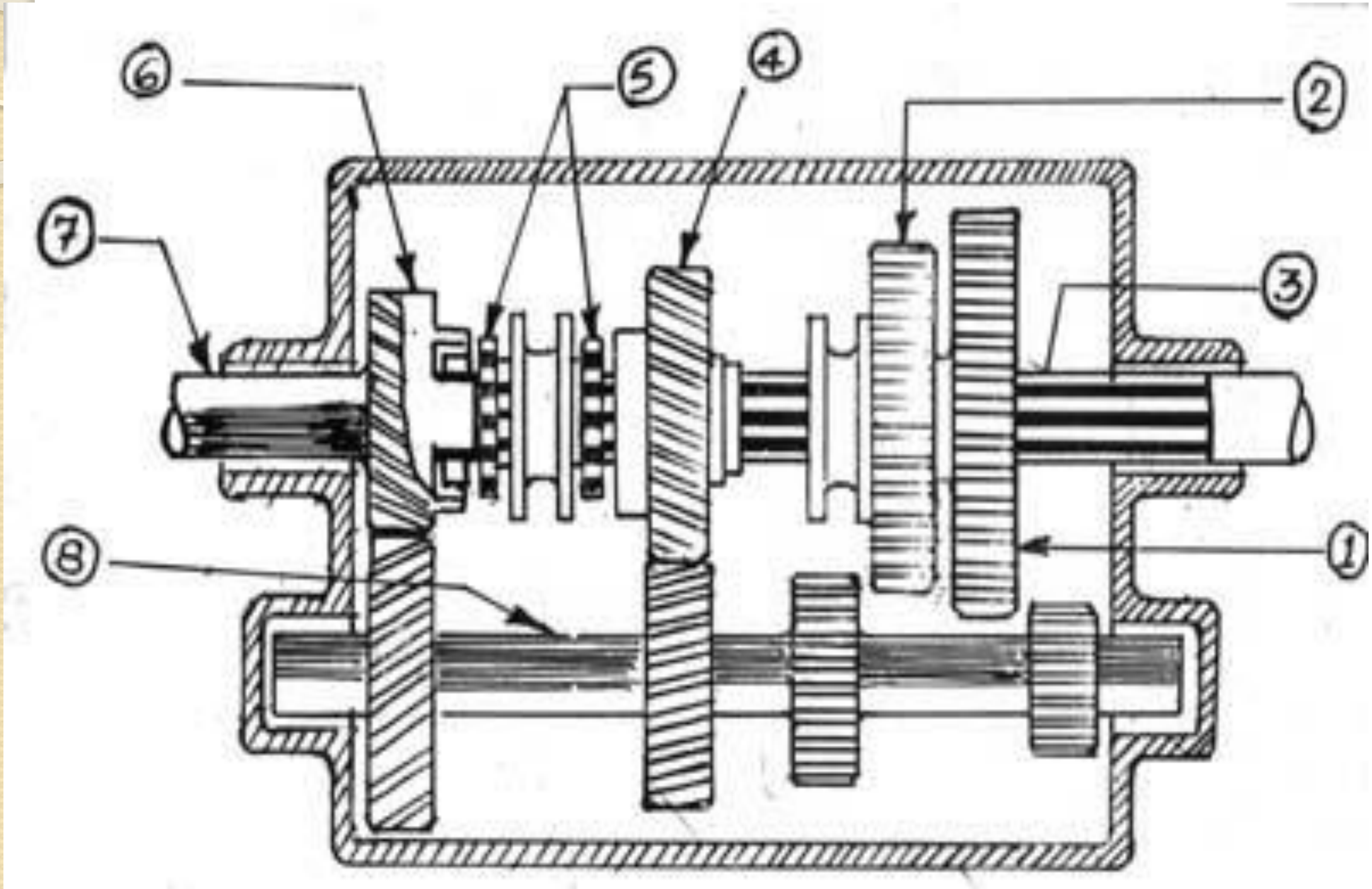


http://www.howstuffworks.com/differential2.htm

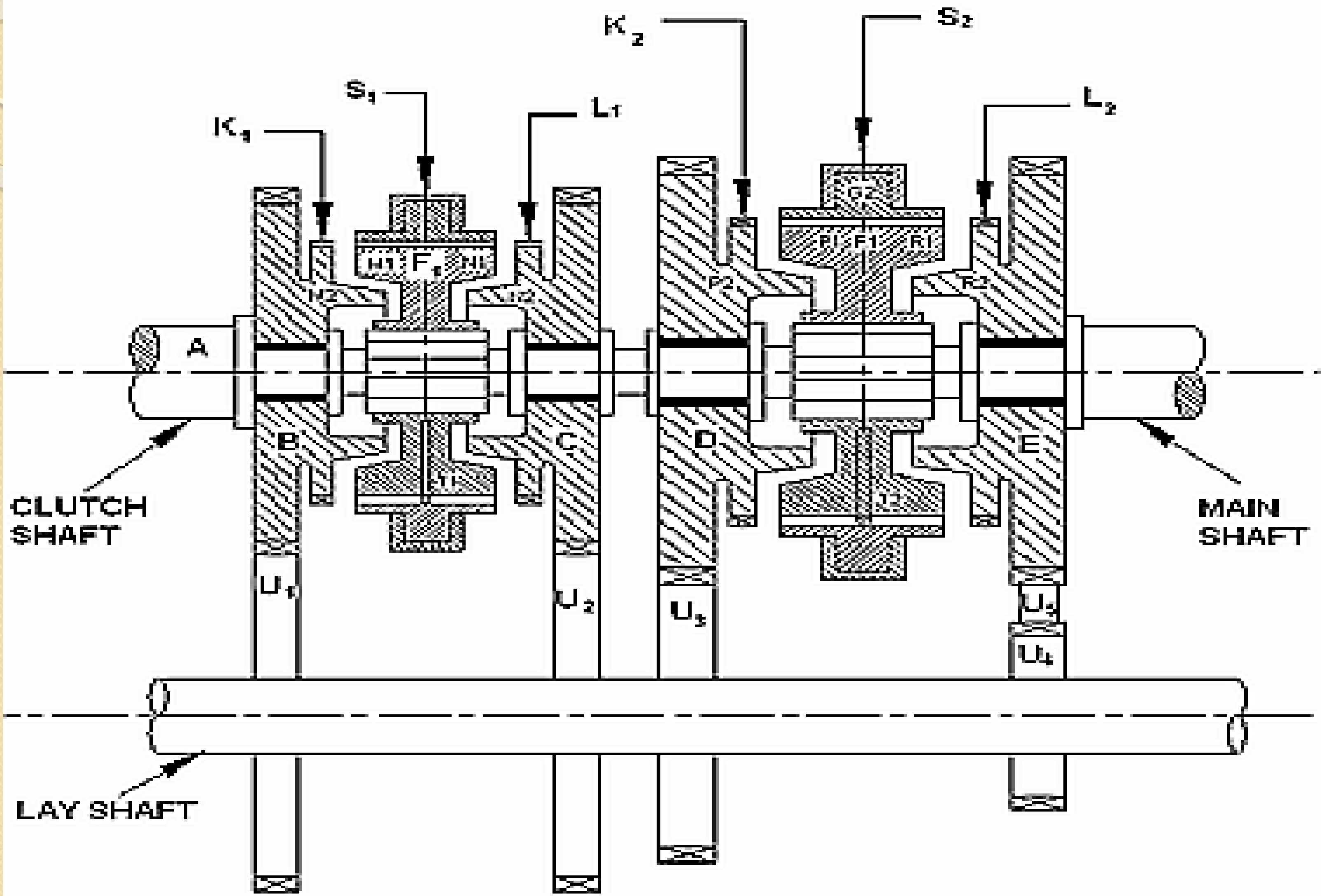
SLIDING MESH GEARBOX



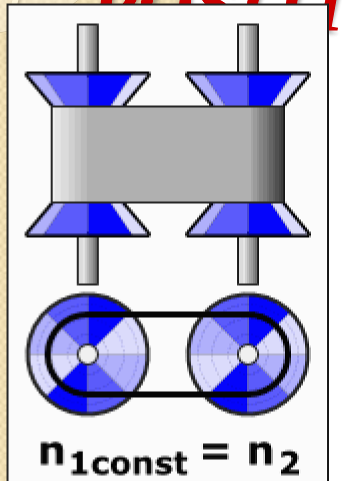
CONSTANT MESH GEARBOX

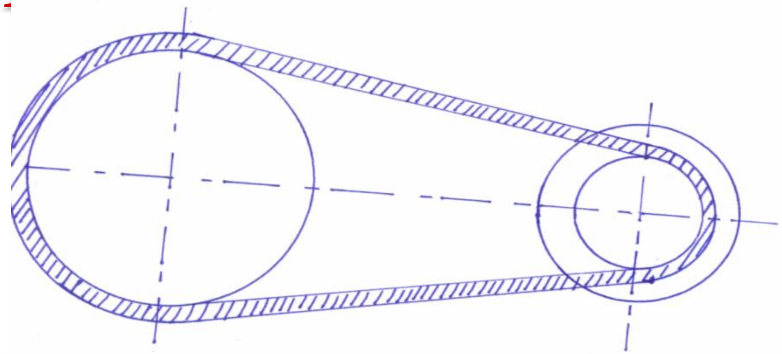


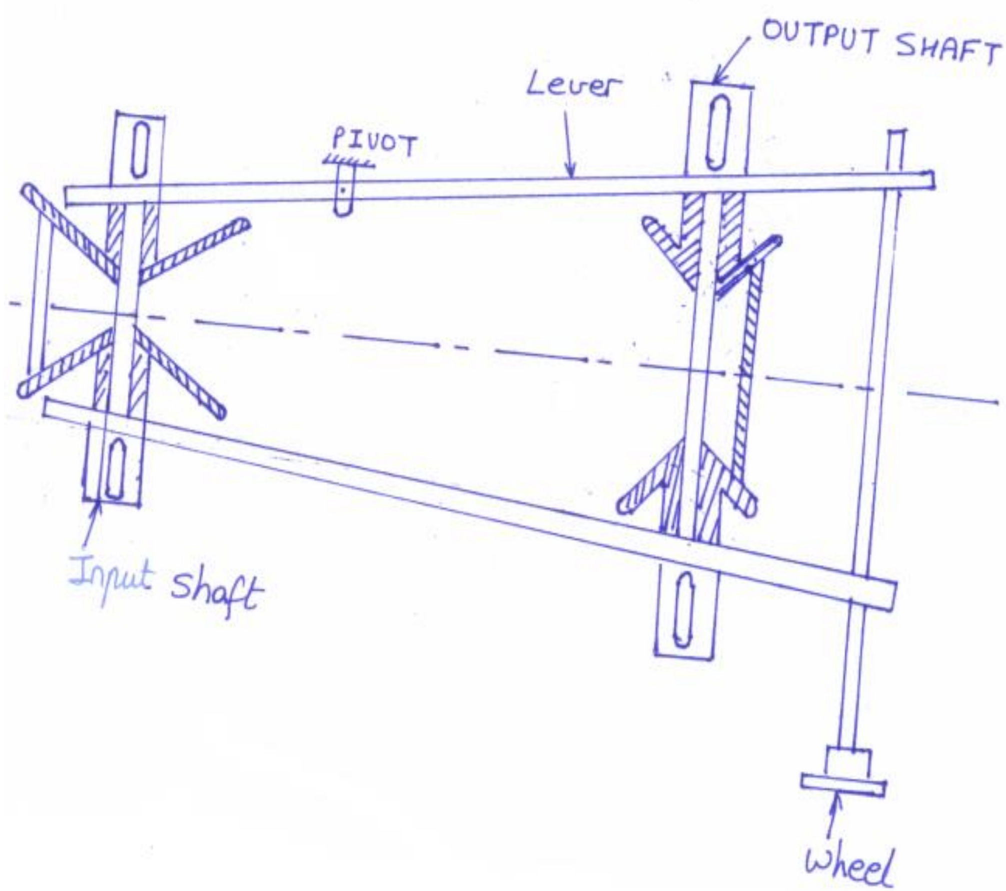
SYNCHROMESH GEARBOX



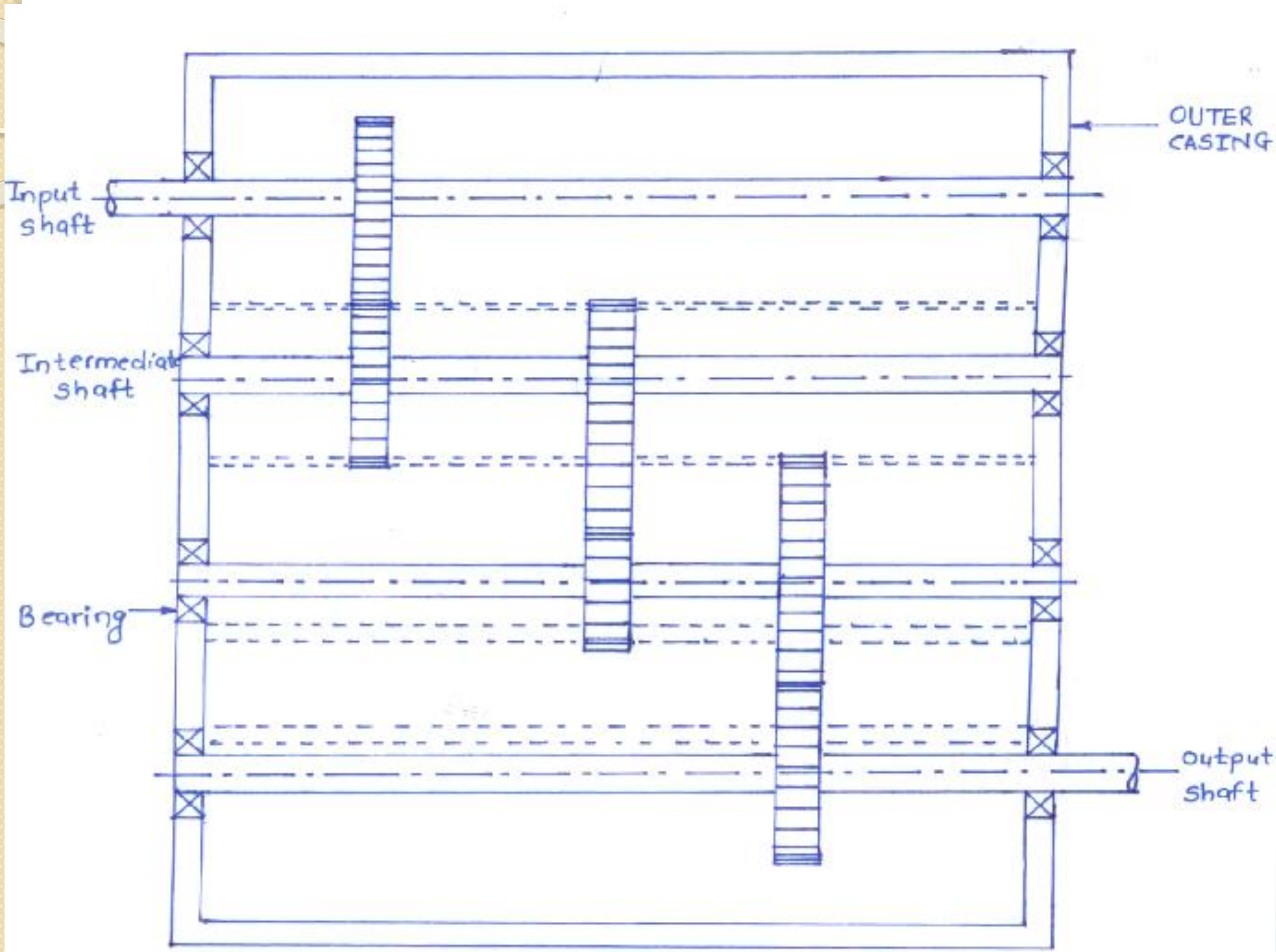
POSITIVE INFINITELY VARIABLE GEARBOX







INDUSTRIAL GEARBOX



DIFFERENTIAL GEARBOX